THE UNIVERSITY OF ZAMBIA
Department of Mathematics and Statistics
MAT2110: Engineering Mathematics I Assignment 1
Due Date: 19 May 2020.

Instructions

- The assignment should be **handwritten**(you may type), scanned and then upload it using Moodle.
- Attempt all questions
- Show all necessary working to earn full marks

1. (a) Find the equation of the circle which passes through through $(2, 3), (3, 2)$ and $(-4, 3)$.

   (b) In each of the following parabolas, find the focus, directrix and hence sketch the graph.

   i. $x^2 = -6y$ ii. $y^2 = 8x$

   (c) Find the foci, vertices and hence sketch of the following ellipses.

   i. $16x^2 + 25y^2 = 400$ ii. $2x^2 + y^2 = 4$

   (d) In each case, find the ellipse's standard form equation from the given information.

   i. Foci $(\pm\sqrt{2}, 0)$, and Vertices $(\pm 2, 0)$. ii. Foci $(0, \pm 4)$, and Vertices $(0, \pm 5)$.

2. (a) Put each of the equations for the hyperbola in standard form and find the hyperbola's foci, vertices, asymptotes and hence sketch the graph.

   i. $9x^2 - 16y^2 = 144$ ii. $8y^2 - 2x^2 = 16$.

   (b) In each of the following hyperbolas centered at the origin, find the hyperbolas standard form equation from the given information.

   i. Foci: $(\pm 2, 0)$, Asymptotes: $y = \pm\frac{1}{\sqrt{3}}x$.

   ii. Vertices: $(\pm 3, 0)$, Asymptotes: $y = \pm\frac{4}{3}x$.

3. (a) Find the parabola's vertex, focus, directrix and hence sketch the parabola, given that

   i. $(y + 2)^2 = 8(x - 1)$ ii. $(x + 1)^2 = -4(y - 3)$

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   (b) Find the foci, vertices, center and hence sketch each of the following ellipses.

   i. $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ ii. $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{25} = 1$

   (c) Find the foci, vertices, center, asymptotes and hence sketch each of the following hyperbolas.

   i. $\frac{(y+2)^2}{4} - \frac{x^2}{5} = 1$ ii. $\frac{(x-2)^2}{16} - \frac{y^2}{9} = 1$

   (d) Find the center, foci, vertices, asymptotes, radius, as appropriate, and sketch the conic section given that:

i. $2x^2 + 2y^2 + 6x - 8y + 12 = 0$

ii. $y^2 - 4y - 8x - 12 = 0$

iii. $4x^2 - 48x + 9y^2 + 72y + 144 = 0$

iv. $x^2 - y^2 + 4x - 6y = 6$

4. (a) In each of the following, find the ellipse's eccentricity, foci and directrices.

   i. $16x^2 + 25y^2 = 400$.

   ii. $169x^2 + 25y^2 = 4225$

   (b) Find the ellipse's standard form equation for each of the following cases.

   i. Foci: $(\pm 8, 0)$, Eccentricity: 0.2

   ii. Vertices: $(0, \pm 70)$, Eccentricity: 0.1

   (c) Find the eccentricity, foci and directrices and sketch the hyperbola if

   i. $9x^2 - 16y^2 = 144$.

   ii. $y^2 - 3x^2 = 3$.

   (d) Find the hyperbol's standard form equation if

   i. Foci: $(\pm 3, 0)$, Eccentricity: 3

   ii. Vertices: $(\pm 2, 0)$, Eccentricity: 2

5. (a) Use the discriminant $B^2 - 4AC$ to decide whether the equation given is a parabola, ellipse, or hyperbola.

   i. $3x^2 - 18xy + 27y^2 - 5x + 7y = -4$

   ii. $3x^2 - 7xy + \sqrt{17}y^2 = 1$

   iii. $xy + y^2 - 3x = 5$

   (b) In each of the following, rotate the coordinate axes to change the given equation into an equation that has no cross product (xy) term. Identify the conic and hence sketch.

   i. $3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0$

   ii. $x^2 - \sqrt{3}xy + 2y^2 = 1$.

6. (a) Identify the particle's path by finding a Cartesian equation for it, and hence sketch the path.

   i. $x = 2\cos t, \quad y = 3\sin t$.

   ii. $y = -3 + 2\cos t, \quad x = \sin t$.

   iii. $x = \sec t, \quad y = \tan t$.

   iv. $x = 1 + t \quad y = 1 - t$.

   (b) Plot each of the following points on the same diagram.

   i. $(2, \frac{\pi}{2})$.

   ii. $(-2, 0)$.

   iii. $(3, \frac{\pi}{4})$.

   iv. $(-3, -\frac{\pi}{4})$.

   (c) Find the Cartesian coordinates of the following points.

   i. $(\sqrt{2}, \frac{\pi}{4})$.

   ii. $(0, \frac{\pi}{2})$.

   iii. $(2\sqrt{3}, \frac{2\pi}{3})$.

7. (a) Change each of the following polar equations to Cartesian and hence describe or identify the graph.

   i. $r = 4\csc\theta$.

   ii. $r^2 = 1$.

   iii. $r^2 = 4r\sin\theta$.

   iv. $r^2 + 2r^2\cos\sin\theta = 1$.

   (b) Find the eccentricity, identify the conic, give an equation of the directrix and hence sketch the conic.

   i. $r = \frac{3}{2+2\cos\theta}$.

   ii. $r = \frac{1}{1+\sin\theta}$.

   iii. $r = \frac{10}{5-6\sin\theta}$.

   iv. $r = \frac{9}{6+2\cos\theta}$.