THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
MAT 2110: Conic Section

Tutorial Sheet 1

* Submit questions 7, 14 and 19. Due date: 06/04/2018

1. Find the coordinates of the focus and the equation of the directrix for each of the following parabolas and sketch the gragh:

   (a) $x^2 = 4y$ (b) $y = \frac{x^2}{32}$ (c) $7y = -5x^2$

2. A parabola has focus at $(3, 4)$ and the x-axis as its directrix. Find the equation of the parabola and sketch it, clearly showing the vertex and y-intercept.

3. Under what conditions is the graph

$$y = Ax^2 + Bx + C$$

   a parabola? If it is indeed a parabola and passes through the points $(1, 3)$, $(2, 4)$ and $(3, 7)$, find the unknown constants.

4. Find the foci and distance sum for of each of the following ellipses and sketch the graph:

   (a) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ (b) $\frac{x^2}{25} + 4y^2 = 1$ (c) $100x^2 + 5100y^2 = 1$

5. An ellipse is centered at the origin and passes through the points $(3, \sqrt{7})$ and $(-\sqrt{3}, 3)$. Find the equation and sketch it.

6. Write down the standard form for the equation of the hyperbola with

   (a) $\pm(|PF_1| - |PF_2|) = 8$ and foci at $(0, \pm 5)$
   (b) $\pm(|PF_1| - |PF_2|) = 1$ and foci at $(0, \pm 5)$

7. Let $k > 0$. Show that the graph of $xy = \frac{k^2}{2}$ is a hyperbola, by showing that this equation is the equation of the hyperbola with foci at (k, k) and $(-k, -k)$ and distance difference $2k$. Sketch the hyperbola when $k = \sqrt{2}$.

8. Find the equation of the hyperbola that goes through the point $(2, 3)$ and has foci at $(\pm 2, 0)$

9. Find the equation of the hyperbola that goes through the points $(\sqrt{6}, 4)$ and $(4, 6)$ if its foci are on the y-axis and the hyperbola has the x-axis as the axis of symmetry.

10. If the vertex of each of the following parabolas is at the origin, what is the equation of the parabola?

(a) focus is at $(8, 0)$ (b) directrix is $y = 11$.

11. Show that the line $5x + 2y = 25$ passes through the focus of the parabola $y^2 = 20x$.

12. Prove that the length of latus rectum of any parabola with vertex at the origin is $4p$. Hence, find the length of latus rectum for each of the following parabolas:

    (i) $2y^2 = 3x$ (ii) $4x^2 = 6y$ (iii) $x^2 = -10y$ (iv) $5x^2 = -2y$ (v) $y^2 = -\frac{3}{10}$

13. The focus of a parabolic mirror is at a distance 10 metres from the vertex. If the mirror is 40 centimetres deep, find the diameter of the outermost circular surface of the mirror.

14. A beam is supported at its ends by two supports which are 16 metres apart. Since the load is concentrated at the center, there is a deflection of 1 metre at the center. How far from the center is the deflection of 0.5 metres?

15. Find the eccentricity of each of the following conics:

    (a) $169x^2 + 25y^2 = 4225$ (b) $y^2 - 4y - 8x - 12 = 0$ (c) $64x^2 - 36y^2 = 2304$ (d) $16x^2 + 25y^2 = 400$

16. Find the equation of the ellipse in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b$, if

    (a) major axis is 10 and $e = \frac{1}{2}$
    (b) minor axis is 8 and $e = \frac{2}{3}$
    (c) minor axis is 10 and foci at $(0, \pm 4)$
    (d) distance sum is 0.2 and foci at $(\frac{1}{\sqrt{102}}, 0)$
    (e) distance between the foci is 32, major axis is along x-axis and $e = \frac{1}{3}$
    (f) distance between the foci is 16 and disatnce between directrices is 30.

17. Find the equation of the hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if

    (i) Transverse axis is 12 and $e = 2$
    (ii) vertices are at $(\pm 2, 0)$ and foci at $(\pm 6, 0)$
    (iii) directrices is $y = 2$ and $e = 5$
    (iv) focus is at $(\sqrt{5}, 0)$ and asymptotes are $2y = \pm x$.

18. Show that the vertical distance between the asymptotes and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ approaches 0.

19. Given that an ellipse centred at the origin is shifted c units to the left so that one focus is at the origin, show that for any given point $P(x, y)$ on the ellipse, the "focus-directrix" equation

$$|PF| = e.|PD|$$

holds.