MAT2110 Tutorial Assignment 10
30 August 2016

1. By using the transformation equations from one coordinate system to another, prove that the discriminant for the general conic section

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

is an invariant:

$$B'^2 - 4A'C' = B^2 - 4AC$$

2. Prove that under a rotation of axes, the straight line

$$ax + by + c = 0$$

remains a straight line.

3. In the following problems, put the equation of the conic into standard form by a suitable rotation . Then characterise the conic section by determining the centre, the focus or foci, the eccentricity and the directrix or directrices. Find also the lines of symmetry. The various quantities must be expressed in terms of the original coordinates. Note that translations of the coordinates may be needed.

(i) $5x^2 - 3xy + 9y^2 = 32$ **(ii)** $4x^2 - 4xy + y^2 + 5x + 10y = 0$

(iii) $x^2 - 2xy + y^2 = 6\sqrt{2}\,(x + y)$ **(iv)** $11x^2 - 24xy + 4y^2 + 40 = 0$