MAT2110 Test 2
3 May 2015
Time: Two Hours
Answer: Any Three Questions

1. (**a**) Determine the eccentricity and the major and minor axes of the ellipse $r = 5/(4 - 2\cos\theta)$.

(**b**) Use the differential arc length ds to prove that the length of a quarter of the circumference of the circle $x^2 + y^2 = 16$ is π.

(**c**) (**i**) Determine the equation of the line that passes through the points $P_1(1, 1, 1)$ and $P_2(2, 0, 3)$.

(**ii**) Determine the equation of the plane perpendicular to this line and passing through the point $P_2(2, 0, 3)$.

2. (**a**) One of the parametric equations of the circle $x^2 + y^2 = 4$ is $x = \sqrt{1 - t^2}$.

(**i**) Determine what the other parametric equation must be.

(**ii**) Determine the curvature of the circle at the points $t = 0.5$ and $t = 0.25$ and comment.

(**iii**) Determine the equation of the line tangent to the circle at the point $t = \sqrt{5/9}$.

(**b**) Find the sum of the series $S = \sum_{k=1}^{\infty} \frac{1}{5^k}$.

(**c**) Obtain the Maclaurin series of $\sin x$.

3 (**a**) Determine the power series expansion of $x/(1 - x^2)$.

(**b**) (**i**) Find the line of intersection of the planes $3x - 4y + z = 10$ and $2x + y + z = 12$.

(**ii**) Determine the angle between the planes.

(**c**) The position vector of a certain particle is given by $\mathbf{R} = (a + mt)\,\widehat{\mathbf{i}} + (b + nt)\,\widehat{\mathbf{j}}$.

(**i**) Show that the particle moves in a straight line

(**ii**) Change the parameter to the arc length s measured from the point $P_0(a, b)$ and verify that your calculation is correct.

4. (**a**) The work done on a particle by the force $\mathbf{F}$ between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is given by $W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$ where $d\mathbf{r} = (dx, dy, dz)$. The force acting on one such particle is $\mathbf{F} = (2x, 3e^{-y}, -2z^2)$ in Newtons.

(**i**) Determine the work done on the particle between the origin $\mathbf{r}_1$ and the point $\mathbf{r}_2 = (1, 1, 1)$

(**ii**) If the mass of the particle is 3 kg, find the acceleration of the particle at the point $(0.5, 1.5, 1)$.

(**b**) (**i**) Show that the polar equation of the line $y = x$ is $\theta = \pi/4$.

(**ii**) Express the intersection of this line with the hyperbola $r = 10/(3 - 4\cos\theta)$ in Cartesian coordinates.

(**c**) Determine the area of the triangle with the vertices $P_1(0, 0, 0)$, $P_2(1, 2, 3)$ and $P_3(1, 1, 1,)$.