**MAT2100 Tutorial Sheet 3**

**1**. Find the Cartesian coordinates of the points whose polar coordinates are **(i)** $(4, 0)$, **(ii)** $(-5, -\pi)$ **(iii)** $(-8, 5\pi/5)$

**2**. Give the polar coordinates of the points whose rectangular coordinates are **(i)** $(-3. -3)$, **(ii)** $(3, 4)$, **(iii)** $(2, -1)$

**3**. Plot the curve $r = 3\sin\theta$ and confirm the shape by expressing the curve in Cartesian coordinates.

**4**. Determine the polar equation of a circle of radius 4 centred on the point $(2, 3)$

**5**. Identify the given conic and characterise it. If it has two directices, give the left-hand one.

$$\textbf{(i)}\ r = \frac{7}{1-\cos\theta}, \quad \textbf{(ii)}\quad r = \frac{10}{4-3\cos\theta} \quad \textbf{(iii)}\ r = \frac{10}{3-4\cos\theta}$$

MAT2110 Tutorial Sheet 3 Solns 19/3/2015

(i) $(r,\theta) = (4,0)$.

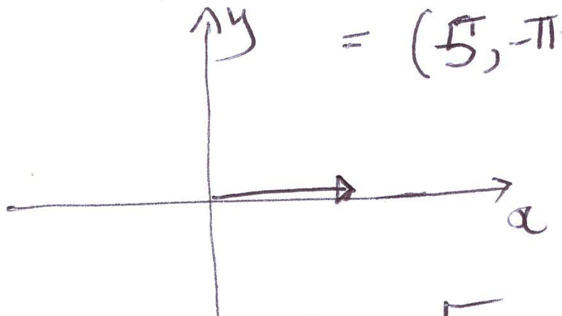
$x = r\cos\theta = 4$

$y = r\sin\theta = 0$

$(x,y) = (4,0)$

(ii) $(r,\theta) = (-5,-\pi)$

$= (5, -\pi+\pi) = (5,0)$




$x = 5\cos 0 = 5$

$y = 5\sin 0 = 0$

$(x,y) = (5,0)$

(iii) $(r,\theta) = (-8, 5\pi/4)$

$= \left(8, \frac{5\pi}{4}+\pi\right)$

$= \left(8, \frac{9\pi}{4}\right)$

$= \left(8, \frac{8\pi}{4}+\frac{\pi}{4}\right)$

$= \left(8, 2\pi+\frac{\pi}{4}\right)$

$= \left(8, \frac{\pi}{4}\right)$

$x = 8\cos \pi/4 = \frac{8}{\sqrt{2}}$

$y = 8\sin\frac{\pi}{4} = \frac{8}{\sqrt{2}}$

$(x,y) = \left(\frac{8}{\sqrt{2}}, \frac{8}{\sqrt{2}}\right)$.

2. $(x,y) = (-3,-3)$

$r = (x^2+y^2)^{1/2} = (9+9)^{1/2}$

$= \sqrt{18} = \sqrt{2\times 9}$

$= 3\sqrt{2}$

$\tan\theta' = \frac{y}{x} = \frac{-3}{-3} = 1$

$\theta' = 45^0$

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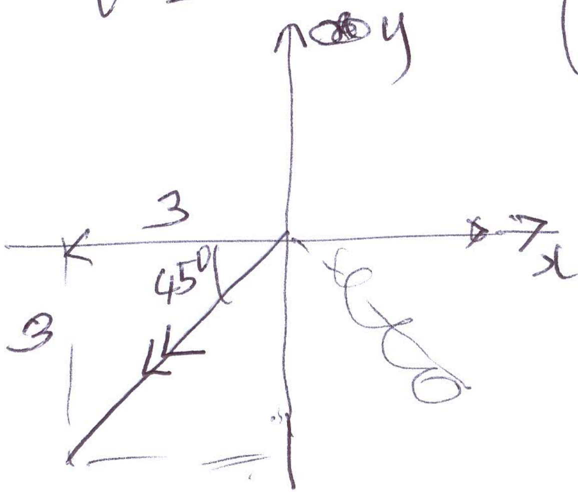



Hence $(r,\theta) = \left(3\sqrt{2}, \pi+\frac{\pi}{4}\right)$

$= \left(3\sqrt{2}, \frac{5\pi}{4}\right)$

2 (ii)

$(x,y) = (3,4)$

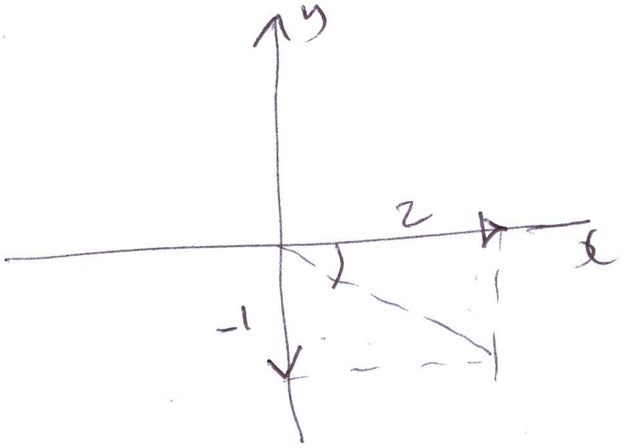
$r = \sqrt{9+16} = 5$

$\tan\theta = \frac{4}{3}$; $\theta = 53.1^\circ$

(iii) $(x,y) = (2,-1)$

$r = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$\tan\theta = -\frac{1}{2}$; $\theta = -26.6^\circ$




$\theta = -26.6^\circ = 333.4^\circ$

3. We have

$r = 3\sin\theta$

$r^2 = 3r\sin\theta$

$r^2 = 3y$

$x^2+y^2 = 3y$

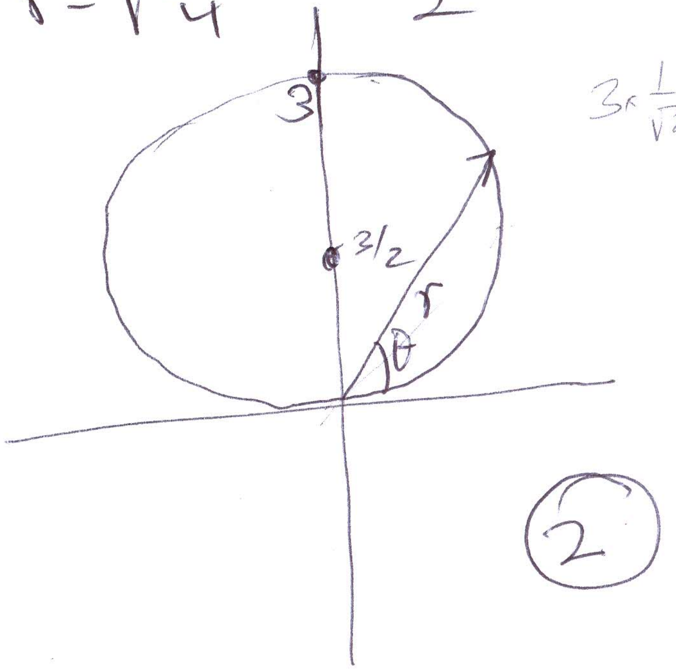
$x^2+(y^2-3y) = 0$

$x^2+y^2-3y+\frac{9}{4}-\frac{9}{4} = 0$

$x^2+\left(y-\frac{3}{2}\right)^2 = \frac{9}{4}$

Hence we have a circle of centre $(0,\frac{3}{2})$ and radius

$r = \sqrt{\frac{9}{4}} = \frac{3}{2}$




(2)

4: This has eqn

$(x-2)^2+(y-3)^2 = 16$

$x^2-4x+4+y^2-6y+9 = 16$

$x^2-4x+y^2-6y = 3$

$x^2+y^2-4x-6y = 3$

$r^2-4r\cos\theta-6r\sin\theta = 3$

This is the eqn in polar coordinates.

5; $r = \frac{7}{1-\cos\theta}$

$r - r\cos\theta = 7$

$r - x = 7$

$r = x + 7$

$r^2 = x^2 + 14x + 49$

$\cancel{x^2} + y^2 = \cancel{x^2} + 14x + 49$

$y^2 = 14x + 49$

$\cancel{y^2 = 14(x+}$

$y^2 = 7(2x+7)$

$= 14(x + \frac{7}{2})$

This is a parabola centred on $(-\frac{7}{2}, 0)$.

The eqn of a conic in polar coordinates has the form

$$r = \frac{ep}{1 - e\cos\theta}$$

Here the eccentricity is $e = 1$ and since

$ep = 7$, $p = 7$

~~In the displaced system,~~

In the translated coordinate system,

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$y'^2 = 14x'$

In that system, the directrix is the line $x' = p$; $x' = 7$

Here $x' = x + \frac{7}{2}$; $x' = 7 \Rightarrow$

$7 = x + \frac{7}{2}$, $x = \frac{7}{2}$

Hence the directrix is $x = \frac{7}{2}$.

(iii) For $r = \frac{10}{4 - 3\cos\theta}$

we have

$$r = \frac{10}{4(1 - \frac{3}{4}\cos\theta)}$$

$$r = \frac{2.5}{1 - 0.75\cos\theta}$$

$\therefore e = 0.75$, $ep = 2.5$

$p = \frac{2.5}{e} = \frac{2.5}{0.75} =$

$$= \frac{250}{75} = \frac{50}{15} = \frac{10}{3}$$

$$= 3.333$$

Since $e < 1$, we have an ellipse. One focus is at the origin by definition. The left directrix is $p$ units or $\frac{10}{3}$ units to the left.

Hence it is the line

$$x = -\frac{10}{3}.$$

$$ep = 10/3$$

$$p = \frac{10/3}{e} = \frac{10/3}{4/3} = 2.5$$

One focus is at the origin and its left directrix is the line $x = -2.5$.

(iii) $$r = \frac{10}{3 - 4\cos\theta} = \frac{10}{3\left(1 - \frac{4}{3}\cos\theta\right)} = \frac{10/3}{1 - \frac{4}{3}\cos\theta}$$

Since $$r = \frac{ep}{1 - e\cos\theta}$$

$e = \frac{4}{3}$, which gives a hyperbola.