

THE UNIVERSITY OF ZAMBIA  
SCHOOL OF ENGINEERING  
DEPARTMENT OF MECHANICAL ENGINEERING

MEC 2309 - PROPERTIES OF ENGINEERING MATERIALS I

END OF TERM TEST

TERM I, 2013

ANSWER BOTH QUESTIONS AND THEY CARRY EQUAL MARKS AS INDICATED

## SOLUTIONS

**Question 1:**

- (a) Describe, with aid of sketches, the interstitial and substitutional solid solutions. Discuss two factors affecting the formation of each type of solid solution. [20 marks]

**Solution:**

### **Types of Solid Solutions**

In a solid solution, the solute atoms are distributed throughout the crystal grains, the crystal structure being the same as that of the pure metal, which is the solvent.

A solid solution may be formed in two ways;

(i) A *substitutional* solid solution, in which solute atoms replace some solvent atoms so that they lie at normal atom sites of the crystal structure (Figure 1a(i)).

(ii) An *interstitial* solid solution, in which the solute atoms are located in the interstices (or inter-atomic spaces) of the solvent lattice (Figure 1a(ii)).

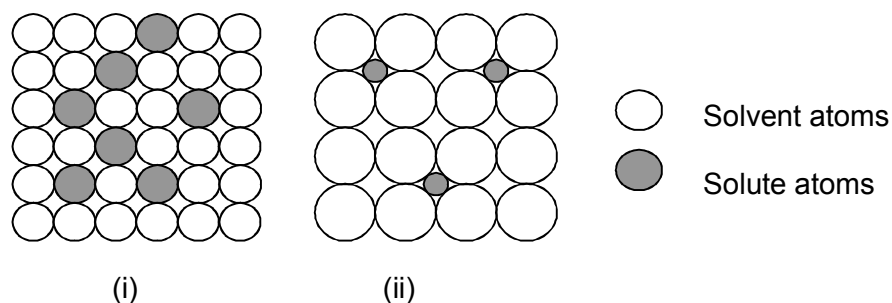


Figure 1a: (a) Substitutional Solid Solution; (b) Interstitial Solid Solution

### **Types of Substitutional Solid Solutions**

In the substitutional solid solution, the solute atoms may replace solvent atoms at random, or may take up a more *ordered* structure or even *clustered* configurations (Figure 1b). The clustered arrangement is in effect a form of the ordered configuration. Such an ordered or clustered structure can exist at a particular composition only. For near-by compositions the structure would be ordered as possible. Where ordered structures (also known as *super lattices*) exist, they usually do so at room temperatures, becoming *disordered* at higher temperatures.

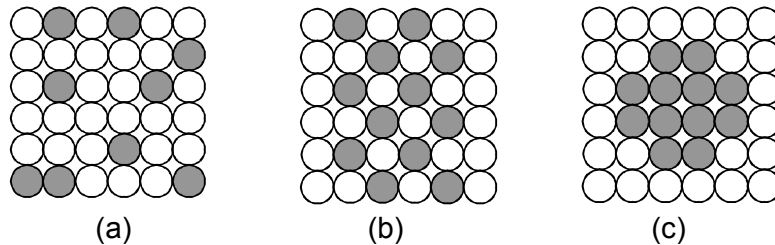


Figure 1b: Types of Substitutional Solid solutions: (a) Random or disordered; (b) Ordered; and (c) Clustered

### Effect of Atomic Size and Crystal Structure

The type of solid solution formed and the limits of stability are governed by the relative sizes of the relative sizes of the solute and solvent atoms.

The distances between metals and between ions in ionic crystals approximately obey an additive law, each atom or ion being packed in a structure as if it were a sphere of definite size. The radius of the sphere is not constant for any given ion or atom, but depends on the number of equally-spaced near neighbours- the *coordination number*. In a close-packed lattice, the number is 12, in a body-centred cubic lattice it is 8, in a diamond it is 4, etc. in passing from 12-fold to 8-fold coordination, there is a contraction of 3% in the radius and a greater contraction when passing to lower coordination numbers. For purposes of comparison, atomic radii are best expressed as the value appropriate to one particular coordination number.

Interstitial solid solutions can form only when the solute atom is small enough to fit into a space between solvent atoms without causing excessive distortion. In a close-packed lattice, there are two kinds of space, one with four atoms and the other with six atoms arranged symmetrically around the void. The centres of these surrounding atoms lie at the corners of rectangular tetrahedra and octahedra, respectively, so that the voids are known as *tetrahedral* and *octahedral* spaces, Figure 1c.

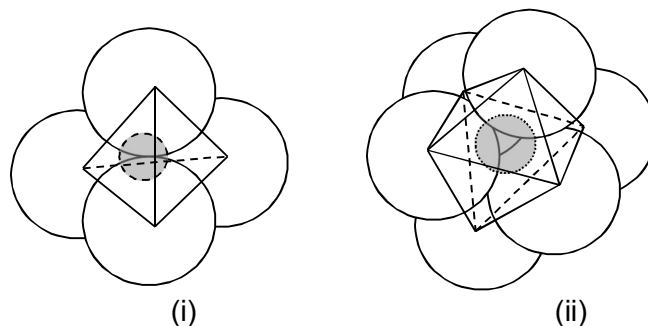


Figure 1c: (i) Tetrahedral and (ii) Octahedral spaces in close-packed lattice, showing interstitial atoms

There can be appreciable solid solubility only if the solute atom of the diameter of the solute atoms is about 0.6 or less of the diameter of the solvent atom. Apart from the inert gases, the alkali and alkaline-earth and some of the rare-earth metals, nearly all atoms have diameters which lie in the range 0.21nm to 0.36nm, and the metals which form the basis of commercially important alloys lie in the range 0.25nm to 0.32nm. The only atoms small enough to form interstitial solid solutions are hydrogen, oxygen, nitrogen, carbon and boron.

(b) Draw the [101] and [102] directions for a cubic structure and determine the angle between? [10 marks]

**Solution:**

Note that from the indices given, there are no intercepts on the y-axis. So, these directions are in the x-z plane.

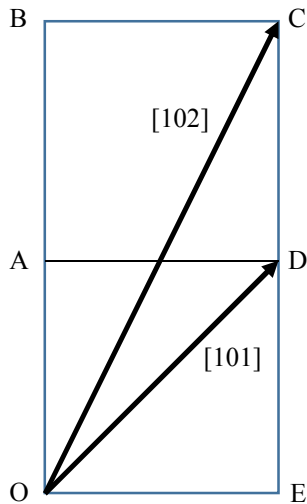


Figure 1d: Directions [101] and [102]

Note: from Figure 1d that

OE is the x-axis and OB is the z-axis.

The two directions are shown.

Since this is a cubic system,  
OA = OE = a.

Therefore by inspection, the angle  
 $E\hat{O}A = 45^\circ$ .

$$E\hat{O}C = \tan^{-1} \frac{2a}{a} = \tan^{-1} 2 = 63.43^\circ$$

So, the angle between the directions is angle  $D\hat{O}C$ .

$$D\hat{O}C = E\hat{O}C - E\hat{O}A = 63.43 - 45$$

$$\boxed{D\hat{O}C = 18.43^\circ}$$

(c) A metal is found to have BCC structure, a lattice parameter,  $a$ , of  $3.31 \times 10^{-10} \text{m}$ , and a density of  $16.6 \text{ Mg/m}^3$ . Determine the atomic mass (in kg) of this element. [10 marks]

**Solution:**

- In bcc, number of atoms per unit cell,  $n = 2$ .
- Density =  $16.6 \text{ Mg/m}^3 = 16,600 \text{ kg/m}^3$ , given.
- $a = 3.31 \times 10^{-10} \text{m}$ , given

$$\text{Density} = \frac{\text{number of atoms per unit cell} \times \text{atomic mass}}{\text{Unit cell volume}}$$

Therefore

$$\text{Atomic mass} = \frac{\text{density} \times \text{unit cell volume}}{\text{number of atoms per unit cell}} = \frac{\rho V}{n} = \frac{16,600 \times (3.31 \times 10^{-10})^3}{2} = 3.01 \times 10^{-25}$$

$$\boxed{\text{Atomic mass} = 3.01 \times 10^{-25} \text{ kg}}$$

- (d) Vanadium (V) has a bcc structure and a lattice parameter,  $a$ , of  $3.08 \times 10^{-10}$  m. Determine the highest linear density of atoms (atoms/m) encountered in vanadium. [10 marks]

**Solution:**

The most close packed direction is the [111] direction.

For  $a = 3.08 \times 10^{-10}$  m, the [111] diagonal is  $a\sqrt{3}$ .

And there are: half a diameter of one corner atom + a full diameter of the body atom + half a diameter of the diagonally opposite corner atom. This gives two diameters, i.e. two atoms on the main diagonal of the unit cell.

Therefore:

Highest linear density (LD) is the number of atoms per unit length of the major diagonal if the atoms were considered pure spheres along that diameter.

$$LD = \frac{\text{Number of atoms on main diagonal}}{\text{length of main diagonal}} = \frac{2}{3.08 \times 10^{-10} \times \sqrt{3}}$$

$$\boxed{LD = 3.75 \times 10^9 \text{ atoms/m}}$$

**Question 2:**

(a) Two hypothetical metals *C* and *D* are dissolved in each other and the following information is available from the experiment:

- The melting temperatures of *C* and *D* are 700°C and 900°C, respectively.
- The metals form a eutectic at a composition of 55% *D* at 400°C.
- The maximum solubilities possible are found to be 20% of *D* in *C* and 25% of *C* in *D*.
- At room temperature, maximum solubility of *D* in *C* is 5% and *C* in *D* is 10%.

Draw the phase diagram to scale and label the main features. All lines may be assumed to be straight. [20 marks]

**Solution:**

The labelled diagram is shown in Figure 2a. Note that the dotted lines 60%*C*/40%*D* (vertical) and *ad* at 500°C (horizontal) are not part of the answer in this part.

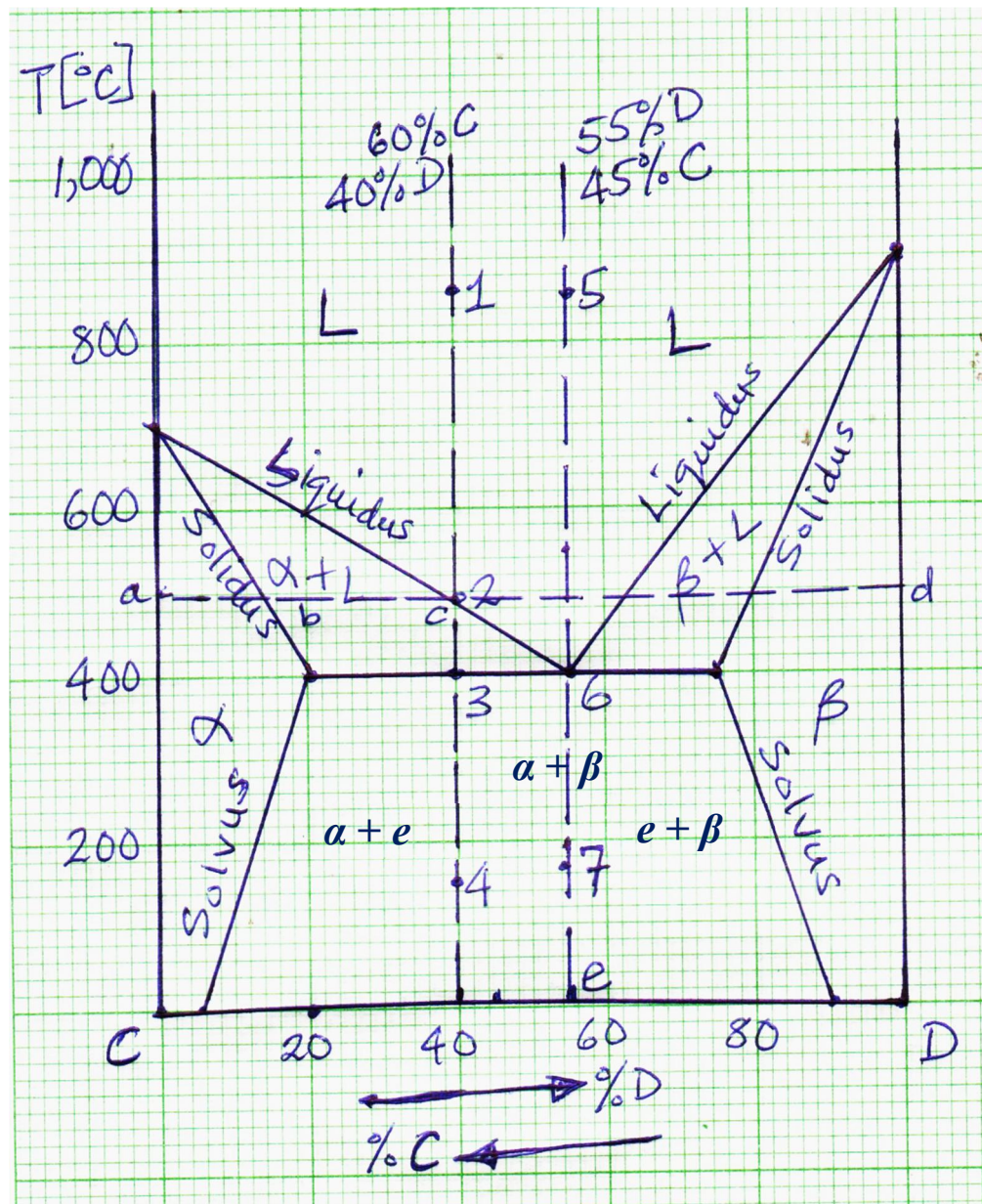


Figure 2a: Labelled phase diagram of metals *C* and *D*.

(b) Describe the changes which would occur when the following alloy compositions are allowed to cool slowly from the liquid state to room temperature. Mention the temperatures where the changes occur.

(i) 45% C / 55% D.

(ii) 60% C / 40% D.

[10 marks]

**Solution:**

**(i) Alloy 60%C/40%D**

When the alloy is cooled slowly (slowly so that equilibrium is achieved at all points) from say Point 1 to Point 4:

- At point 1, the alloy is all liquid composed of 60%C and 40%D.
- As the temperature falls, at point 2 (500°C), some solid  $\alpha$  phase begins to precipitate from the liquid. This solid phase is rich in C.
- As the temperature falls further, the solid  $\alpha$  phase increases and the liquid phase decreases until point 3.
- At point 3 (400°C), the liquid all turns into solid  $\alpha$  phase in the neighbourhood of point 3, but just above point 3.
- In the neighbourhood of point 3, but just below point 3, the solid phase changes to  $\alpha + \beta$  or  $\alpha + e$  (e is the eutectic composition).
- This composition and phase remains unchanged to room temperature, point 4.

**(ii) Alloy 60%C/40%D**

When the alloy is cooled slowly (slowly so that equilibrium is achieved at all points) from say Point 1 to Point 4:

- At 1, the alloy is all liquid composed of 55%D and 45%C.
- As the temperature falls, at point 6 (the eutectic point), the liquid turns straight into the solid phase  $\alpha + \beta$  or phase e (the eutectic composition).
- This composition and phase remains unchanged to room temperature, point 7.

(c) For 60% C / 40% D, give the phases present, their relative proportions and their compositions at 500°C [20 marks]

**Solution:**

(i) At point 2: (Strictly speaking, we first mean in the neighbourhood of Point 2, but above Point 2):

The alloy is all liquid in the neighbourhood of but above Point 2:

Composition = 60% C and 40% D.

(ii) At point 2: (Strictly speaking, we then mean, in the neighbourhood of Point 2, but below Point 2):

In the neighbourhood of but below Point 2, the solid  $\alpha$  phase is just beginning to precipitate, but its quantity is still negligible.

Therefore phases present are  $\alpha$  phase and liquid phase.

- **Relative proportions by Lever Rule:**

(a)  $\alpha$  phase =  $\frac{cc}{bc} \times 100\% = \frac{0}{40-14} \times 100\% = \frac{0}{26} \times 100\% = 0\%$

**$\alpha$  phase = 0%**

(b) L phase =  $\frac{bc}{bc} \times 100\% = \frac{40-14}{40-14} \times 100\% = \frac{26}{26} \times 100\% = 100\%$

**L phase = 100%**

- **Compositions:**

(a) In the  $\alpha$ -phase:

$$\% C = \frac{d-b}{d-a} \times 100 = \frac{100-14}{100-0} \times 100 = \frac{86}{100} \times 100 = 86\%$$

$$\% D = \frac{b-a}{d-a} \times 100 = \frac{14-0}{100-0} \times 100 = \frac{14}{100} \times 100 = 14\%$$

**% C = 86%**

(b) In the L-phase:

$$\% C = \frac{d-c}{d-a} \times 100 = \frac{100-40}{100-0} \times 100 = \frac{60}{100} \times 100 = 60\%$$

$$\% D = \frac{c-a}{d-a} \times 100 = \frac{40-0}{100-0} \times 100 = \frac{40}{100} \times 100 = 40\%$$

**% D = 40%**

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**END OF MEC 2309 TERM I TEST**

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December, 2013*