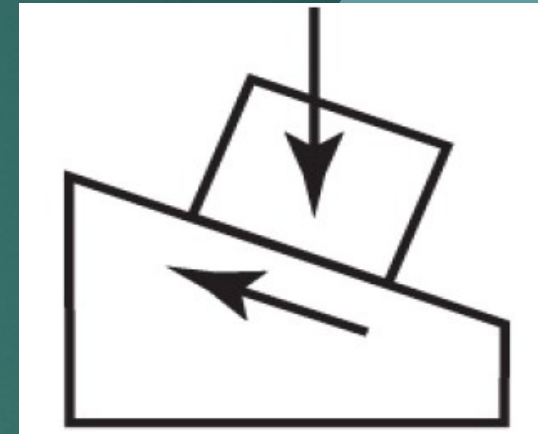


# CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

## Lecture A8

1

JQL  
14/05/2021



❖ FRICTION

✓ Introduction to Friction

✓ WEDGES

## TOPIC (CHAPTER) OBJECTIVES

2

14/05/2021

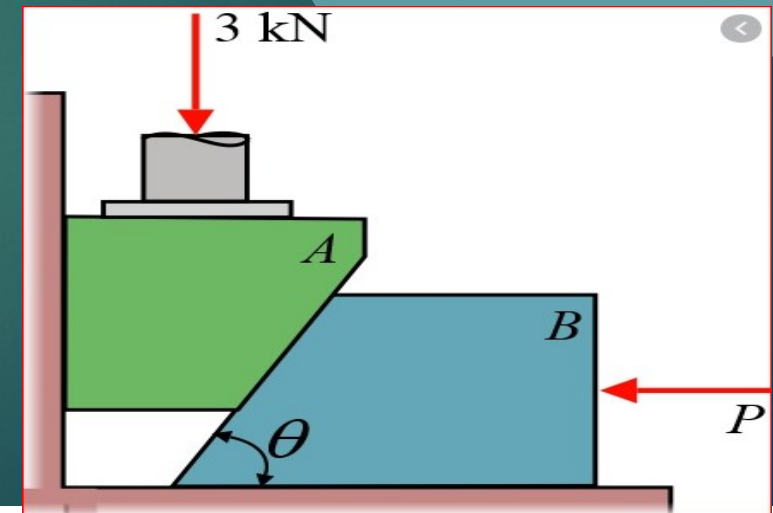
JOL

Upon completion of this topic (Friction), readers will be able to:



- ❖ Define friction and the forces associated with friction
- ❖ Calculate the coefficient of static friction between objects
- ❖ Calculate the angle of static friction

❖ Solve motion-impending friction problems using the equilibrium equations from the previous lectures



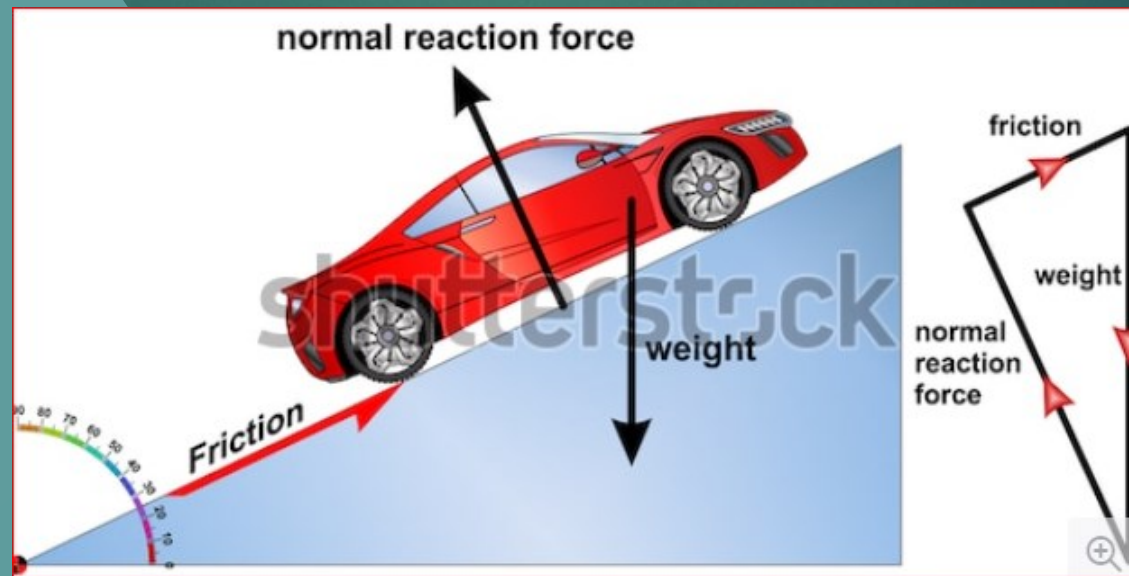
# TOPIC (CHAPTER) OBJECTIVES

3

14/05/2021

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- ❖ *Discuss the use of wedges and be able to calculate forces in a wedge system*
- ❖ *Calculate belt tension in belts wrapped around cylindrical surfaces.*
- ❖ *Calculate the forces in a square-threaded screw.*
- ❖ *To investigate the concept of rolling resistance.*



# OBJECTIVES OF THIS LECTURE

4

14/05/2021

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- ❖ *Define friction and the forces associated with friction*
- ❖ *To introduce the concept of dry friction and show how to analyze the equilibrium of rigid bodies subjected to this force*



- ❖ *Calculate the angle of static friction*
- ❖ *To present specific applications of frictional force analysis on wedges*

## CHAPTER INTRODUCTION

- ▶ In the preceding sections, it was assumed that surfaces in contact were either frictionless or rough.
- ▶ If they were frictionless, the force each surface exerted on the other was normal to the surfaces and the two surfaces could move freely with respect to each other.
- ▶ If they were rough, it was assumed that tangential forces could develop to prevent the motion of one surface with respect to the other.
- ▶ This view was a simplified one.

## CHAPTER INTRODUCTION

- ▶ Actually, no perfectly friction-less surface exists.
- ▶ When two surfaces are in contact, tangential forces, called friction forces, will always develop if one attempts to move one surface with respect to the other.
- ▶ On the other hand, these friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.
- ▶ *The distinction between frictionless and rough surfaces is thus a matter of degree.*
- ▶ This will be seen more clearly in the following sections, which are devoted to the study of friction and of its applications to common engineering situations.

## CHAPTER INTRODUCTION

- ▶ In some types of machines and processes we want to minimize the retarding effect of friction forces.
- ▶ Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere.
- ▶ In other situations we wish to maximize the effects of friction, as in brakes, clutches, belt drives, and wedges.
- ▶ Wheeled vehicles depend on friction for both *starting and stopping*, and ordinary walking depends on friction between the *shoe and the ground*.

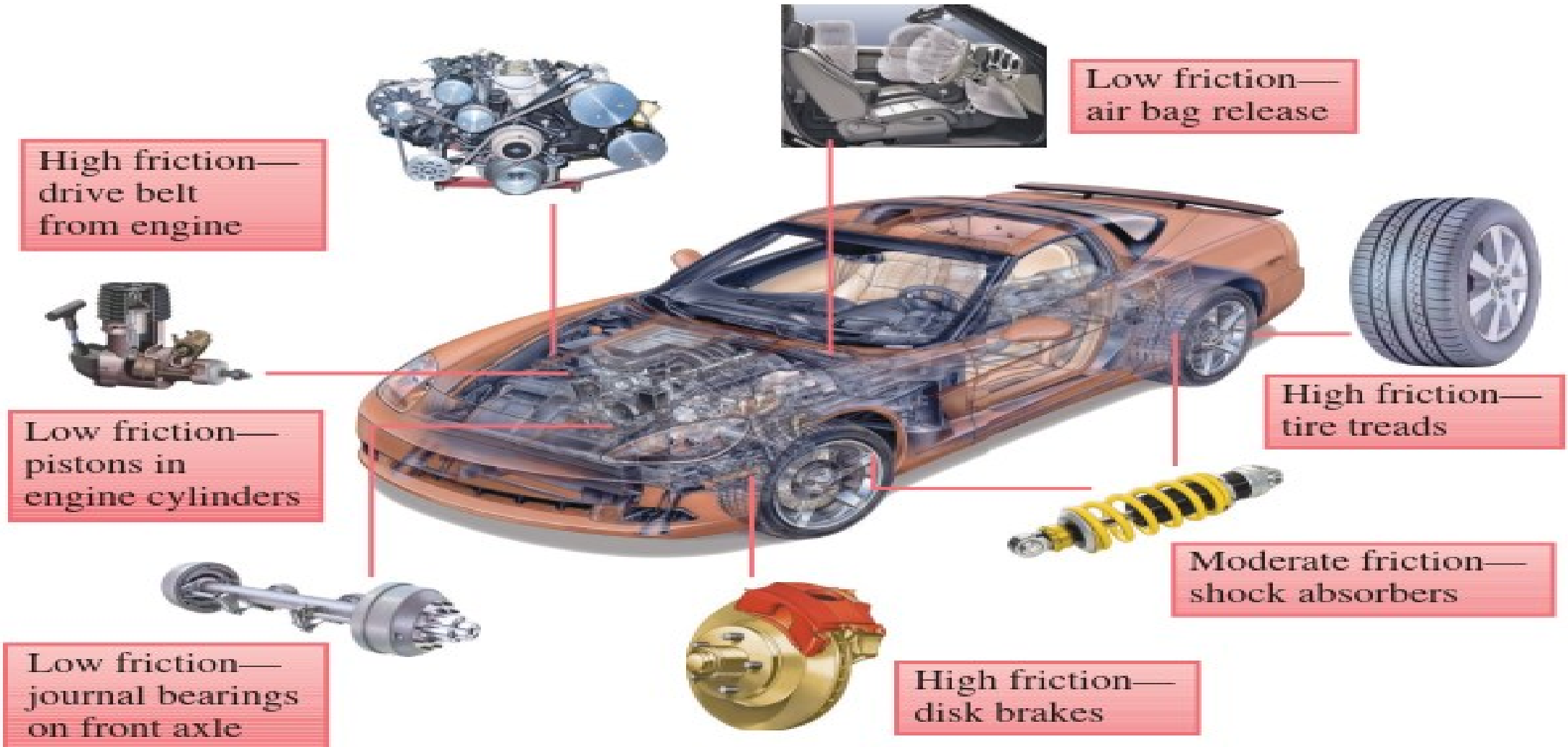
## CHAPTER INTRODUCTION

- ▶ Friction forces are present throughout nature and exist in all machines no matter how accurately constructed or carefully lubricated.
- ▶ A machine or process in which friction is small enough to be neglected is said to be ideal.
- ▶ When friction must be taken into account, the machine or process is termed real.
- ▶ In all cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat.
- ▶ Tear & Wear is another effect of friction.

# TYPES OF FRICTION

- ▶ In this study we briefly discuss the types of frictional resistance encountered in mechanics
  - ❖ dry
  - ❖ fluid &
  - ❖ internal friction.
- ▶ But basically there are **two** main types of friction:
- ▶ Dry friction, sometimes called Coulomb friction, and
- ▶ Fluid friction

# TYPES OF FRICTION



**Photo 4.5** Examples of friction in an automobile. Depending upon the application, the degree of friction is controlled by design engineers.

# TYPES OF FRICTION

## Dry Friction

- ▶ Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide.
- ▶ A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place.
- ▶ The direction of this friction force always opposes the motion or impending motion.
- ▶ The principles of dry or Coulomb friction were developed largely from the experiments of Coulomb in 1781 and from the work of Morin from 1831 to 1834.

# TYPES OF FRICTION

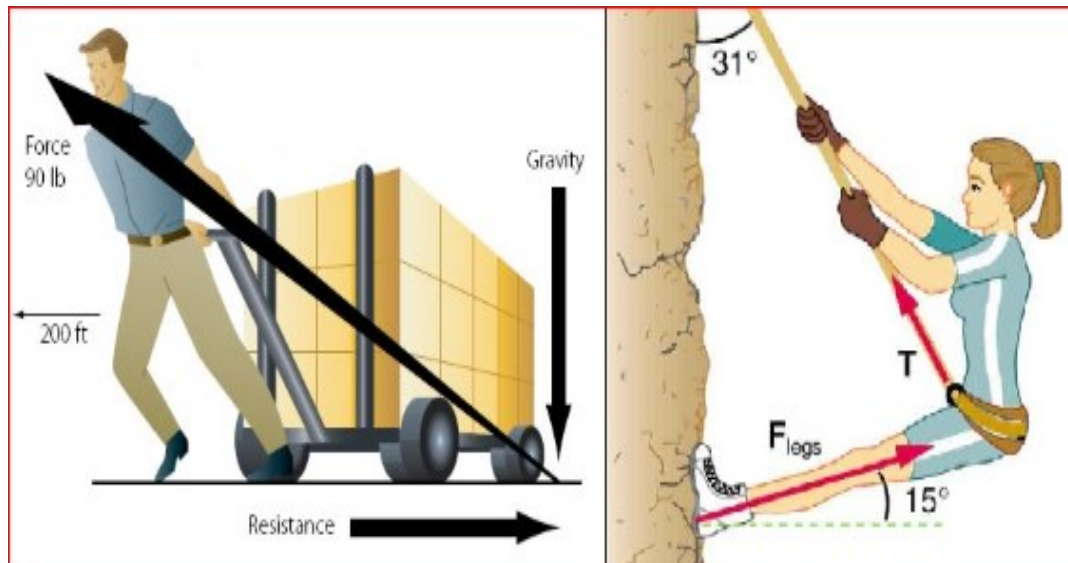
## Fluid Friction

- ▶ Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. OR just Fluid friction develops between layers of fluid moving at different velocities.
- ▶ This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers.
- ▶ When there is no relative velocity, there is no fluid friction.
- ▶ Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers.

# TYPES OF FRICTION

## Fluid Friction

- ▶ Fluid friction is of great importance in problems involving the flow of fluids through pipes and orifices or dealing with bodies immersed in moving fluids. It is also basic in the analysis of the motion of lubricated mechanisms.
- ▶ Such problems are considered in texts on fluid mechanics.



- ▶ *This lecture and the CEE 2219 study is limited to dry friction, i.e., to problems involving rigid bodies which are in contact along nonlubricated surfaces.*

# DRY FRICTION

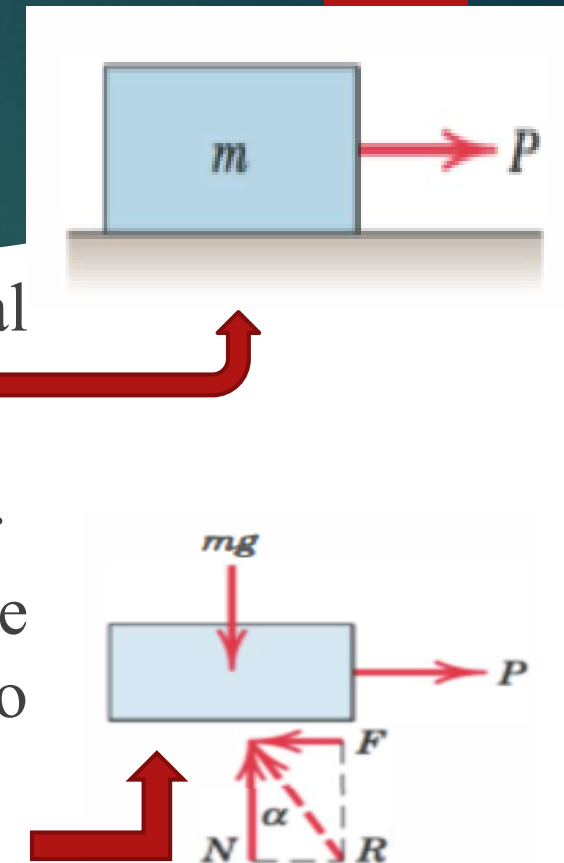
## Mechanism of Dry Friction

► Consider a solid block of mass  $m$  resting on a horizontal surface, as shown in Fig.

► We assume that the contacting surfaces have some roughness.

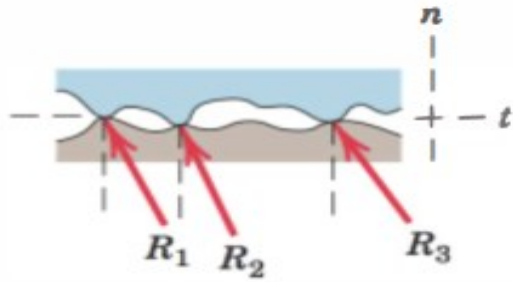
► The experiment involves the application of a horizontal force  $P$  which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity.

► The FBD of the block for any value of  $P$  is shown in Fig., where the tangential friction force exerted by the plane on the block is labeled  $F$ .



## DRY FRICTION

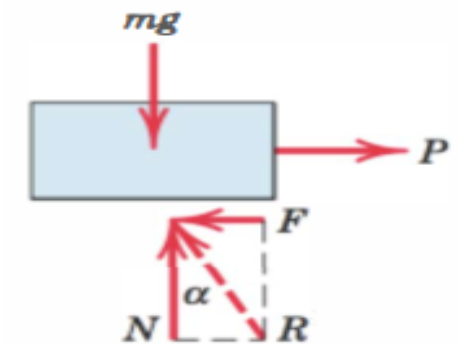
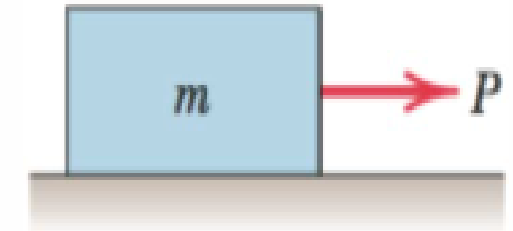
### Mechanism of Dry Friction



► This friction force acting on the body will always be in a direction to oppose motion or the tendency toward motion of the body.

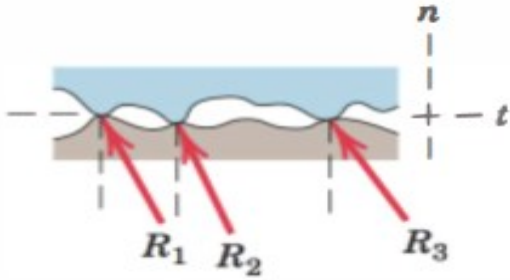
► There is also a normal force  $N$  which in this case equals  $mg$ , and the total force  $R$  exerted by the supporting surface on the block is the resultant of  $N$  and  $F$ .

► A magnified view of the irregularities of the mating surfaces, Fig. above, helps us to visualize the mechanical action of friction.

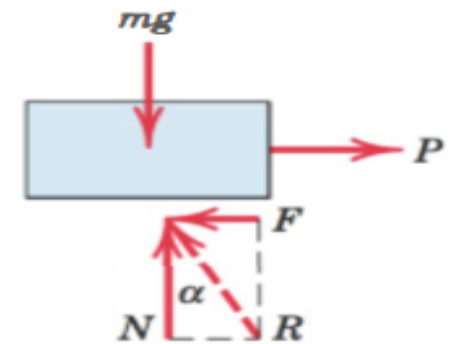
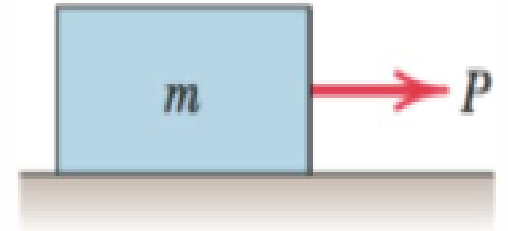


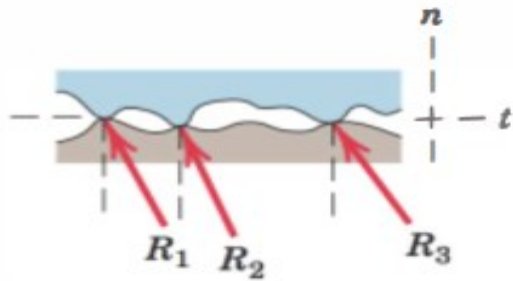
# DRY FRICTION

## Mechanism of Dry Friction



- Support is necessarily intermittent and exists at the mating humps.
- The direction of each of the reactions on the block,  $R_1$ ,  $R_2$ ,  $R_3$ , etc., depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point.
- The total normal force  $N$  is the sum of the  $n$ -components of the  $R$ 's, and the total frictional force  $F$  is the sum of the  $t$ -components of the  $R$ 's.

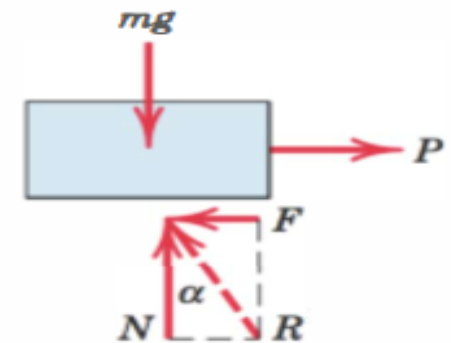
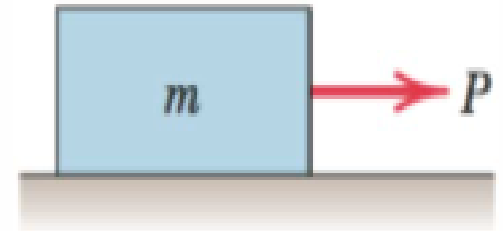


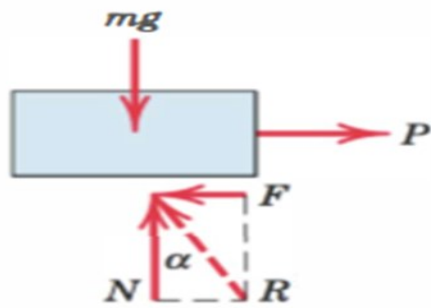


## DRY FRICTION

### Mechanism of Dry Friction


- ▶ When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and
- ▶ the  $t$ -components of the  $R$ 's are smaller than when the surfaces are at rest relative to one another.
- ▶ This observation helps to explain the well known fact that the force  $P$  necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.

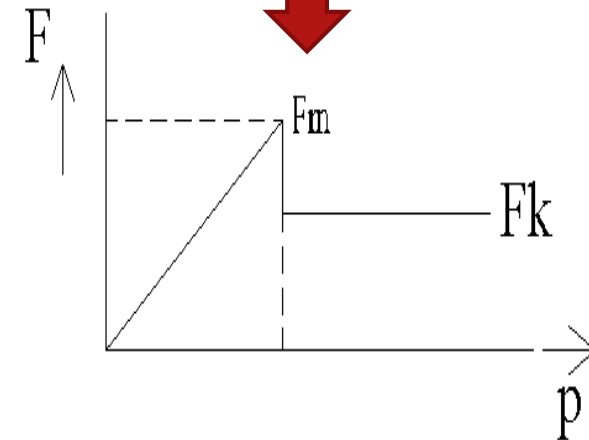
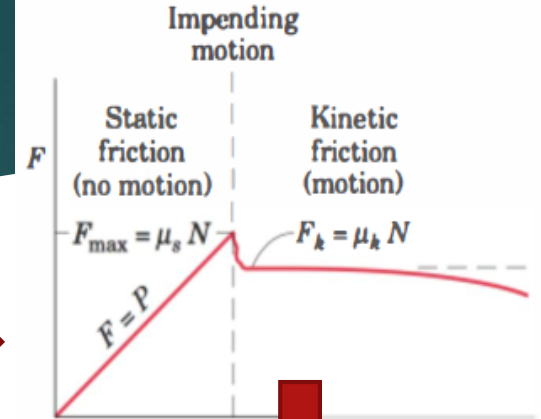




## DRY FRICTION

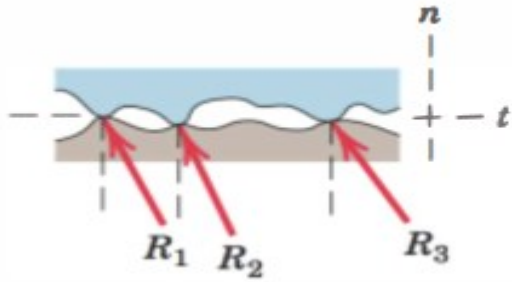
### Mechanism of Dry Friction

- ▶ If we perform the experiment and record the friction force  $F$  as a function of  $P$ , we obtain the relation shown in Fig. 
- ▶ When  $P$  is zero, equilibrium requires that there be no friction force.
- ▶ As  $P$  is increased, the friction force must be equal and opposite to  $P$  as long as the block does not slip.
- ▶ During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations.
- ▶ Finally, we reach a value of  $P$  which causes the block to slip and to move in the direction of the applied force.



# DRY FRICTION

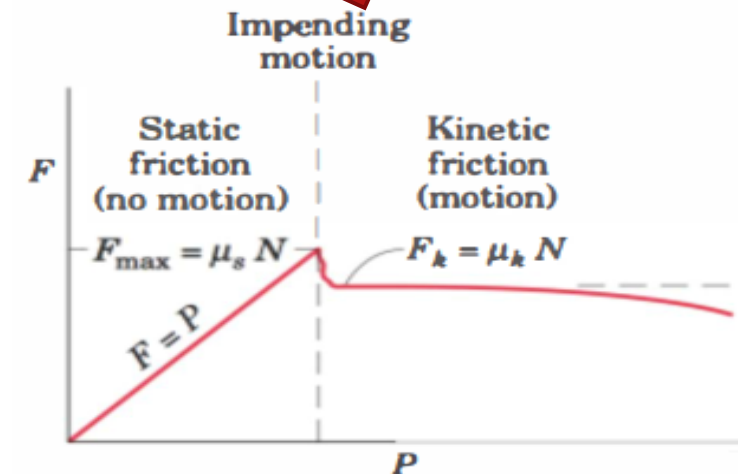
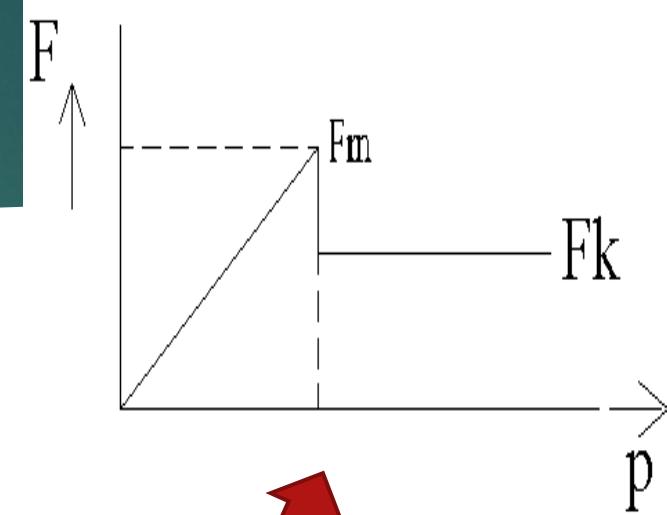
## Mechanism of Dry Friction

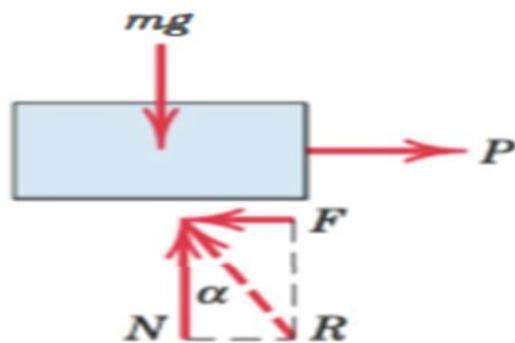


► At this same time the friction force decreases slightly and abruptly.

► It then remains essentially constant for a time but then decreases still more as the velocity increases.

► At this step we introduce Static friction and Kinetic friction





## DRY FRICTION

### Mechanism of Dry Friction

- ▶ Experiment shows that  $F_m$  and  $F_k$  are proportional to  $N$  and the constants of proportionality are  $\mu_S$  and  $\mu_K$ , where S and K stand for static and kinetic, respectively

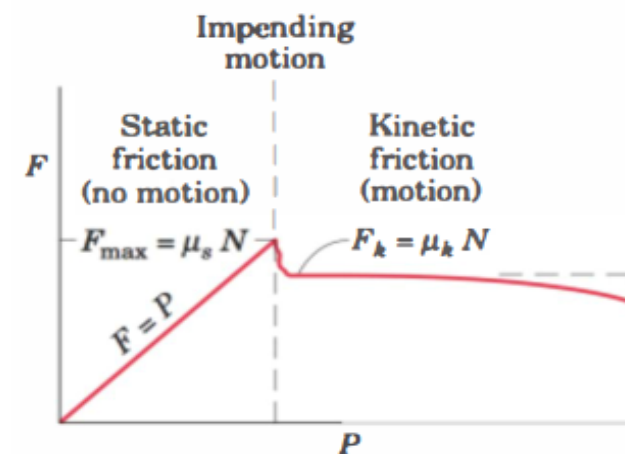
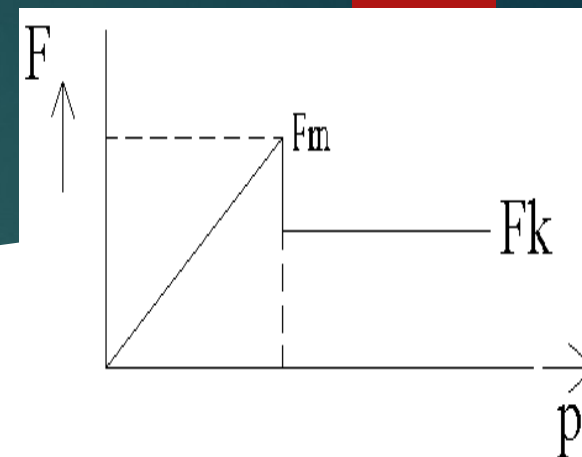
$\mu_S$  = Coefficient of static friction

$\mu_K$  = Coefficient of kinetic friction

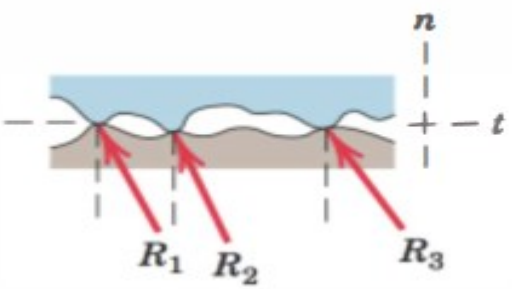
- ▶ These coefficients

-Do not depend on the area of the surfaces in contact

-Depend strongly on the nature of the surfaces in contact (generation of high local temperatures, relative hardness and the presence of thin surface films of oxide, oil, dirt,)



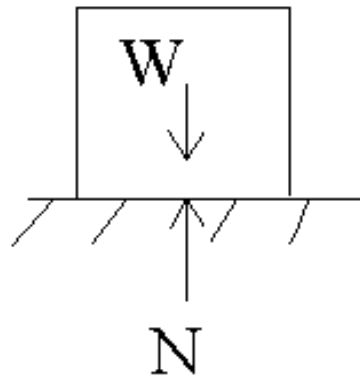
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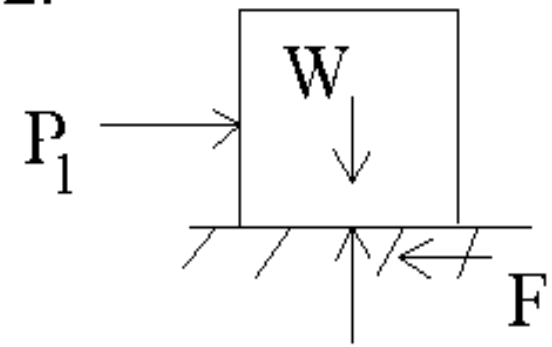
# DRY FRICTION

## Mechanism of Dry Friction

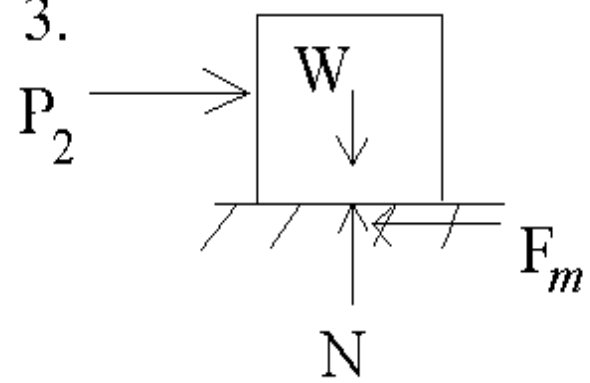
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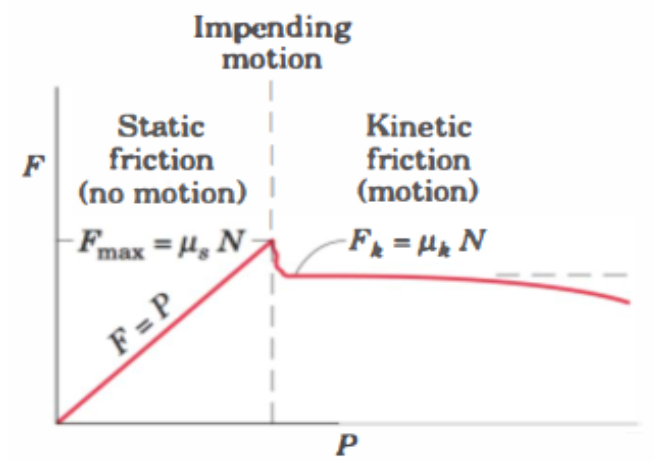
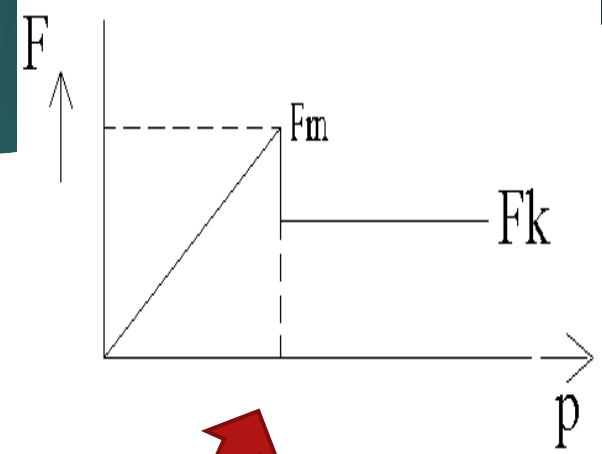
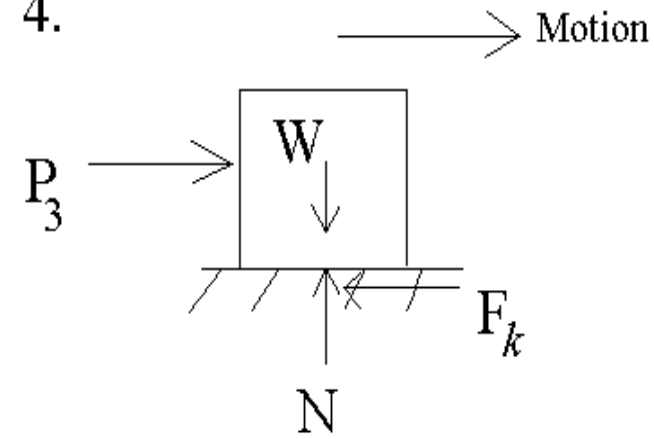
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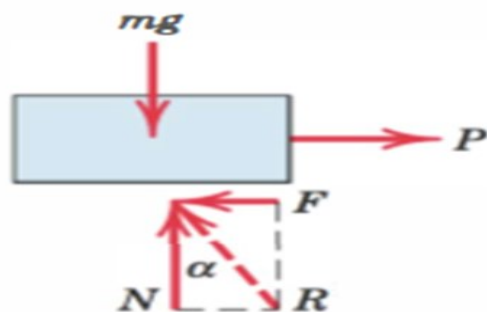


3.



4.





## DRY FRICTION

### Mechanism of Dry Friction

► **Static friction**, the friction force is determined by the equations of equilibrium.

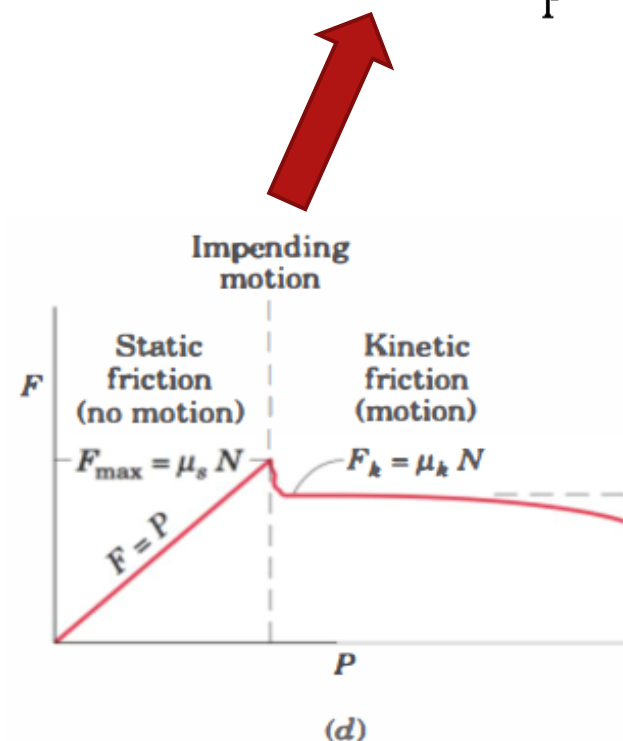
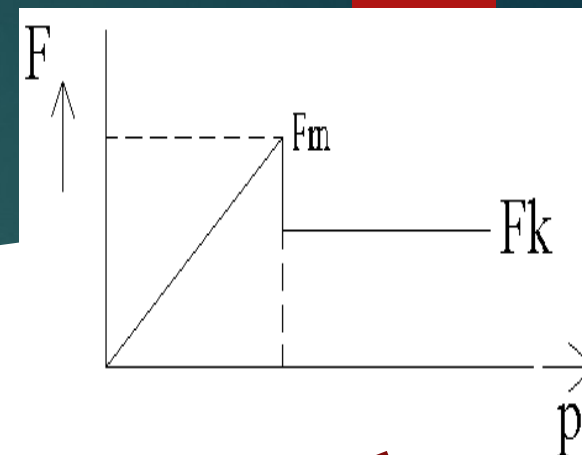
► This friction force may have any value from zero up to and including the maximum value ( $F_M$ ).

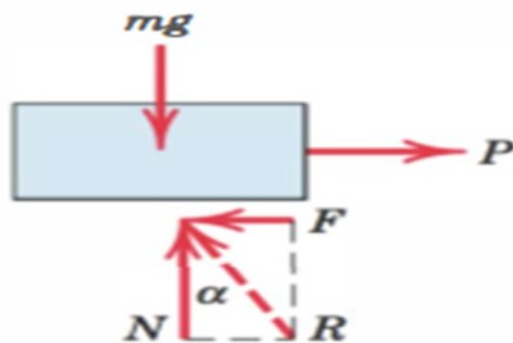
► For a given pair of mating surfaces the experiment shows that this maximum value of static friction  $F_{max}$  is proportional to the normal force  $N$ .

► Thus, we may write

$$F_{max} = \mu_s N$$

► where  $\mu_s$  is the proportionality constant called Coefficient of static friction





## DRY FRICTION

### Mechanism of Dry Friction

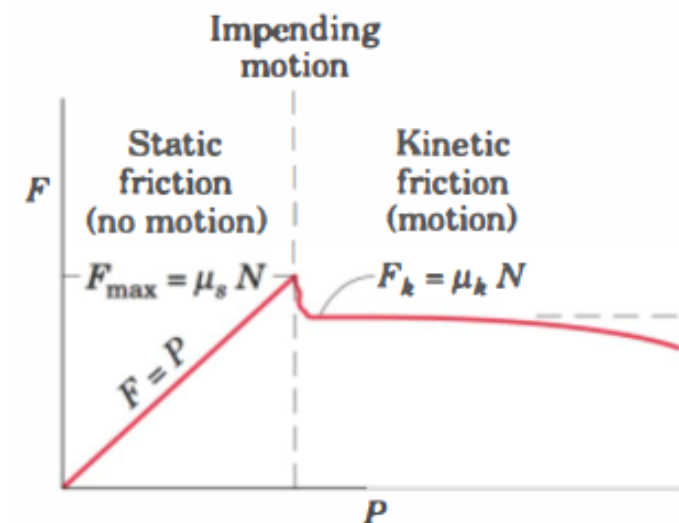
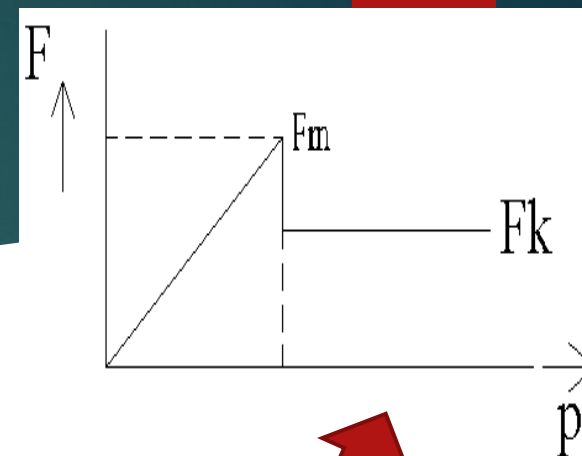
$$F_{\max} = \mu_s N$$

► Be aware that Eq. above describes only the limiting or maximum value of the static friction force and not any lesser value.

► Thus, the equation applies only to cases where motion is impending with the friction force at its peak value.

► For a condition of static equilibrium when motion is ***not impending***, the static friction force is

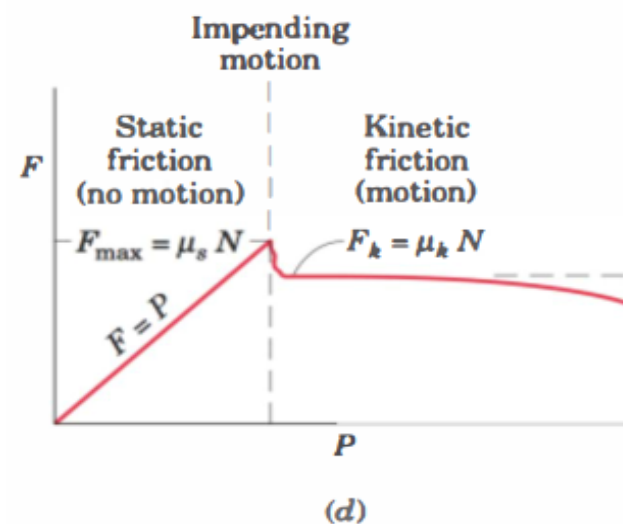
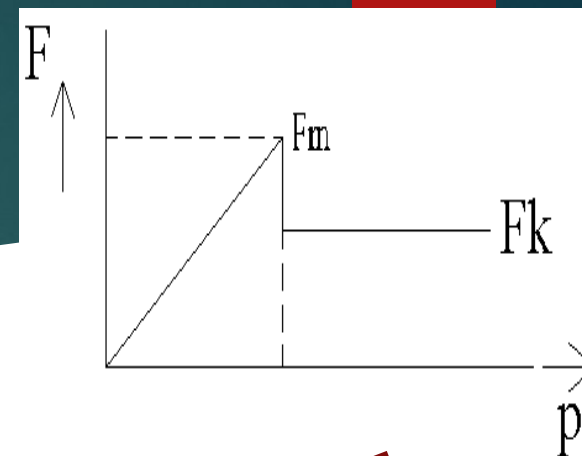
$$F < \mu_s N$$



# DRY FRICTION

## Mechanism of Dry Friction

- ▶ **Kinetic friction**, After slippage occurs, a condition of kinetic friction go along with the ensuing motion.
- ▶ Kinetic friction force is usually somewhat less than the maximum static friction force.
- ▶ The kinetic friction force  $F_k$  is also proportional to the normal force. Thus,  $\longrightarrow$   $F_k = \mu_k N$
- ▶ where  $\mu_k$  is the Coefficient of kinetic friction
- ▶ It follows that  $\mu_k$  is generally less than  $\mu_s$ . As the velocity of the block increases, the kinetic friction decreases somewhat, and at high velocities, this decrease may be significant.



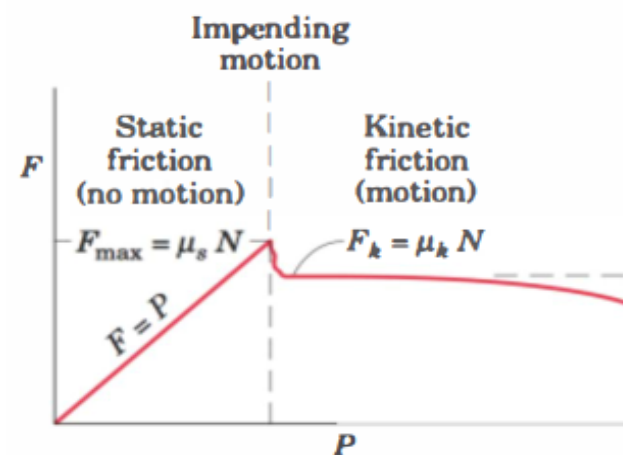
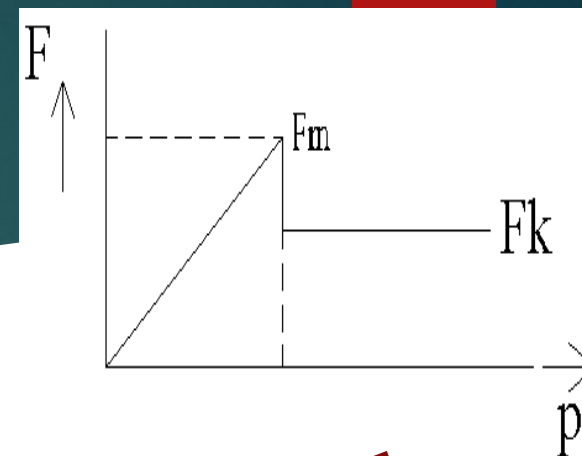
$$F_k = \mu_k N$$

$$F_{\max} = \mu_s N$$

## DRY FRICTION

### Mechanism of Dry Friction

- ▶ **Kinetic friction**, In the engineering literature we frequently find expressions for maximum static friction and for kinetic friction written simply as  $F = \mu N$ .
- ▶ It is understood from the problem at hand whether maximum static friction or kinetic friction is described.
- ▶ Although we will frequently distinguish between the static and kinetic coefficients, in other cases no distinction will be made, and the friction coefficient will be written simply as  $\mu$ .
- ▶ In such cases you must decide which of the friction conditions, maximum static friction for impending motion or kinetic friction, is involved.



$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1} \mu_s$$

## DRY FRICTION

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1} \mu_k$$

$$F_k = \mu_k N$$

### Mechanism of Dry Friction

$$F_{\max} = \mu_s N$$

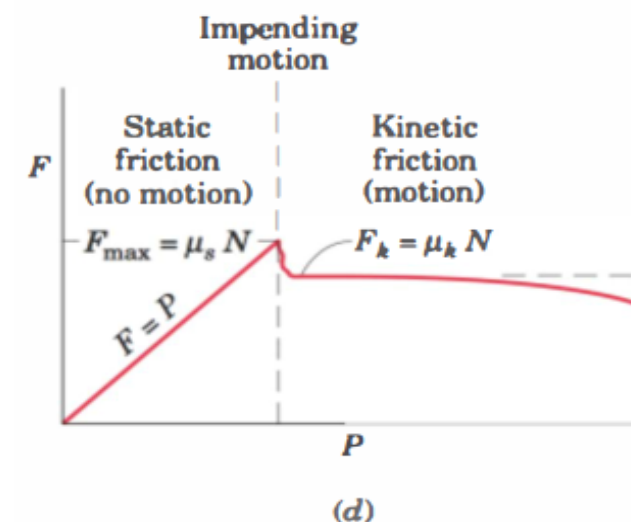
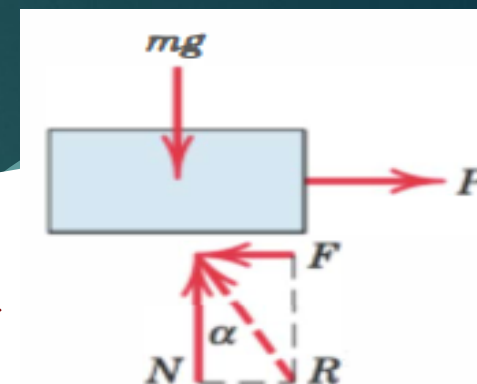
► **Friction angle**, The direction of the resultant  $R$  in Fig. measured from the direction of  $N$  is specified by  $\tan \alpha = F/N$ .

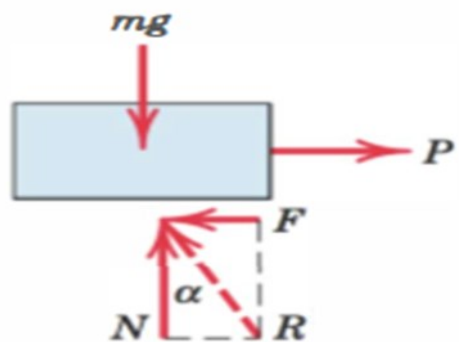
► When the friction force reaches its limiting static value  $F_{\max}$ , the angle  $\alpha$  reaches a maximum value  $\phi_s$ . Thus  **$\tan \phi_s = \mu_s$**

► When slippage is occurring, the angle  $\alpha$  has a value  $\phi_k$  corresponding to the kinetic friction force. In like manner,

$$\mathbf{\tan \phi_k = \mu_k}$$

► In practice we often see the expression  $\tan \phi = \mu$ , in which the coefficient of friction may refer to either the static or the kinetic case, depending on the particular problem.





## DRY FRICTION

### TYPES OF FRICTION PROBLEMS

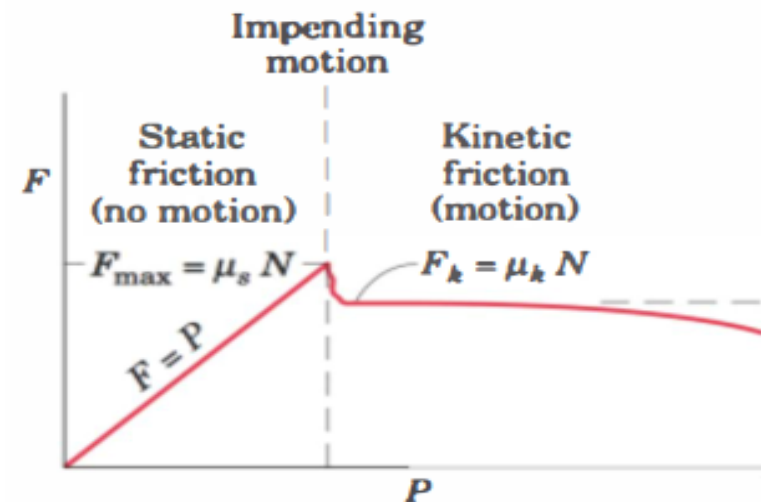
► We can now recognize the following three types of problems encountered in applications involving dry friction.

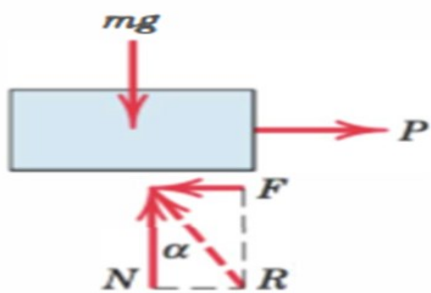
1) The first step in solving a friction problem is to identify its type.

► In the first type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping, and the friction force equals the limiting static friction

$$F_{\max} = \mu_s N$$

► The equations of equilibrium will, of course, also hold.





## DRY FRICTION

### TYPES OF FRICTION PROBLEMS

$$F_{\max} = \mu_s N$$

- 2) In the second type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force  $F$  necessary for equilibrium. Three outcomes are possible:

(a)  $F < (F_{\max} = \mu_s N)$

(b)  $F = (F_{\max} = \mu_s N)$

(c)  $F > (F_{\max} = \mu_s N)$

► For outcomes (a) & (b) equation of equilibrium are valid while in (c) the equilibrium is invalid and here motion occurs.

► Therefore the valid equation is

$$F_k = \mu_k N$$

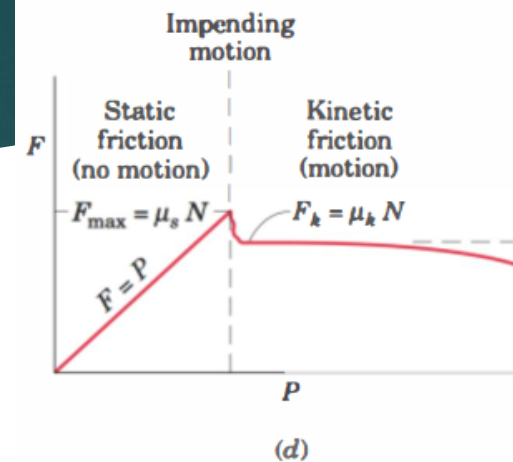


Table 8-1 Typical Values for  $\mu_s$

Contact Materials	Coefficient of Static Friction ( $\mu_s$ )
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Copper on copper	0.74–1.21

# DRY FRICTION

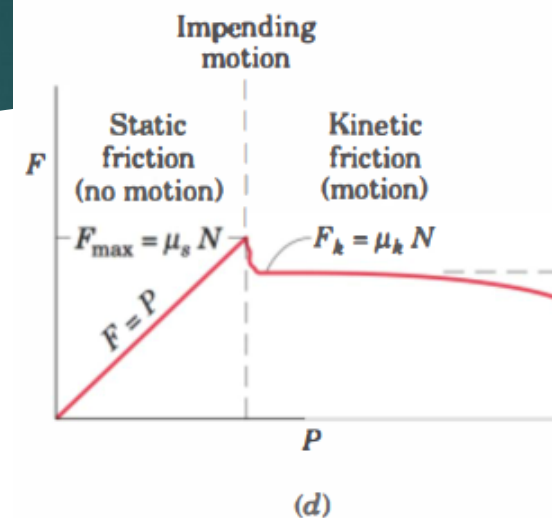
## TYPES OF FRICTION PROBLEMS

3) In the third type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies.

► For this problem type, Eq. below always gives the kinetic friction force directly.

$$F_k = \mu_k N$$

Table 8-1 Typical Values for $\mu_s$	
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Copper on copper	0.74–1.21



# DRY FRICTION

## Procedure for Analysis

Equilibrium problems involving dry friction can be solved using the following procedure.

### Free-Body Diagrams.

- Draw the necessary free-body diagrams, and unless it is stated in the problem that impending motion or slipping occurs, *always* show the frictional forces as unknowns (i.e., *do not assume*  $F = \mu N$ ).
- Determine the number of unknowns and compare this with the number of available equilibrium equations.
- If there are more unknowns than equations of equilibrium, it will be necessary to apply the frictional equation at some, if not all, points of contact to obtain the extra equations needed for a complete solution.

- If the equation  $F = \mu N$  is to be used, it will be necessary to show  $\mathbf{F}$  acting in the correct sense of direction on the free-body diagram.

### Equations of Equilibrium and Friction.

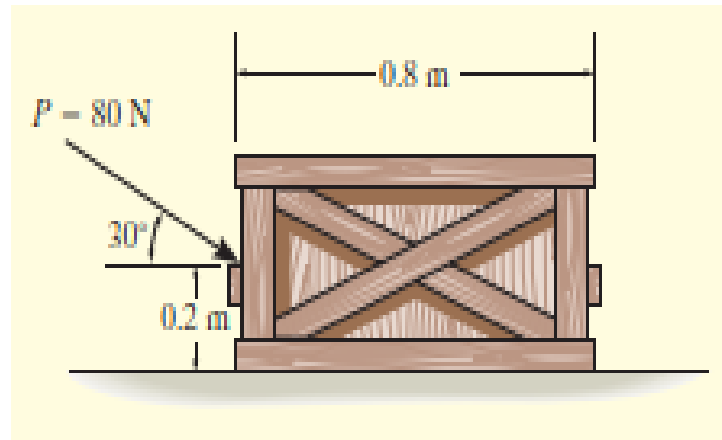
- Apply the equations of equilibrium and the necessary frictional equations (or conditional equations if tipping is possible) and solve for the unknowns.
- If the problem involves a three-dimensional force system such that it becomes difficult to obtain the force components or the necessary moment arms, apply the equations of equilibrium using Cartesian vectors.

# DRY FRICTION

## Example 8.1

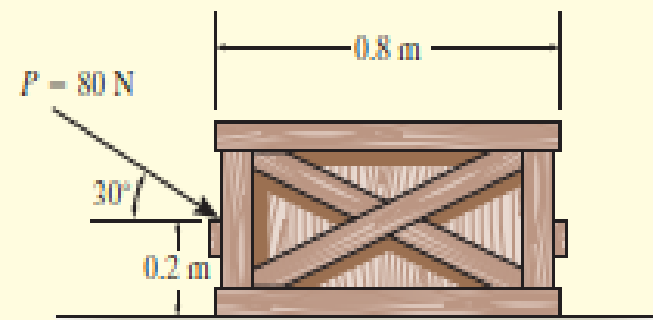
### Question

- The uniform crate shown in Fig. has a mass of 20 kg. If a force  $P = 80 \text{ N}$  is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is  $\mu_s = 0.3$ .



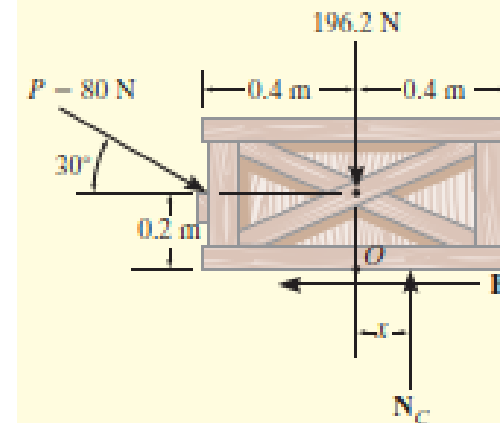
## DRY FRICTION

### Example 8.1



### Solution

- ▶ As shown on FBD, the resultant normal force  $N_C$  must act a distance  $x$  from the crate's center line in order to counteract the tipping effect caused by  $P$ .
- ▶ There are three unknowns,  $F$ ,  $N_C$ , and  $x$ , which can be determined strictly from the three equations of equilibrium.



### Equations of Equilibrium.

$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad 80 \cos 30^\circ \text{ N} - F = 0 \\ + \uparrow \Sigma F_y = 0; & \quad -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} = 0 \\ \zeta + \Sigma M_O = 0; & \quad 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) = 0 \end{aligned}$$

Solving,

$$\begin{aligned} F &= 69.3 \text{ N} \\ N_C &= 236.2 \text{ N} \\ x &= -0.00908 \text{ m} = -9.08 \text{ mm} \end{aligned}$$

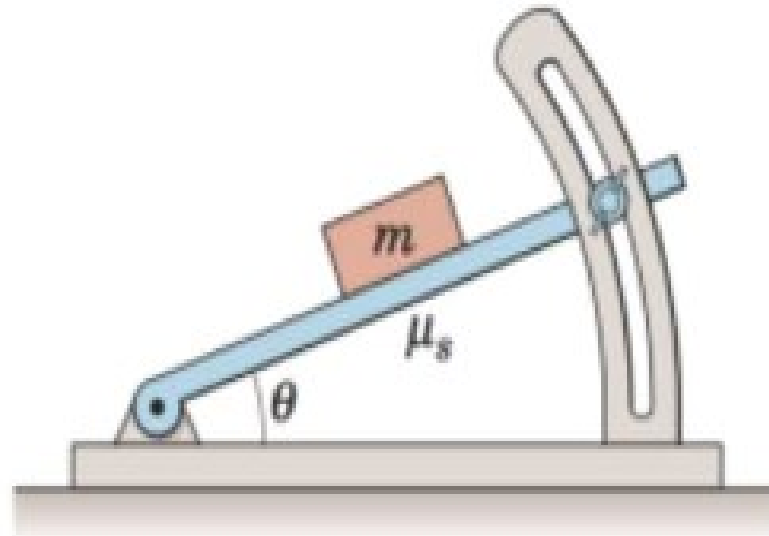
Since  $x$  is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since  $x < 0.4 \text{ m}$ . Also, the *maximum* frictional force which can be developed at the surface of contact is  $F_{\max} = \mu_s N_C = 0.3(236.2 \text{ N}) = 70.9 \text{ N}$ . Since  $F = 69.3 \text{ N} < 70.9 \text{ N}$ , the crate will *not slip*, although it is very close to doing so.

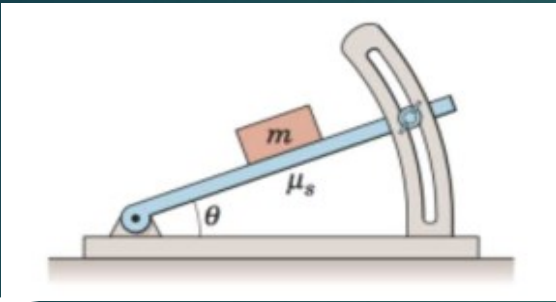
# DRY FRICTION

## Example 8.2

### Question

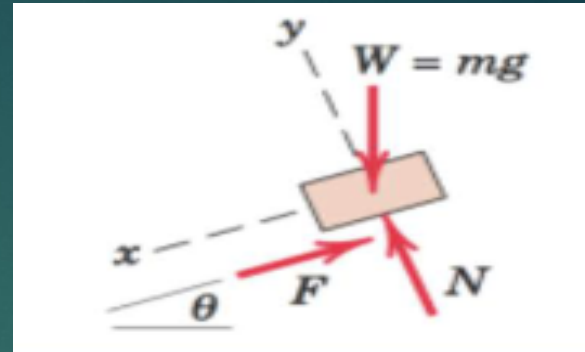
► Determine the maximum angle  $\theta$  which the adjustable incline may have with the horizontal before the block of mass  $m$  begins to slip. The coefficient of static friction between the block and the inclined surface is  $\mu_s$ .





## DRY FRICTION

### Example 8.2



### Solution

- ▶ The free-body diagram of the block shows its weight  $W = mg$ , the normal force  $N$ , and the friction force  $F$  exerted by the incline on the block.
- ▶ The friction force acts in the direction to oppose the slipping which would occur if no friction were present. Equilibrium in the  $x$ - and  $y$ -directions requires

$$[\Sigma F_x = 0] \quad mg \sin \theta - F = 0 \quad F = mg \sin \theta$$

$$[\Sigma F_y = 0] \quad -mg \cos \theta + N = 0 \quad N = mg \cos \theta$$

Dividing the first equation by the second gives  $F/N = \tan \theta$ . Since the maximum angle occurs when  $F = F_{\max} = \mu_s N$ , for impending motion we have

$$\mu_s = \tan \theta_{\max} \quad \text{or} \quad \theta_{\max} = \tan^{-1} \mu_s \quad \text{Ans.}$$

# DRY FRICTION

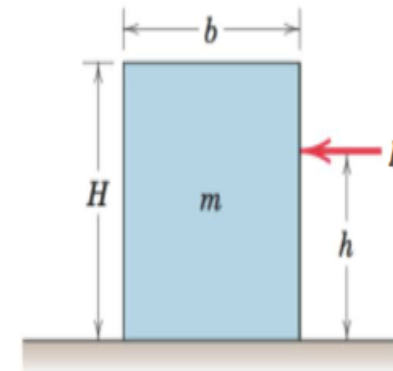
## Example 8.3

### Question

► The homogeneous rectangular block of mass  $m$ , width  $b$ , and height  $H$  is placed on the horizontal surface and subjected to a horizontal force  $P$  which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is  $\mu_s$ .

Determine

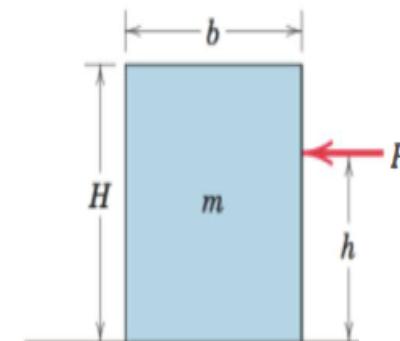
- the greatest value which  $h$  may have so that the block will slide without tipping over and
- the location of a point  $C$  on the bottom face of the block through which the resultant of the friction and normal forces acts if  $h = H/2$ .



- ① Recall that the equilibrium equations apply to a body moving with a constant velocity (zero acceleration) just as well as to a body at rest.

## DRY FRICTION

### Example 8.3



### Solution

**Solution.** (a) With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A. The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value  $\mu_k N$ , and the angle  $\theta$  becomes  $\theta = \tan^{-1} \mu_k$ . The resultant of  $F_k$  and  $N$  passes through a point B through which  $P$  must also pass, since three coplanar forces in equilibrium are concurrent. Hence, from the geometry of the block

$$\tan \theta = \mu_k = \frac{b/2}{h} \quad h = \frac{b}{2\mu_k}$$

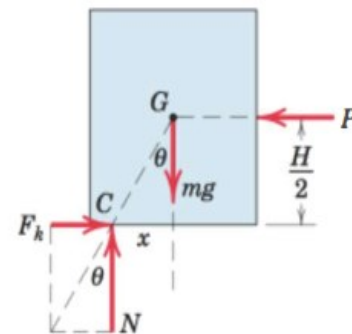
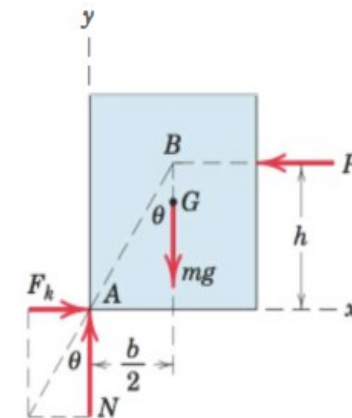
$$\text{Ans. } [\Sigma M_A = 0] \quad Ph - mg \frac{b}{2} = 0 \quad h = \frac{mgb}{2P} = \frac{mgb}{2\mu_k mg} = \frac{b}{2\mu_k} \quad \text{Ans.}$$

If  $h$  were greater than this value, moment equilibrium about A would not be satisfied, and the block would tip over.

Alternatively, we may find  $h$  by combining the equilibrium requirements for the  $x$ - and  $y$ -directions with the moment-equilibrium equation about A. Thus,

$$[\Sigma F_y = 0] \quad N - mg = 0 \quad N = mg$$

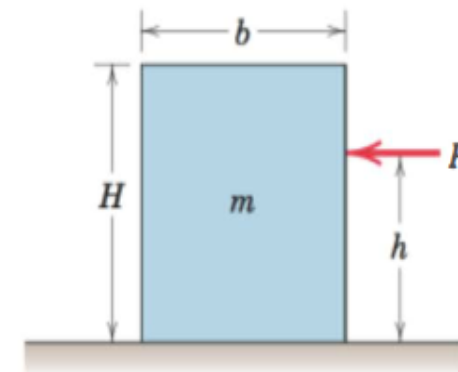
$$[\Sigma F_x = 0] \quad F_k - P = 0 \quad P = F_k = \mu_k N = \mu_k mg$$



- ② Alternatively, we could equate the moments about  $G$  to zero, which would give us  $F(H/2) - Nx = 0$ . Thus, with  $F_k = \mu_k N$  we get  $x = \mu_k H/2$ .

## DRY FRICTION

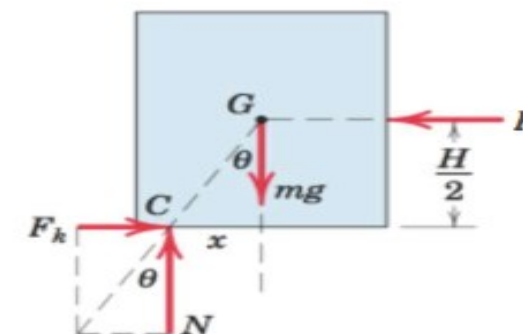
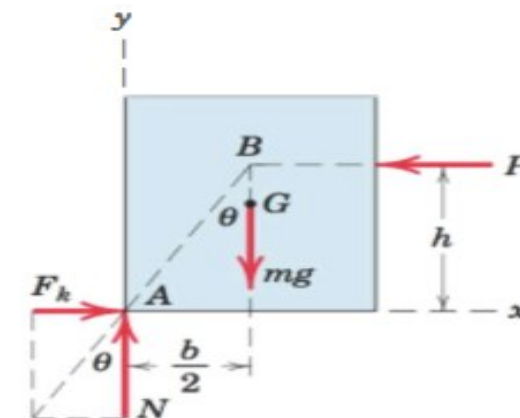
### Example 8.3



(b) With  $h = H/2$  we see from the free-body diagram for case (b) that the resultant of  $F_k$  and  $N$  passes through a point  $C$  which is a distance  $x$  to the left of the vertical centerline through  $G$ . The angle  $\theta$  is still  $\theta = \phi = \tan^{-1} \mu_k$  as long as the block is slipping. Thus, from the geometry of the figure we have

$$\frac{x}{H/2} = \tan \theta = \mu_k \quad \text{so} \quad x = \mu_k H/2 \quad \text{Ans.}$$

If we were to replace  $\mu_k$  by the static coefficient  $\mu_s$ , then our solutions would describe the conditions under which the block is (a) on the verge of tipping and (b) on the verge of slipping, both from a rest position.



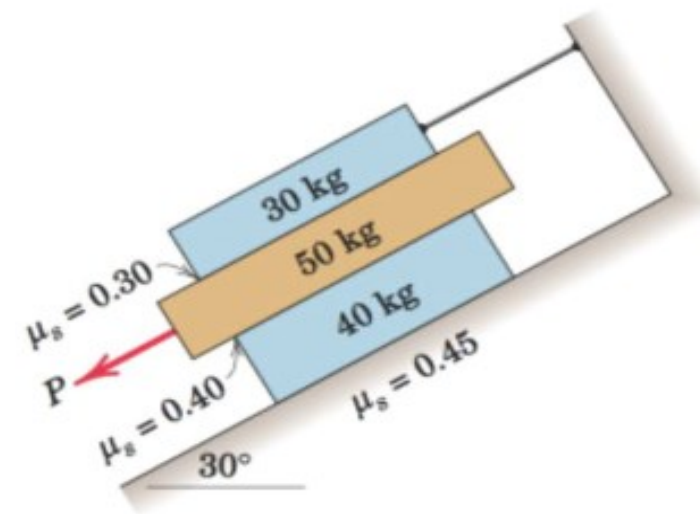
# DRY FRICTION

## Example 8.4

### Question

► The three flat blocks are positioned on the  $30^\circ$  incline as shown, and a force  $P$  parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown.

Determine the maximum value which  $P$  may have before any slipping takes place.

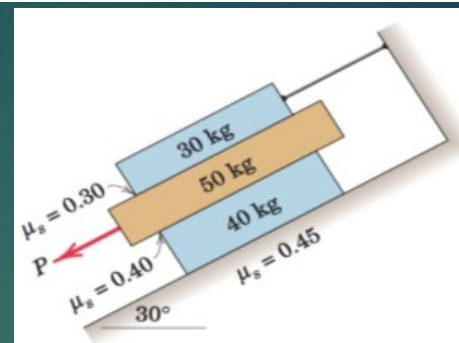


# DRY FRICTION

## Example 8.4

① In the absence of friction the middle block, under the influence of  $P$ , would have a greater movement than the 40-kg block, and the friction force  $F_2$  will be in the direction to oppose this motion as shown.

② We see now that  $F_2$  is less than  $\mu_s N_2 = 272 \text{ N}$ .



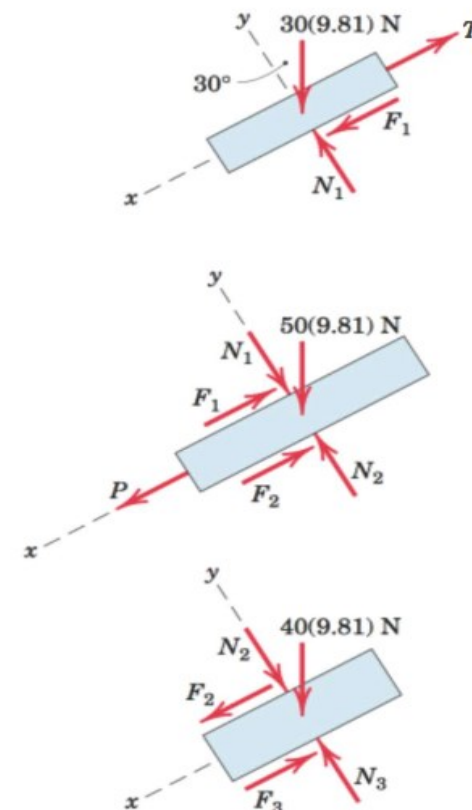
### Solution

► The FBD of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50-kg and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

► The normal forces, which are in y-direction, may be determined without reference to the friction forces, which are all in the x-direction.

Thus,

$(\Sigma F_y = 0)$	(30-kg)	$N_1 - 30(9.81) \cos 30^\circ = 0$	$N_1 = 255 \text{ N}$
	(50-kg)	$N_2 - 50(9.81) \cos 30^\circ - 255 = 0$	$N_2 = 680 \text{ N}$
	(40-kg)	$N_3 - 40(9.81) \cos 30^\circ - 680 = 0$	$N_3 = 1019 \text{ N}$

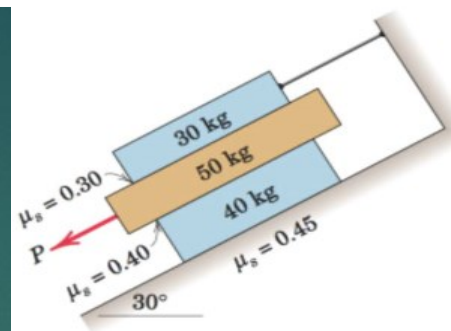


① In the absence of friction the middle block, under the influence of  $P$ , would have a greater movement than the 40-kg block, and the friction force  $F_2$  will be in the direction to oppose this motion as shown.

② We see now that  $F_2$  is less than  $\mu_s N_2 = 272 \text{ N}$ .

## DRY FRICTION

### Example 8.4



### Solution

► We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

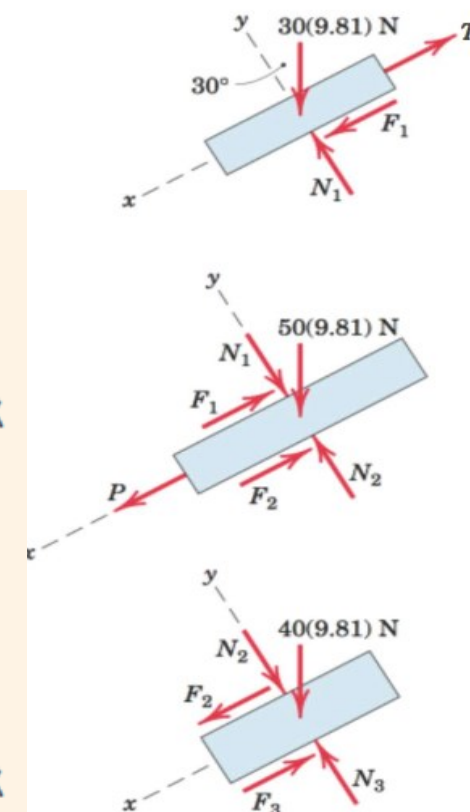
$$[F_{\max} = \mu_s N] \quad F_1 = 0.30(255) = 76.5 \text{ N} \quad F_2 = 0.40(680) = 272 \text{ N}$$

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$$[\Sigma F_x = 0] \quad P - 76.5 - 272 + 50(9.81) \sin 30^\circ = 0 \quad P = 103.1 \text{ N}$$

We now check on the validity of our initial assumption. For the 40-kg block with  $F_2 = 272 \text{ N}$  the friction force  $F_3$  would be given by

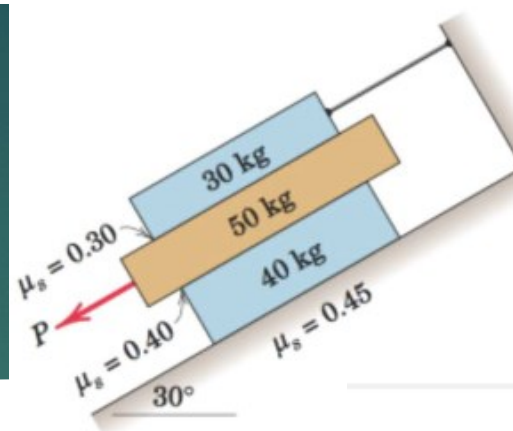
$$[\Sigma F_x = 0] \quad 272 + 40(9.81) \sin 30^\circ - F_3 = 0 \quad F_3 = 468 \text{ N}$$



# DRY FRICTION

## Example 8.4

- ① In the absence of friction the middle block, under the influence of  $P$ , would have a greater movement than the 40-kg block, and the friction force  $F_2$  will be in the direction to oppose this motion as shown.
- ② We see now that  $F_2$  is less than  $\mu_s N_2 = 272 \text{ N}$ .



### Solution

But the maximum possible value of  $F_3$  is  $F_3 = \mu_s N_3 = 0.45(1019) = 459 \text{ N}$ . Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value  $F_3 = 459 \text{ N}$ , equilibrium of the 40-kg block for its impending motion requires

$$[\Sigma F_x = 0] \quad F_2 + 40(9.81) \sin 30^\circ - 459 = 0 \quad F_2 = 263 \text{ N}$$

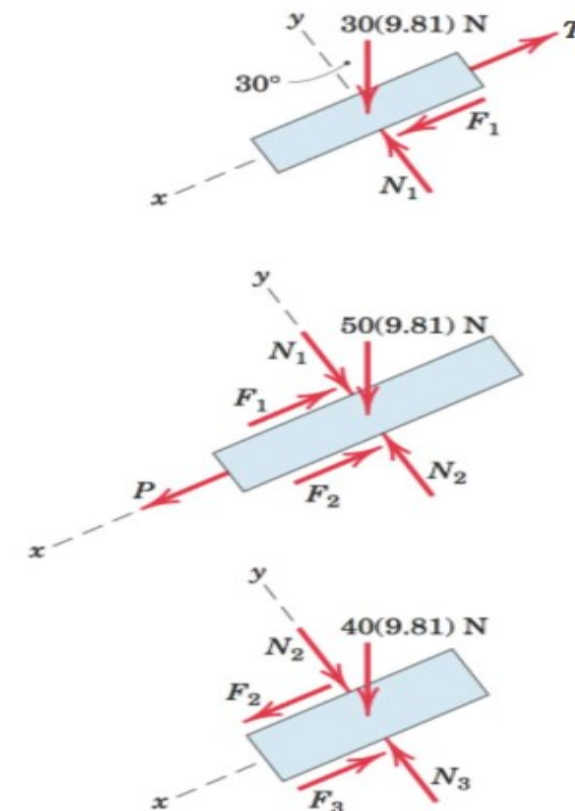
Equilibrium of the 50-kg block gives, finally,

$$[\Sigma F_x = 0] \quad P + 50(9.81) \sin 30^\circ - 263 - 76.5 = 0$$

$$P = 93.8 \text{ N}$$

*Ans.*

Thus, with  $P = 93.8 \text{ N}$ , motion impends for the 50-kg and 40-kg blocks as a unit.



► In the next section, we will explore the application of friction in various machine. Note that most of these applications are normally either limiting static or kinetic friction. The machines that will be discussed in CEE2219 include the following:

- ❖ Wedges
- ❖ Screws
- ❖ Belts\*
- ❖ Collar & Pivot Bearings
- ❖ Disks\*
- ❖ Journal Bearings and
- ❖ Rolling Resistance\*

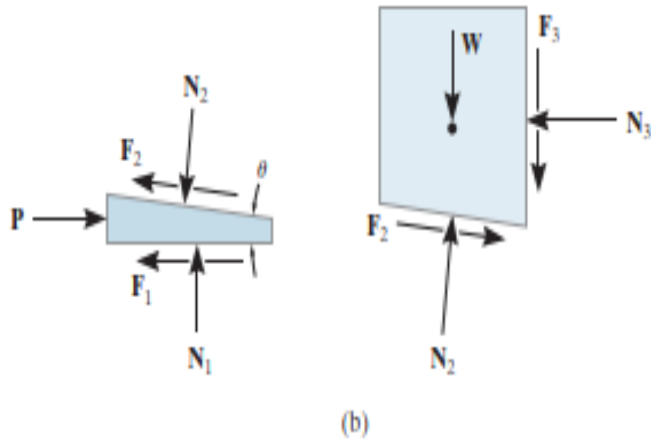
## WEDGES

- ▶ A wedge is one of the simplest and most useful machines. A wedge is used to produce small adjustments in the position of a body or to apply large forces. These loads may be raised by applying to the wedge a force usually considerably smaller than the weight of the load.
- ▶ Wedges largely depend on friction to function. Because of the friction between the surfaces in contact, a wedge, if properly shaped, will remain in place after being forced under the load.
- ▶ When sliding of a wedge is impending, the resultant force on each sliding surface of the wedge will be inclined from the normal to the surface by an amount equal to the friction angle.

# PRICTICAL APPLICATIONS OF FRICTION

## WEDGES

44

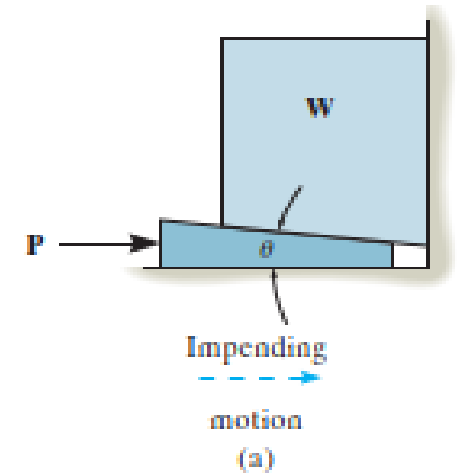


► The component of the resultant along the surface is the friction force, which is always in the direction to oppose the motion of the wedge relative to the mating surfaces.

► Consider, for example, the wedge shown in Fig (A)., which is used to lift the block by applying a force to the wedge.

► FBD of the block and wedge are shown in Fig. below. Here we have excluded the weight of the wedge since it is usually small compared to the weight  $W$  of the block.

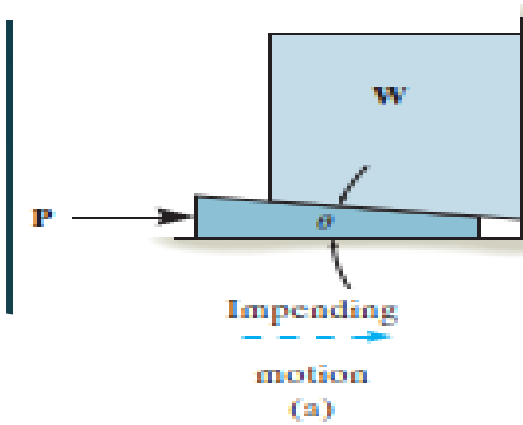
► Also, note that the frictional forces  $F_1$  and  $F_2$  must oppose the motion of the wedge.



# PRICTICAL APPLICATIONS OF FRICTION

## WEDGES

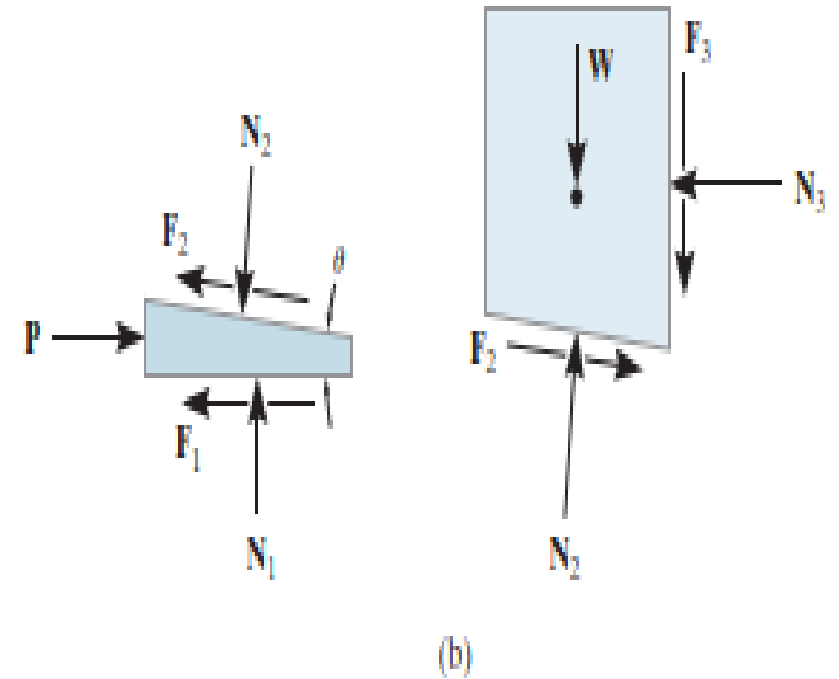
45



► Likewise, the frictional force  $F_3$  of the wall on the block must act downward so as to oppose the block's upward motion.

► The locations of the resultant normal forces are not important in the force analysis since neither the block nor wedge will “tip.”

► Hence the moment equilibrium equations will not be considered. There are seven unknowns, consisting of the applied force  $P$ , needed to cause motion of the wedge, and six normal and frictional forces.

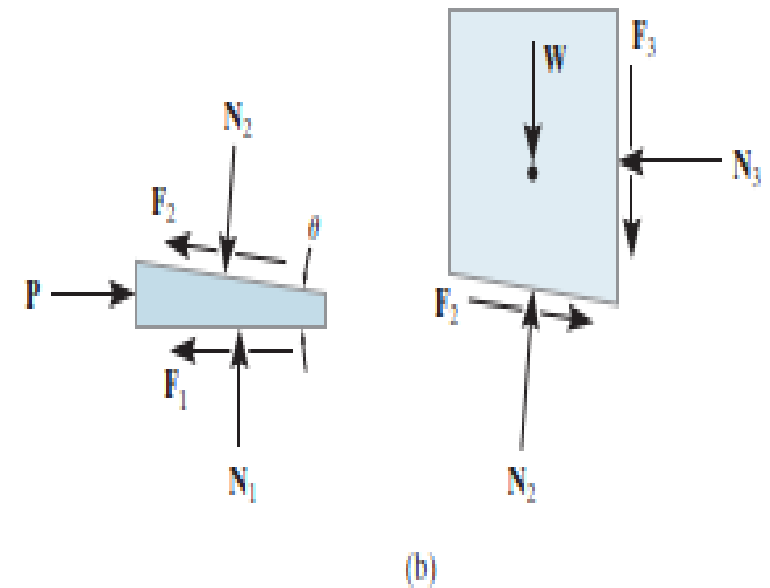
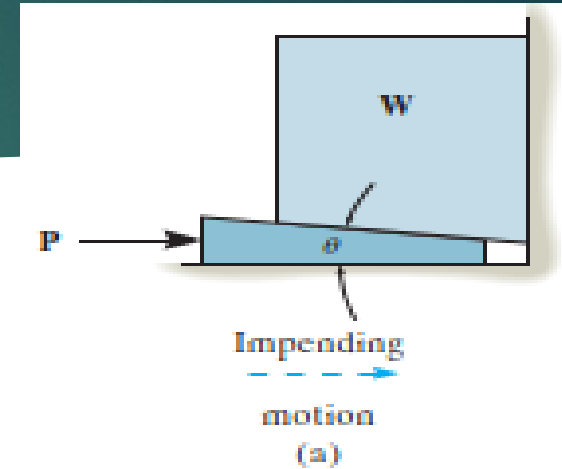


# PRICTICAL APPLICATIONS OF FRICTION

## WEDGES

► The seven available equations consist of four force equilibrium equations,  $\sum F_x = 0$ ,  $\sum F_y = 0$  applied to the wedge and block, and three frictional equations,  $F = \mu N$ , applied at each surface of contact.

► If the block is to be lowered, then the frictional forces will all act in a sense opposite to that shown in Fig. (b).

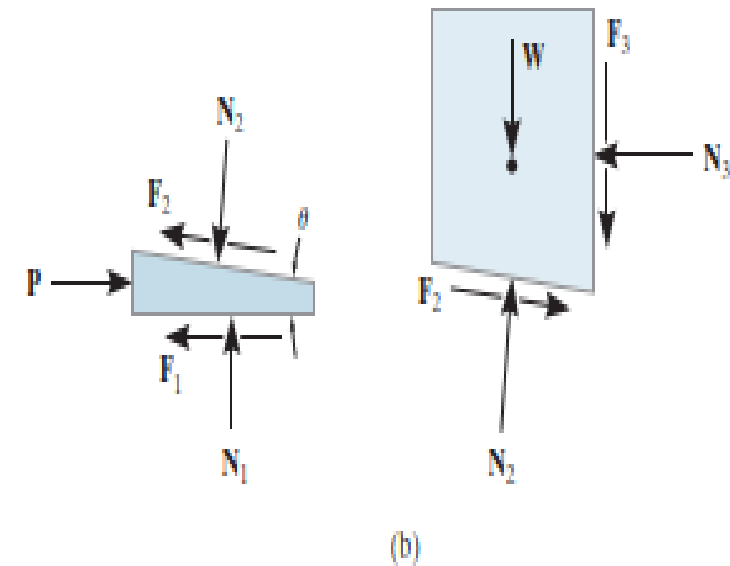
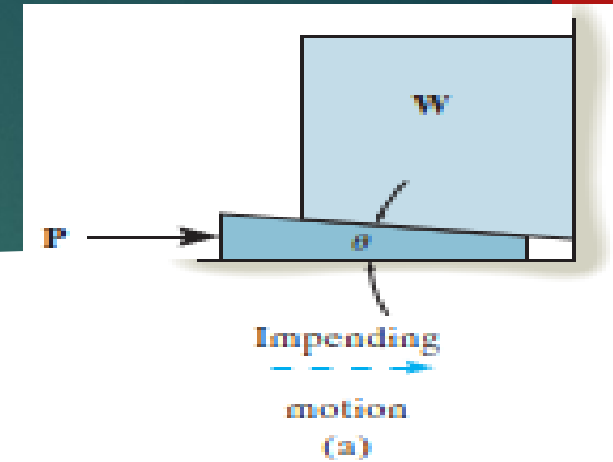


# PRICTICAL APPLICATIONS OF FRICTION

## WEDGES

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- ▶ Provided the coefficient of friction is very small or the wedge angle  $\theta$  is large, then the applied force  $P$  must act to the right to hold the block.
- ▶ Otherwise,  $P$  may have a reverse sense of direction in order to pull on the wedge to remove it.
- ▶ If  $P$  is not applied and friction forces hold the block in place, then the wedge is referred to as self-locking.

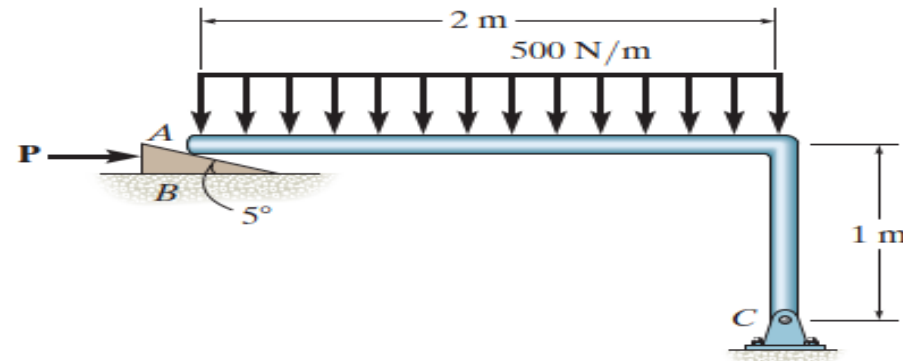


# WEDGES

## Example 8.5

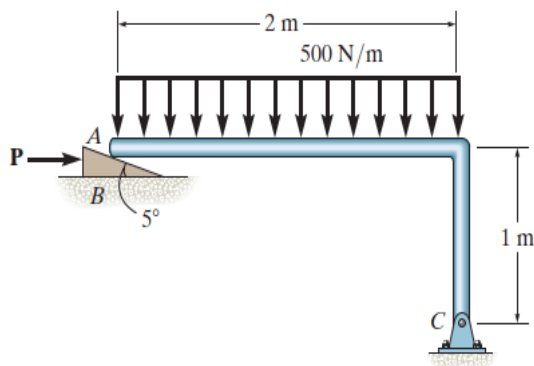
### Question

► The wedge is used to level the member. Determine the horizontal force  $P$  that must be applied to begin to push the wedge forward. The coefficient of static friction between the wedge and the two surfaces of contact is  $\mu_s = 0.2$ . Neglect the weight of the wedge.



# WEDGES

## Example 8.5



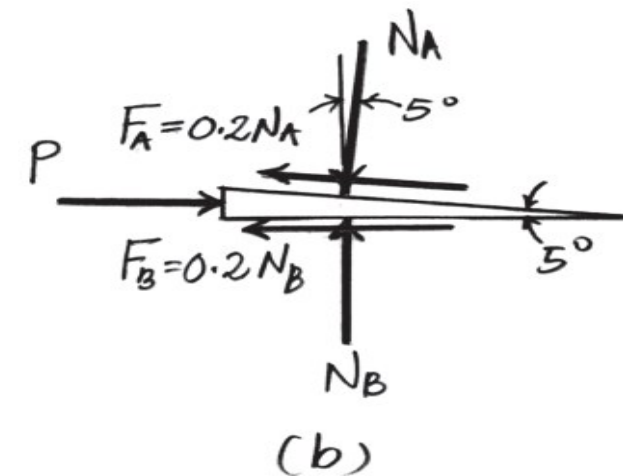
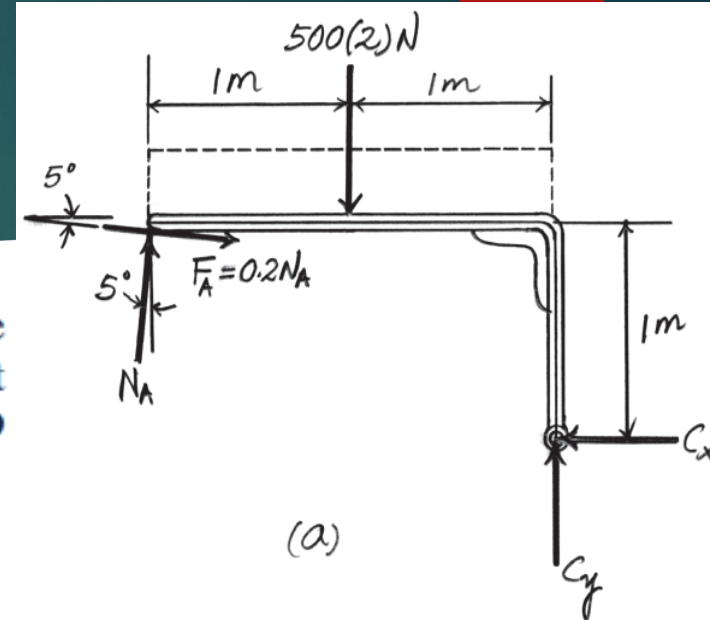
### Solution

**Equations of Equilibrium and Friction.** Since the wedge is required to be on the verge to slide to the right, then slipping will have to occur at both of its contact surfaces. Thus,  $F_A = \mu_s N_A = 0.2 N_A$  and  $F_B = \mu_s N_B$ . Referring to the *FBD* diagram of member *AC* shown in Fig. *a*

$$\begin{aligned} \zeta + \Sigma M_C = 0; \quad & 500(2)(1) - N_A \cos 5^\circ(2) - N_A \sin 5^\circ(1) \\ & - 0.2 N_A \cos 5^\circ(1) + 0.2 N_A \sin 5^\circ(2) = 0 \\ & N_A = 445.65 \text{ N} \end{aligned}$$

Using this result and the *FBD* of the wedge, Fig. *b*,

$$\begin{aligned} + \uparrow \Sigma F_y = 0; \quad & N_B - 445.65 \cos 5^\circ + 0.2(445.65) \sin 5^\circ = 0 \\ & N_B = 436.18 \text{ N} \\ \pm \rightarrow \Sigma F_x = 0; \quad & P - 0.2(445.65) \cos 5^\circ - 445.65 \sin 5^\circ - 0.2(436.18) = 0 \\ & P = 214.87 \text{ N} = 215 \text{ N} \end{aligned}$$



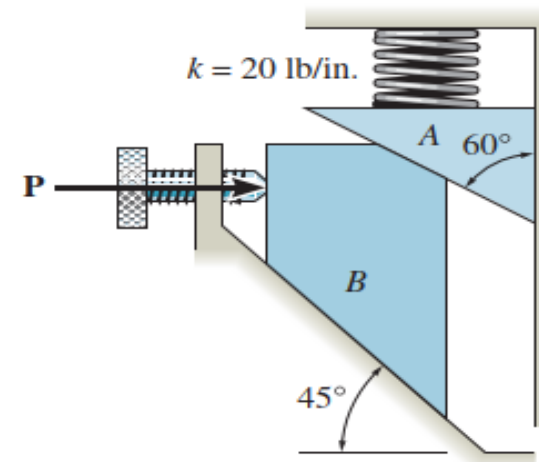
**Ans.**

# WEDGES

## Example 8.6

### Question

► The two blocks used in a measuring device have negligible weight. If the spring is compressed 5 in. when in the position shown, determine the smallest axial force  $P$  which the adjustment screw must exert on  $B$  in order to start the movement of  $B$  downward. The end of the screw is smooth and the coefficient of static friction at all other points of contact is  $\mu_s = 0.3$ .



# WEDGES

## Example 8.6

### Solution

Note that when block  $B$  moves downward, block  $A$  will also come downward.

Block  $A$ :

$$\rightarrow \Sigma F_x = 0; \quad N' \cos 60^\circ + 0.3 N' \sin 60^\circ - N_A = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 0.3 N_A - 0.3 N' \cos 60^\circ + N' \sin 60^\circ - 100 = 0$$

Block  $B$ :

$$\rightarrow \Sigma F_x = 0; \quad N_B \sin 45^\circ - N_B \sin 45^\circ + P - 0.3 N' \sin 60^\circ - N' \cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \quad N_B \cos 45^\circ + 0.3 N_B \cos 45^\circ + 0.3 N' \cos 60^\circ - N' \sin 60^\circ = 0$$

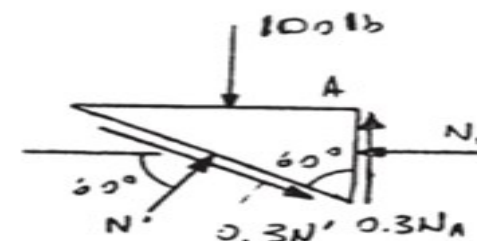
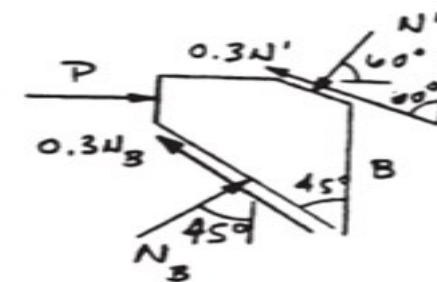
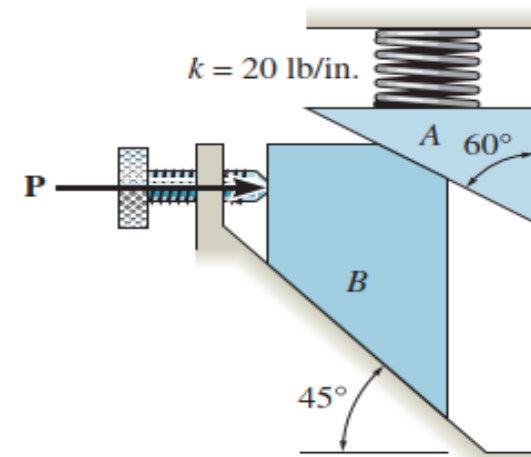
Solving,

$$N' = 105.9 \text{ lb}$$

$$N_B = 82.5 \text{ lb}$$

$$N_A = 80.5 \text{ lb}$$

$$P = 39.6 \text{ lb}$$



Ans.

## HOME WORK EXERCISE

8-4, 8-6, 8-8, 8-14, 8-15, 8-17, 8-21, 8-23, 8-30, 8-36, 8-56, 8-59, 8-62, 8-64 , 8-66

# FRICION

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***THE END – THANK YOU***