CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

JQL
07/04/2021

Lecture A4 (part ii)

❖ INTRODUCTION TO MOMENTS, COUPLES & RESULTANT FORCES

- ✓ CROSS PRODUCT
- ✓ TRIPLE SCALAR PRODUCT
- ✓ INTERNAL & EXTERNAL FORCES

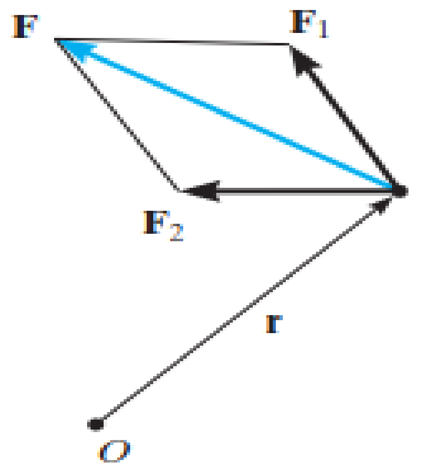
LECTURE OBJECTIVES

- ❖To discuss the concept of the moment of a force and show how to calculate it in 2D & 3D.
- ❖To provide a method for finding the moment of a force about a specified axis.
- ❖To define the moment of a couple.
- ❖To show how to find the resultant effect of a non-concurrent force system.

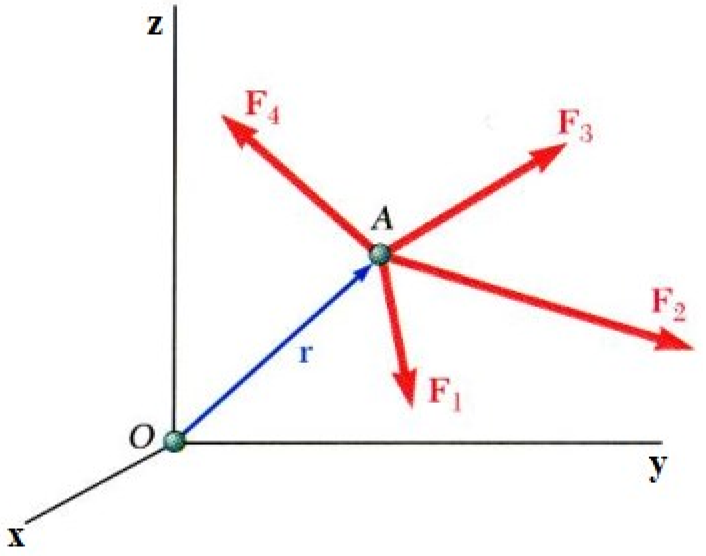
CHAPTER INTRODUCTION

- In this lecture you will learn how the forces can be replaced with a simpler equivalent system:
  - moment of a force about a point
  - moment of a force about an axis
  - moment due to a couple
- *Note that the determination of these quantities (M) involves the computation of **vector products** and [**scalar products**] of two vectors,*

MOMENT OF A FORCE
Varignon's Theorem (Principles of Moments)



- It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point
- Or the moment about a give point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O
- This theorem can be proven easily using the vector cross product since the cross product obeys the distributive law:
- $M_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 \ldots) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \ldots\ldots\ldots$



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE

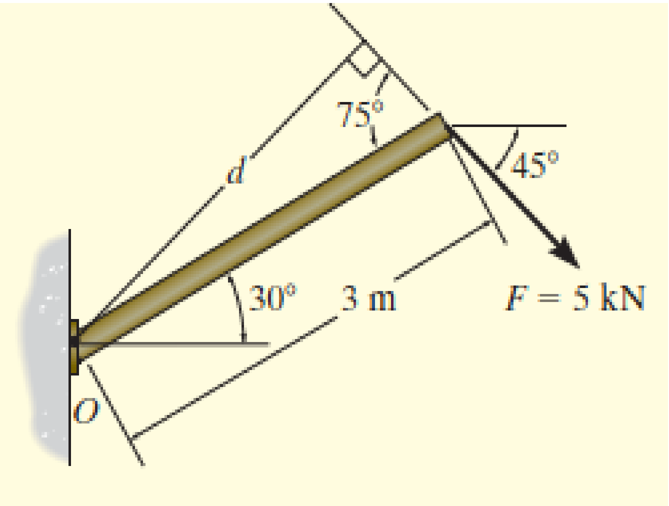
Example 4.8

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Question

- Determine the moment of the force in Fig. 4–18a about point O.

- Three different type of solutions are available!



(a)

MOMENT OF A FORCE
Example 4.8

Solution

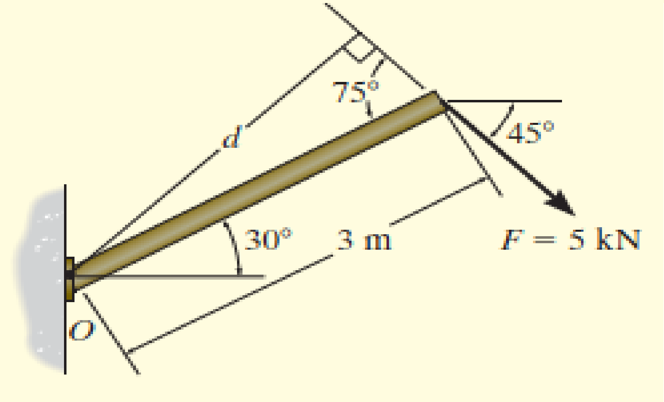
The moment arm d in Fig. 4–18*a* can be found from trigonometry.

$$d = (3\ \text{m}) \sin 75^\circ = 2.898\ \text{m}$$

Thus,

$$M_O = Fd = (5\ \text{kN})(2.898\ \text{m}) = 14.5\ \text{kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.



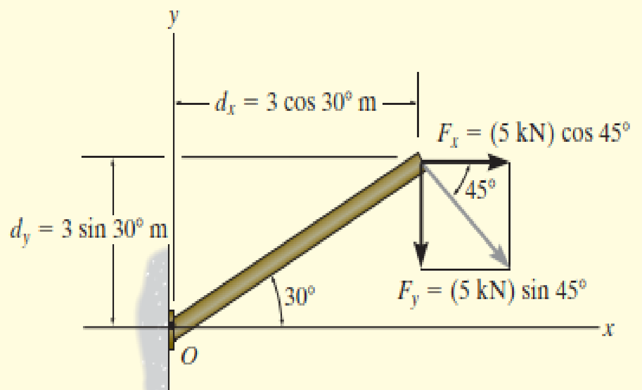
(a)

The x and y components of the force are indicated in Fig. 4–18*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$

$$= -(5 \cos 45^\circ\ \text{kN})(3 \sin 30^\circ\ \text{m}) - (5 \sin 45^\circ\ \text{kN})(3 \cos 30^\circ\ \text{m})$$

$$= -14.5\ \text{kN}\cdot\text{m} = 14.5\ \text{kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$



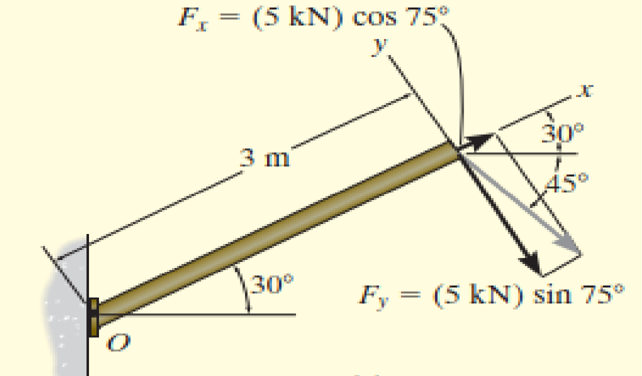
(b)

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4–18*c*. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\circlearrowleft + M_O = -F_y d_x$$

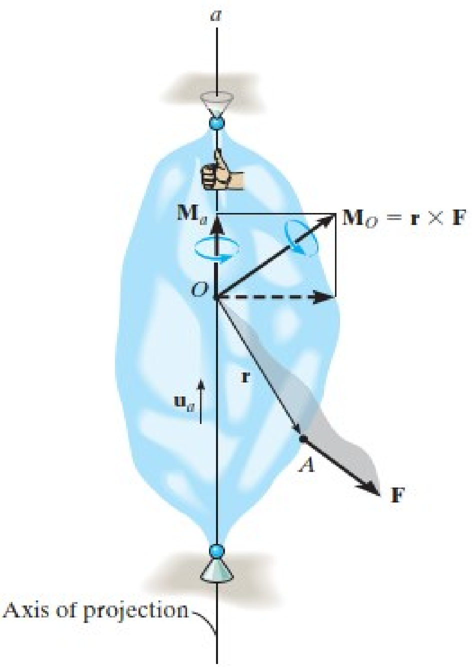
$$= -(5 \sin 75^\circ\ \text{kN})(3\ \text{m})$$

$$= -14.5\ \text{kN}\cdot\text{m} = 14.5\ \text{kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$



(c)

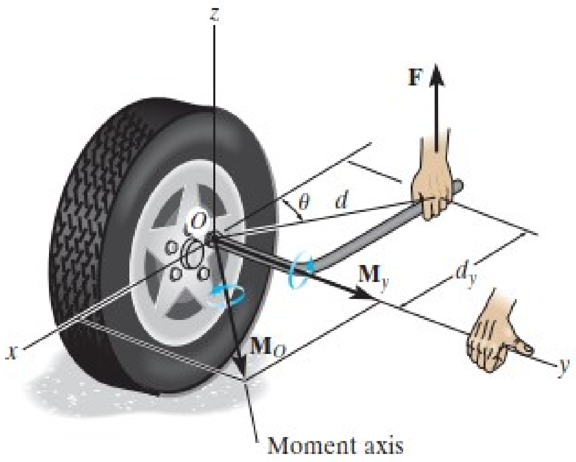
MOMENT OF A FORCE
ABOUT A SPECIFIED AXIS

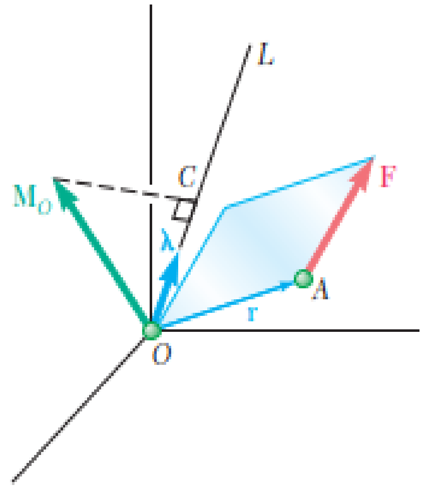


$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$





$$\mathbf{M}_a = M_a\mathbf{u}_a$$

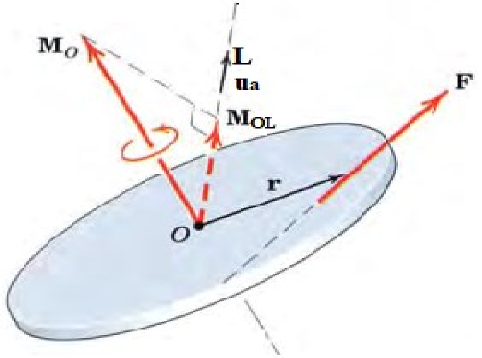
Triple Scalar Product

$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O = \boldsymbol{\lambda} \cdot (\mathbf{r} \times \mathbf{F})$$

$u_{a_x}, u_{a_y}, u_{a_z}$ represent the x, y, z components of the unit vector defining the direction of the a axis

r_x, r_y, r_z represent the x, y, z components of the position vector extended from *any point O* on the a axis to *any point A* on the line of action of the force

F_x, F_y, F_z represent the x, y, z components of the force vector.

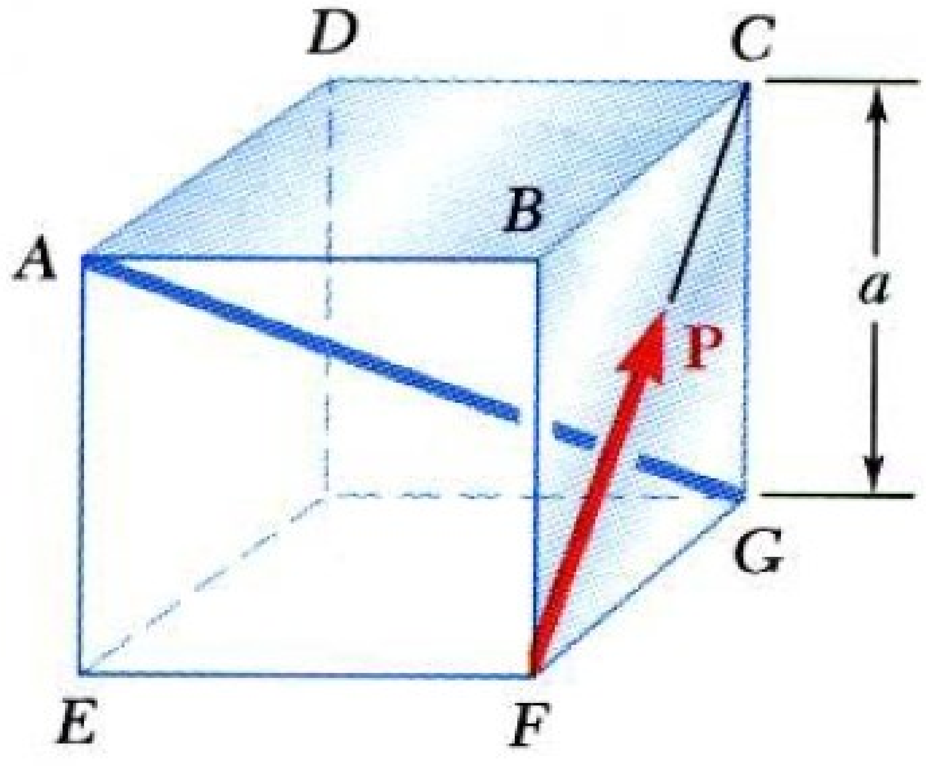


$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE
ABOUT A SPECIFIED AXIS
Example 4.9

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Question



A cube is acted on by a force ***P*** as shown. Determine the moment of ***P***

a) about *A*

b) about the edge *AB* and

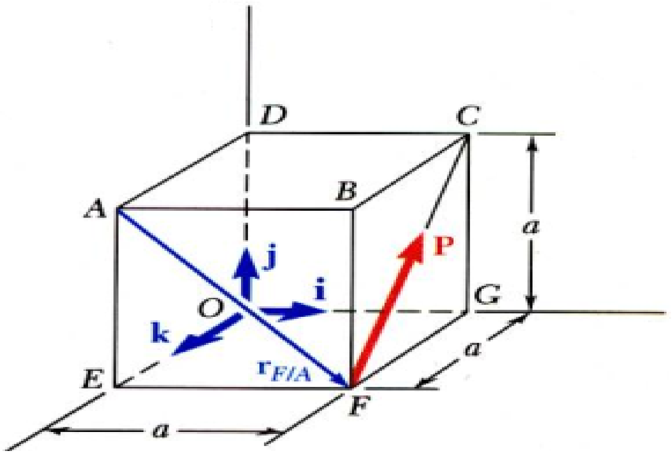
c) about the diagonal *AG* of the cube.

d) Determine the perpendicular distance between *AG* and *FC*.

MOMENT OF A FORCE
ABOUT A SPECIFIED AXIS
Example 4.9

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Solution



a. Moment about A. Choosing x, y, and z axes as shown, we resolve into rectangular components the force **P** and the vector $\mathbf{r}_{F/A} = \overrightarrow{AF}$ drawn from A to the point of application F of **P**.

$$\mathbf{r}_{F/A} = a\mathbf{i} - a\mathbf{j} = a(\mathbf{i} - \mathbf{j})$$
$$\mathbf{P} = (P/\sqrt{2})\mathbf{j} - (P/\sqrt{2})\mathbf{k} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k})$$

The moment of **P** about A is

$$\mathbf{M}_A = \mathbf{r}_{F/A} \times \mathbf{P} = a(\mathbf{i} - \mathbf{j}) \times (P/\sqrt{2})(\mathbf{j} - \mathbf{k})$$
$$\mathbf{M}_A = (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

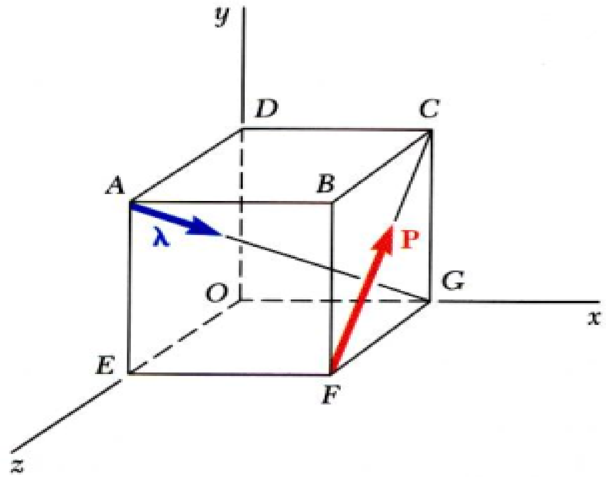
b. Moment about AB. Projecting $\mathbf{M}_A$ on AB, we write

$$M_{AB} = \mathbf{i} \cdot \mathbf{M}_A = \mathbf{i} \cdot (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$M_{AB} = aP/\sqrt{2}$$

MOMENT OF A FORCE
ABOUT A SPECIFIED AXIS
Example 4.9

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Solution



c. Moment about Diagonal AG. The moment of **P** about AG is obtained by projecting $\mathbf{M}_A$ on AG. Denoting by $\boldsymbol{\lambda}$ the unit vector along AG, we have

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AG}}{AG} = \frac{a\mathbf{i} - a\mathbf{j} - a\mathbf{k}}{a\sqrt{3}} = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})$$

$$M_{AG} = \boldsymbol{\lambda} \cdot \mathbf{M}_A = (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k}) \cdot (aP/\sqrt{2})(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$M_{AG} = (aP/\sqrt{6})(1 - 1 - 1) \qquad M_{AG} = -aP/\sqrt{6}$$

Alternative Method. The moment of **P** about AG can also be expressed in the form of a determinant:

$$M_{AG} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{F/A} & y_{F/A} & z_{F/A} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ a & -a & 0 \\ 0 & P/\sqrt{2} & -P/\sqrt{2} \end{vmatrix} = -aP/\sqrt{6}$$

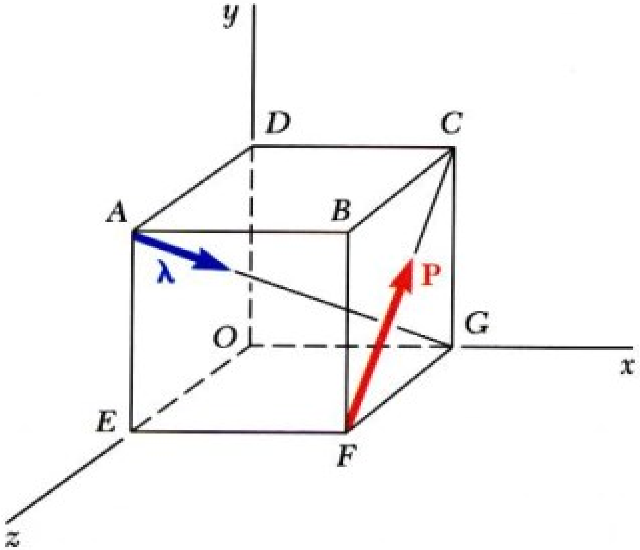
MOMENT OF A FORCE

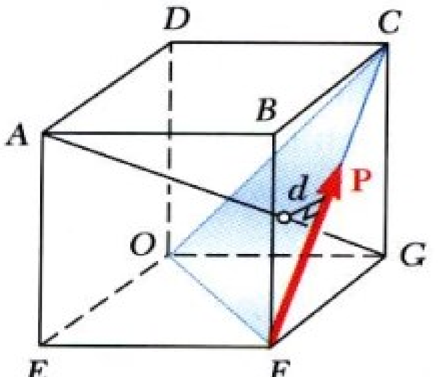
ABOUT A SPECIFIED AXIS

Example 4.9

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Solution





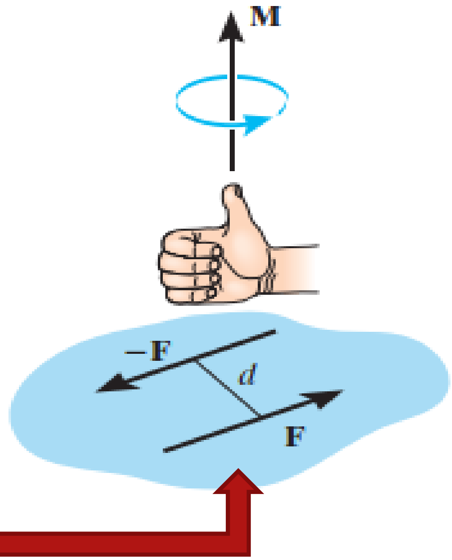
d. Perpendicular Distance between AG and FC. We first observe that **P** is perpendicular to the diagonal AG. This can be checked by forming the scalar product $\mathbf{P} \cdot \boldsymbol{\lambda}$ and verifying that it is zero:

$$\mathbf{P} \cdot \boldsymbol{\lambda} = (P/\sqrt{2})(\mathbf{j} - \mathbf{k}) \cdot (1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k}) = (P\sqrt{6})(0 - 1 + 1) = 0$$

The moment M_{AG} can then be expressed as $-Pd$, where d is the perpendicular distance from AG to FC. (The negative sign is used since the rotation imparted to the cube by **P** appears as clockwise to an observer at G.) Recalling the value found for M_{AG} in part c,

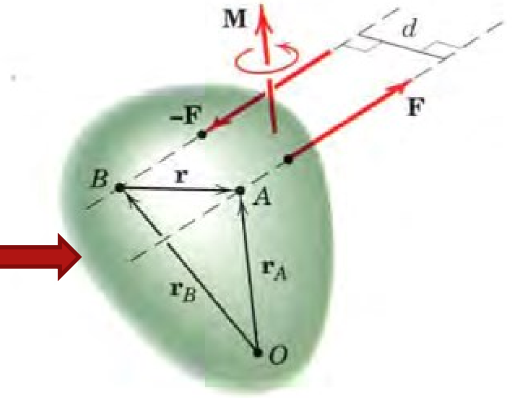
$$M_{AG} = -Pd = -aP/\sqrt{6} \qquad d = a/\sqrt{6}$$

MOMENT OF A COUPLE

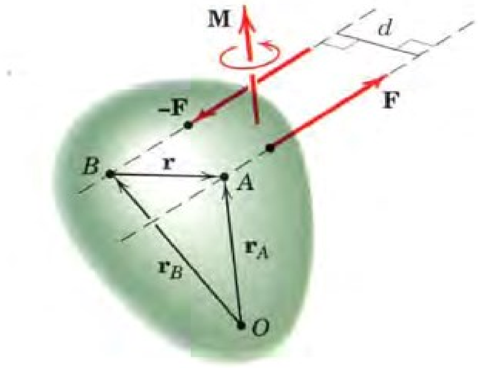


- A couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.
- Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction
- The couple moment determined about O is therefore:

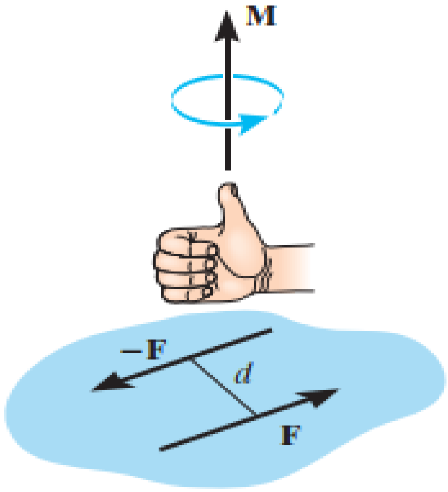
$M = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times -\mathbf{F} = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$

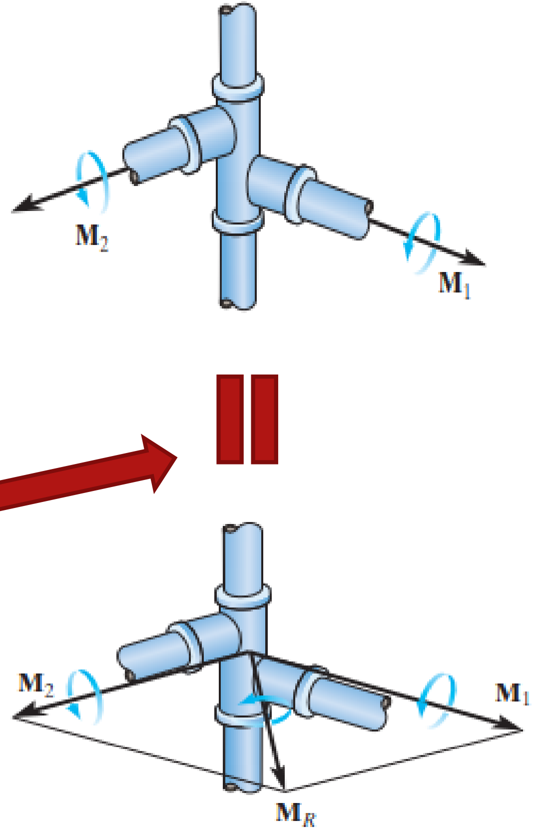


- However $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, so that $M = r \times \mathbf{F}$

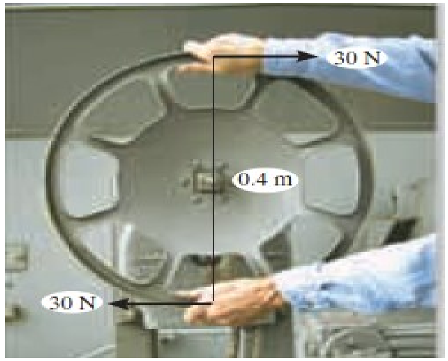


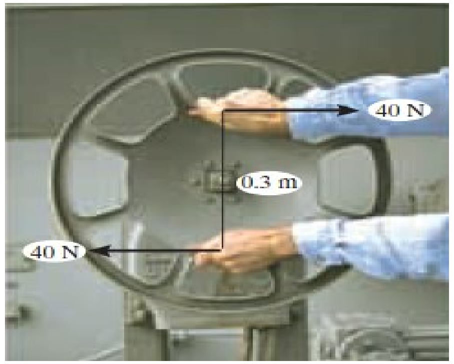
MOMENT OF A COUPLE





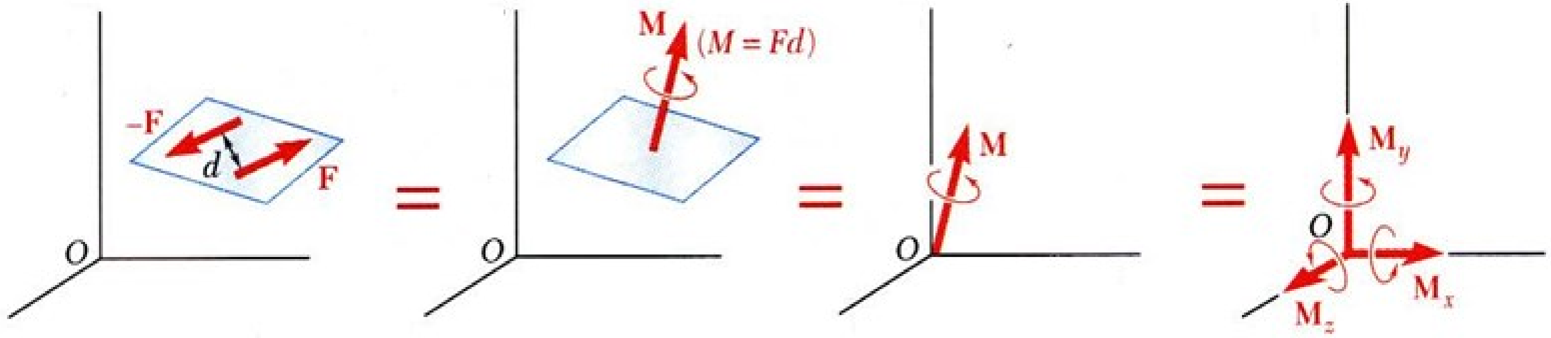
- Scalar formulation M = Fd
- Vector formulation M = r * F
- **Resultant Couple Moment.** Since couple moments are vectors, their resultant can be determined by vector a $\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$
- **Equivalent Couples.** If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent.





MOMENT OF A COUPLE
Representing Couple Moment as Vector

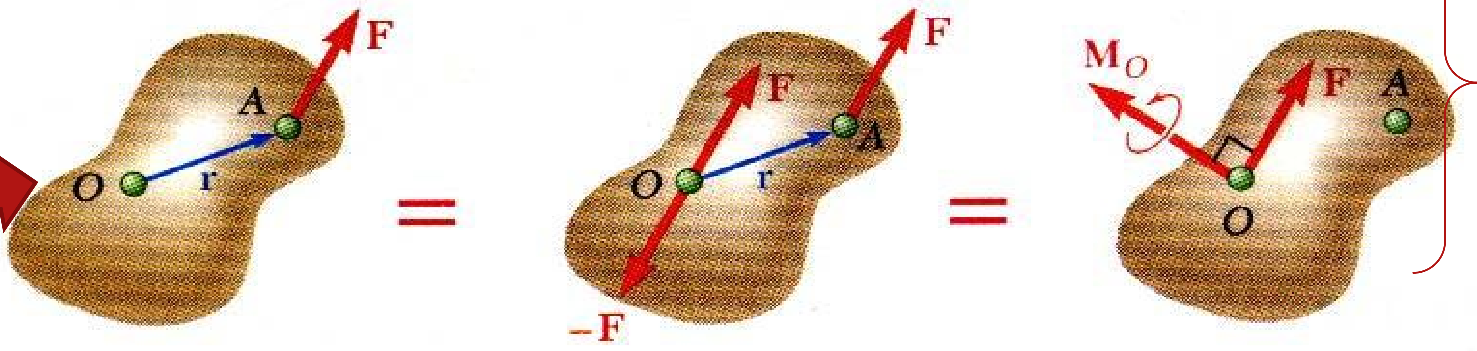
- A couple can be represented by a vector with ***magnitude*** and ***direction*** equal to the moment of the couple.
- *Couple moment vectors* obey the law of addition of vectors.
- Couple moment vectors are free vectors, i.e., the point of application is not significant
- Couple moment vectors may be resolved into component vectors x, y & z

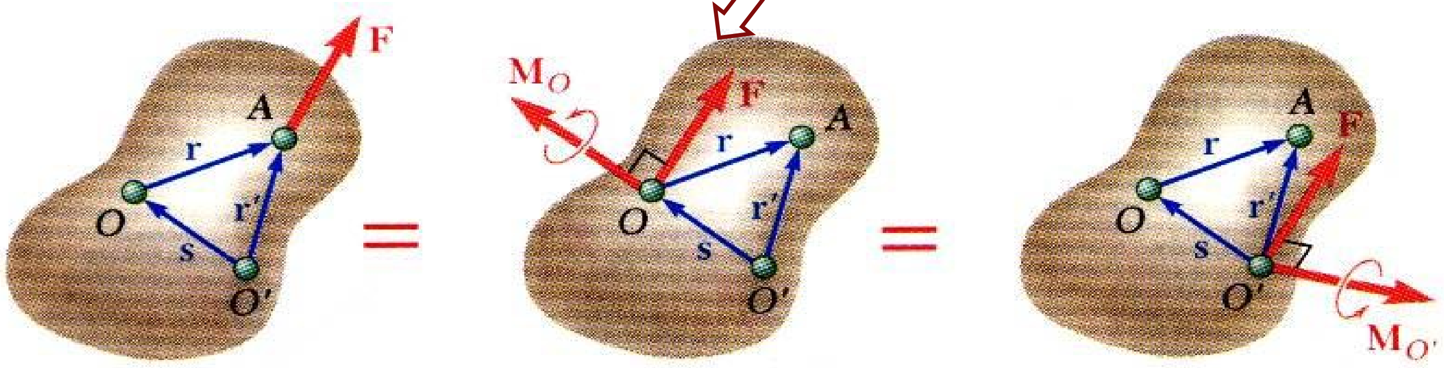


MOMENT OF A COUPLE

Resolution of a Force Into a Force at *O* and a Couple

- Force vector ***F*** can simply be moved to *O* by modifying its action on the body <u>if not along the line of action</u>
- Attaching equal and opposite force vectors at *O* produces no net effect on the body





- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a *force-couple system*

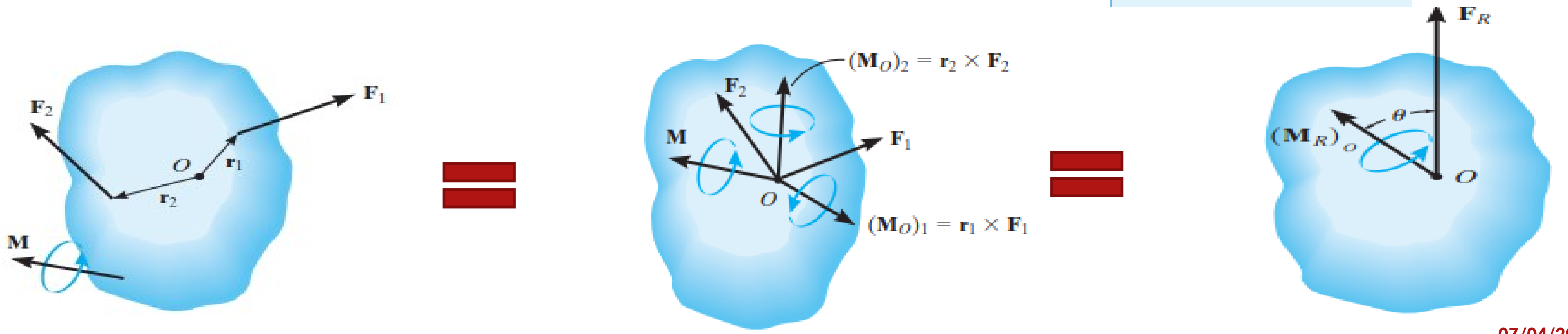
MOMENT OF A COUPLE

Resolution of a Force and a Couple System into Equivalent Resultant Force and Couple Moments

- A system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point O and a resultant couple moment by applying the following eqns:

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$



MOMENT OF A COUPLE

Example 4.10

EXAMPLE 4.10

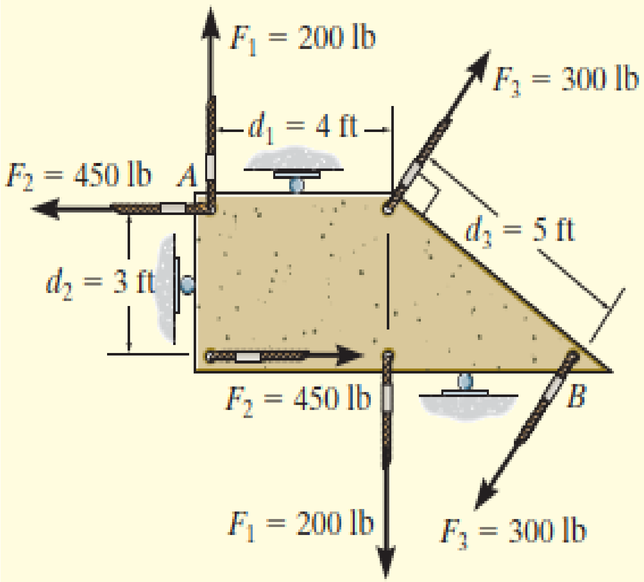


Fig. 4–30

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft + M_R = \Sigma M; \quad M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200\text{ lb})(4\text{ ft}) + (450\text{ lb})(3\text{ ft}) - (300\text{ lb})(5\text{ ft})$$

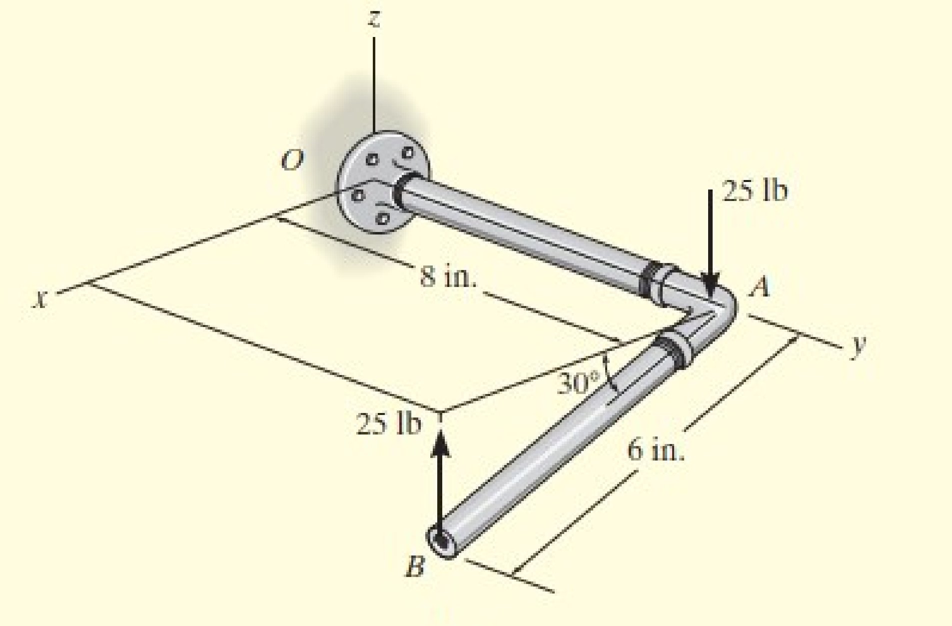
$$= -950\text{ lb}\cdot\text{ft} = 950\text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

Example 4.11

Question

- Determine the couple moment acting on the pipe shown in Fig. Segment AB is directed 30° below the x–y plane.



(a)

- **4 ways of solving this problem!**

Solution

4 different solutions!

The moment of the two couple forces can be found about *any point*. If point O is considered, Fig. 4–32b, we have

$$\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})$$
$$= (8\mathbf{j}) \times (-25\mathbf{k}) + (6 \cos 30°\mathbf{i} + 8\mathbf{j} - 6 \sin 30°\mathbf{k}) \times (25\mathbf{k})$$
$$= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}$$
$$= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.} \quad \textit{Ans.}$$

It is *easier* to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point A, Fig. 4–32c. In this case the moment of the force at A is zero, so that

$$\mathbf{M} = \mathbf{r}_{AB} \times (25\mathbf{k})$$
$$= (6 \cos 30°\mathbf{i} - 6 \sin 30°\mathbf{k}) \times (25\mathbf{k})$$
$$= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.} \quad \textit{Ans.}$$

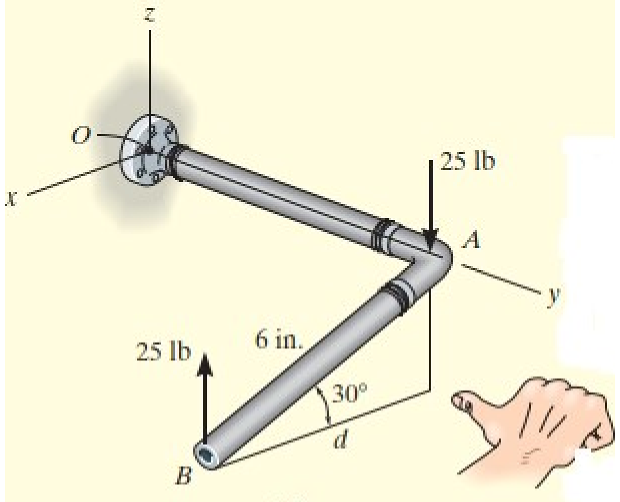
SOLUTION II *(SCALAR ANALYSIS)*

Although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation $M = Fd$. The perpendicular distance between the lines of action of the couple forces is $d = 6 \cos 30° = 5.196$ in., Fig. 4–32d. Hence, taking moments of the forces about either point A or point B yields

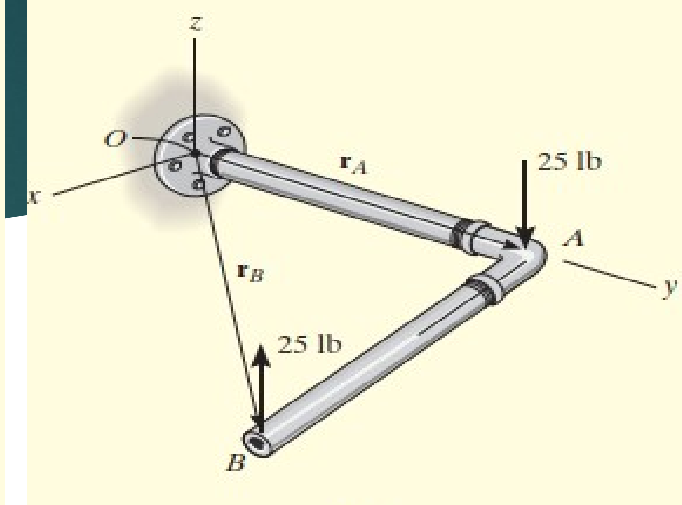
$$M = Fd = 25 \text{ lb } (5.196 \text{ in.}) = 129.9 \text{ lb} \cdot \text{in.}$$

Applying the right-hand rule, $\mathbf{M}$ acts in the $-\mathbf{j}$ direction. Thus,

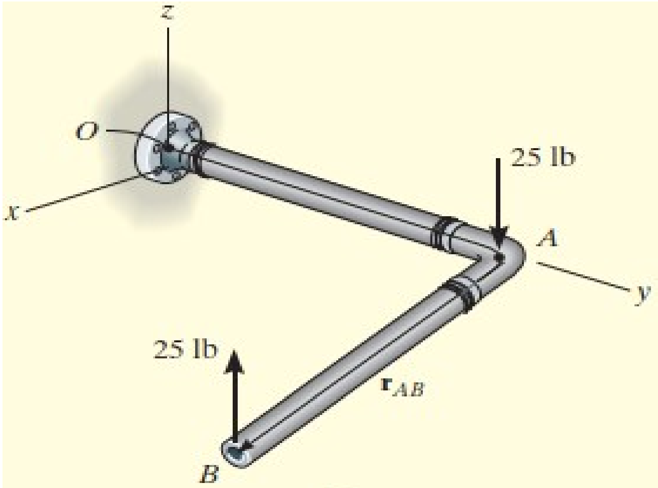
$$\mathbf{M} = \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.} \quad \textit{Ans.}$$



(d)



(b)



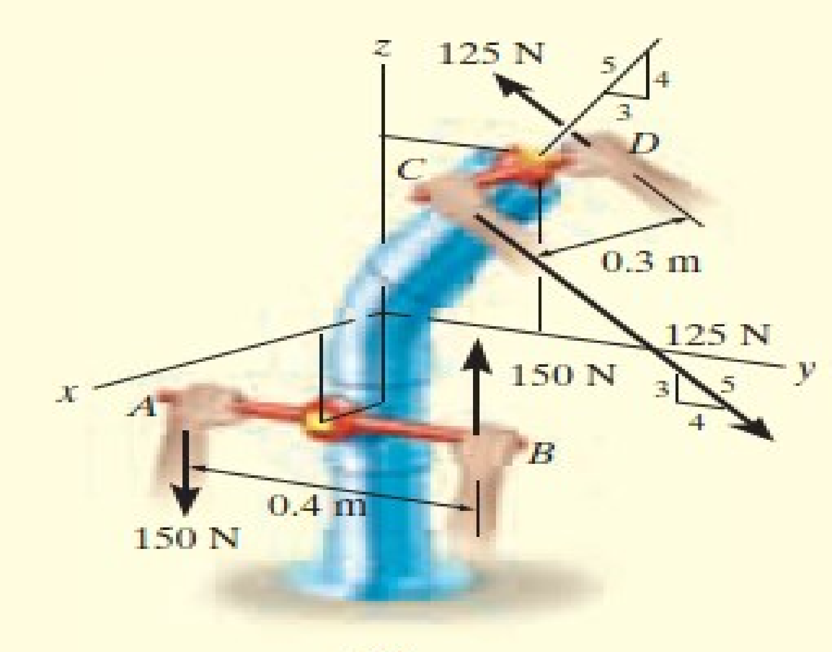
(c)

Example 4.12

Question

- Replace the two couples acting on the pipe column in Fig. by a resultant couple moment

- **2 ways of solving this problem!**



(a)

MOMENT OF A COUPLE

Solution

The couple moment $\mathbf{M}_1$, developed by the forces at A and B, can easily be determined from a scalar formulation.

$$M_1 = Fd = 150\text{ N}(0.4\text{ m}) = 60\text{ N}\cdot\text{m}$$

By the right-hand rule, $\mathbf{M}_1$ acts in the $+\mathbf{i}$ direction, Fig. 4–33b. Hence,

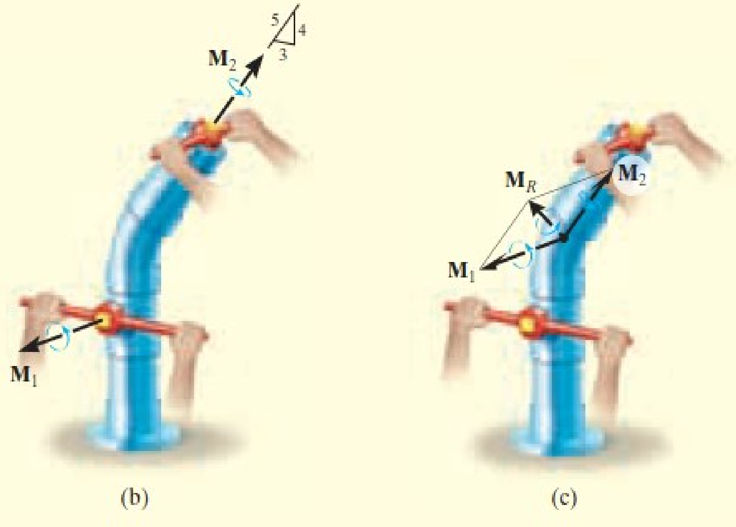
$$\mathbf{M}_1 = \{60\mathbf{i}\}\text{ N}\cdot\text{m}$$

Vector analysis will be used to determine $\mathbf{M}_2$, caused by forces at C and D. If moments are calculated about point D, Fig. 4–33a, $\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C$, then

$$\begin{aligned}\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C &= (0.3\mathbf{i}) \times \left[125\left(\tfrac{4}{5}\right)\mathbf{j} - 125\left(\tfrac{3}{5}\right)\mathbf{k}\right] \\ &= (0.3\mathbf{i}) \times [100\mathbf{j} - 75\mathbf{k}] = 30(\mathbf{i} \times \mathbf{j}) - 22.5(\mathbf{i} \times \mathbf{k}) \\ &= \{22.5\mathbf{j} + 30\mathbf{k}\}\text{ N}\cdot\text{m}\end{aligned}$$

Since $\mathbf{M}_1$ and $\mathbf{M}_2$ are free vectors, they may be moved to some arbitrary point and added vectorially, Fig. 4–33c. The resultant couple moment becomes

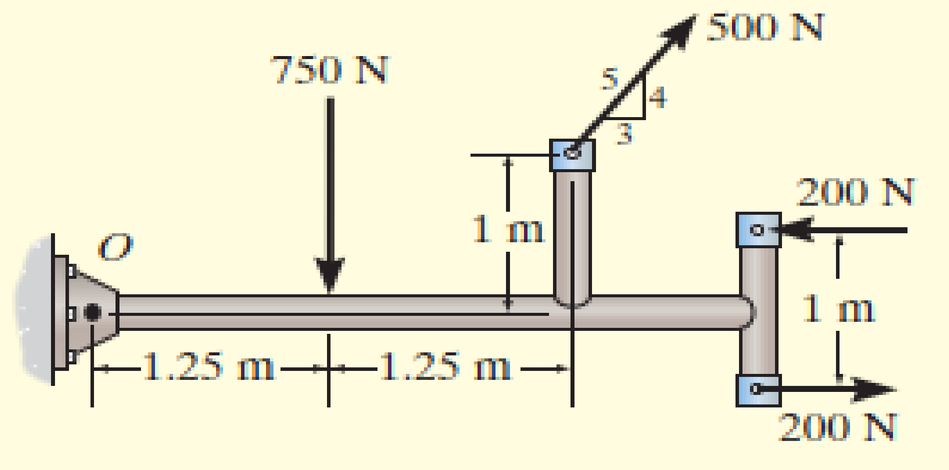
$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = \{60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$



Example 4.13

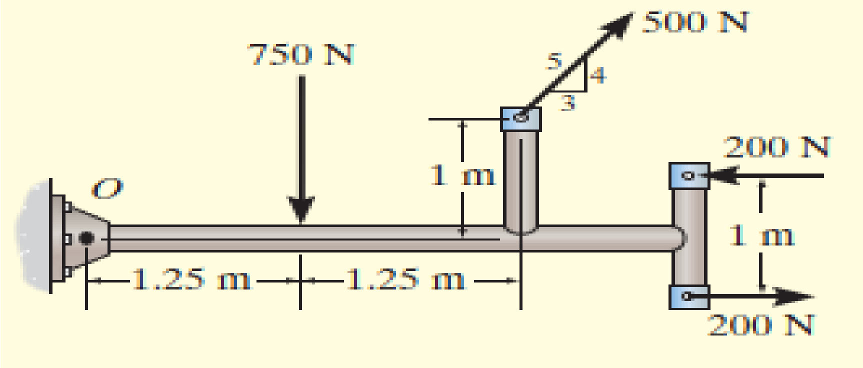
Question

- Replace the force and couple system acting on the member in Fig. 4–38a by an equivalent resultant force and couple moment acting at point O.
- *In Cartesian notation or as a vector*
- **2 ways of solving this problem!**



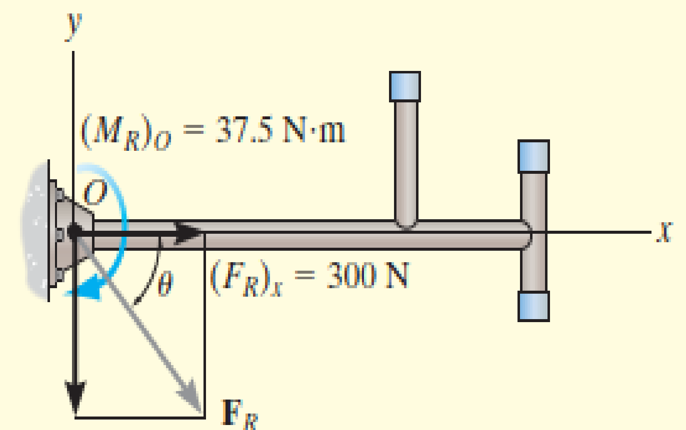
(a)

MOMENT OF A COUPLE



(a)

Example 4.13



(b)

Solution

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its x and y components, thus,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x;\quad (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38*a*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

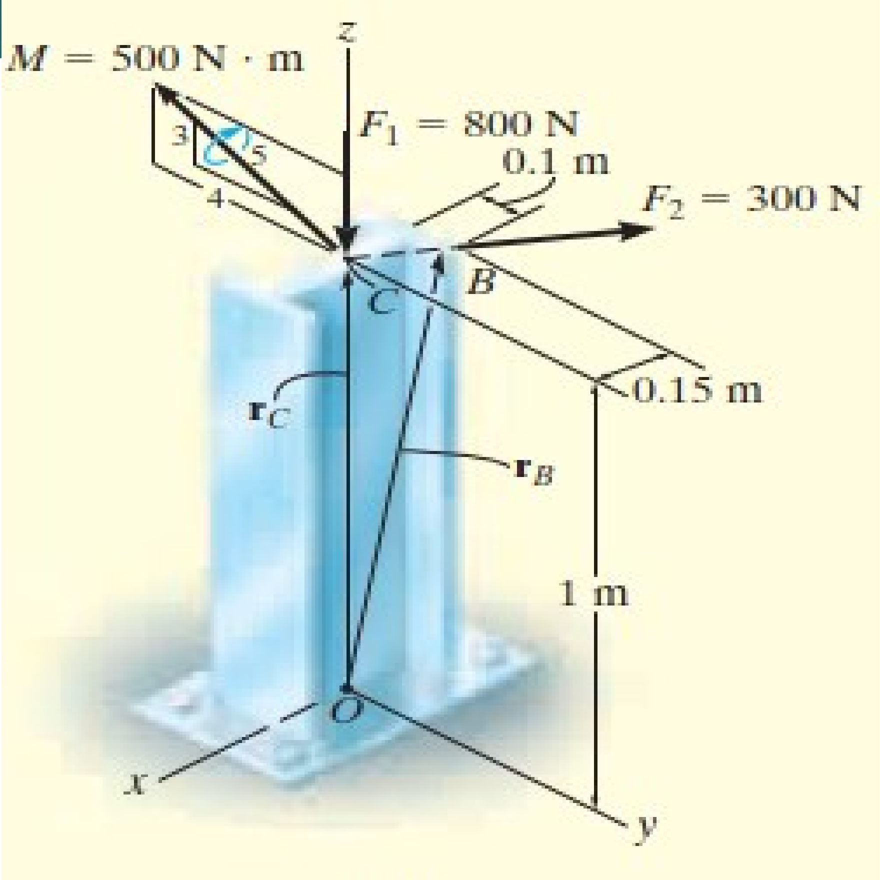
This clockwise moment is shown in Fig. 4–38*b*.

MOMENT OF A COUPLE

Example 4.14

Question for Critical Visual & Analysis

- The structural member is subjected to a couple moment M and forces F1 and F2 in Fig. 4–39a. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.



(a)

MOMENT OF A COUPLE

Example 4.14

Solution

The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have

$$\mathbf{F}_1 = \{-800\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = (300\text{ N})\mathbf{u}_{CB}$$

$$= (300\text{ N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

$$= 300\text{ N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\}\text{ m}}{\sqrt{(-0.15\text{ m})^2 + (0.1\text{ m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\}\text{ N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\}\text{ N}\cdot\text{m}$$

Force Summation.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800\mathbf{k} - 249.6\mathbf{i} + 166.4\mathbf{j}$$

$$= \{-250\mathbf{i} + 166\mathbf{j} - 800\mathbf{k}\}\text{ N}$$

Moment Summation.

$$(\mathbf{M}_R)_o = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

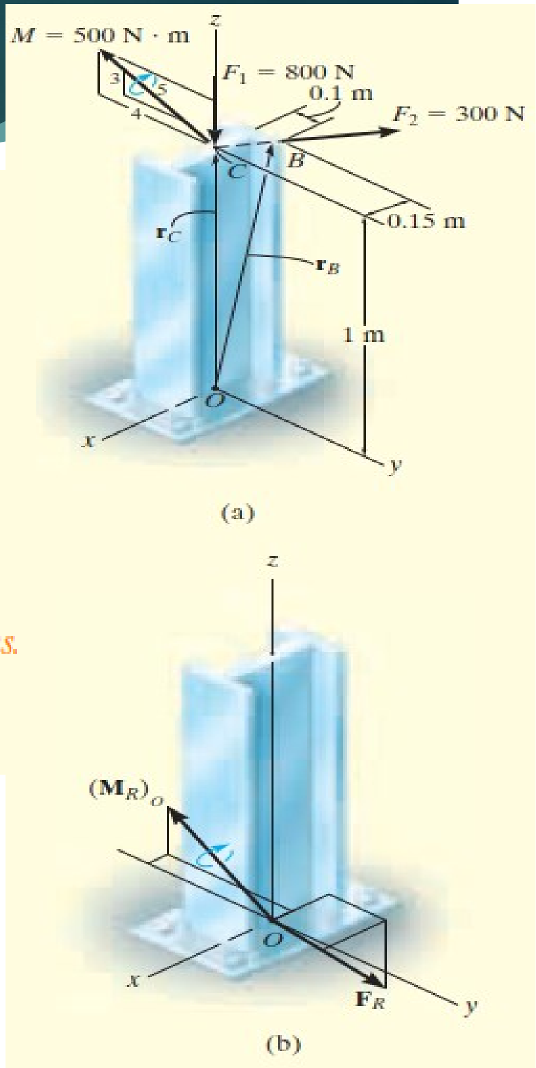
$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

$$= (-400\mathbf{j} + 300\mathbf{k}) + (0) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–39*b*.



HOME WORK EXERCISE

4-4, 4-12, 4-29, 4-34, 4-36, 4-38, 4-43, 4-46, 4-48, 4-50, 4-51, 4-54, 4-56, 4-58, 4-66, 4-69, 4-72, 4-80, 4-83, 4-96, 4-97, 4-102, 4-108, 4-115 & 4-134.