



Lecture A6

CEE 2219: STATICS AND INTRODUCTION TO MECHANICS OF MATERIALS

STRUCTURAL ANALYSIS





CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

Lecture A6

- ❖ **STRUCTURAL ANALYSIS**
- ✓ **PLANE TRUSSES (2D)**
- ✓ **SPACE TRUSSES (3D)**



LECTURE OBJECTIVES



- ❖ To show how to determine the forces in the members of a truss using the method of joints and the method of sections.





CHAPTER INTRODUCTION

- Structure refers a system of connected parts used to support loads, such as buildings, bridges, dams, towers etc.
- Structural analysis is the prediction (design) of the performance of a given structure under prescribed loads and/or other external effects such as support movements and temperature changes
- *Structural Engineers*

- Analyze and design new structures
- Investigate the capacity and serviceability of existing structures
- Develop retrofit methods for existing structures with inadequate capacity
- Forensic investigations
- Research and development





CHAPTER INTRODUCTION





CHAPTER INTRODUCTION

- In the previous lecture we discussed the equilibrium of a rigid body i.e a system of connected members treated as a single rigid body.
- We first drew a FBD of the body showing all forces external to the isolated body, and then we applied the force and moment equations of equilibrium.
- In lecture A6 & A7 we will focus on the determination of the forces internal to a structure - that is, forces of action and reaction between the connected members.
- An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.





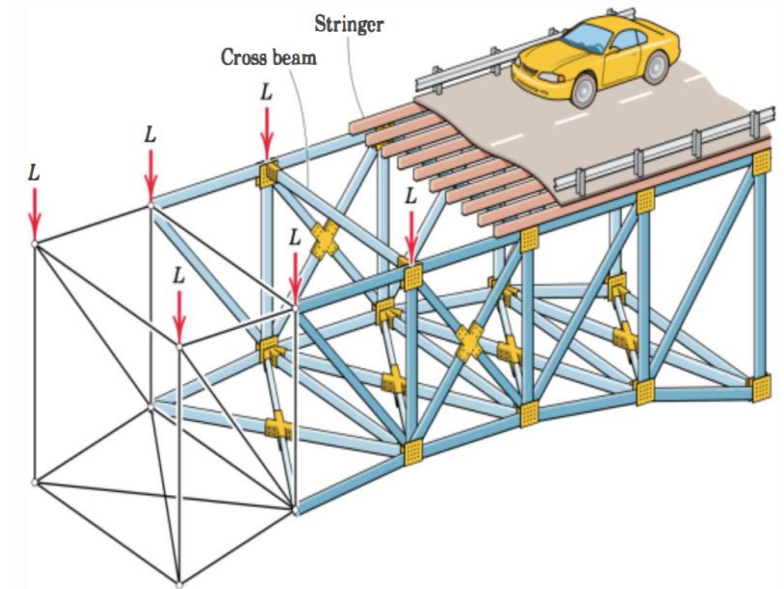
CHAPTER INTRODUCTION

- To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate FBD of individual members or combinations of members
- This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.
- In this chapter we will analyze the internal forces acting in several types of structures (trusses, frames, and machines)
- In this treatment we consider only **statically determinate structures**, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration

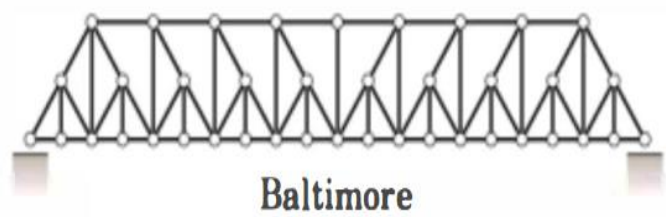
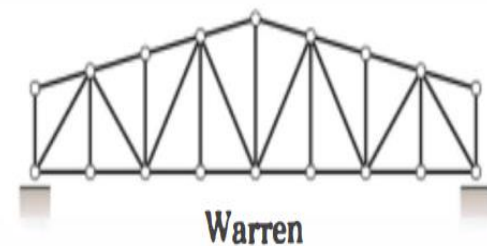
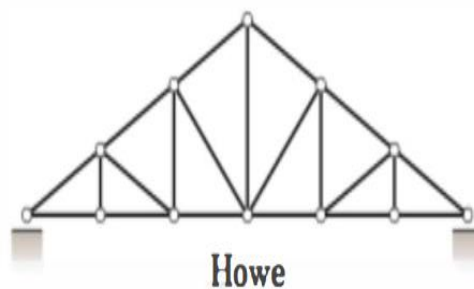
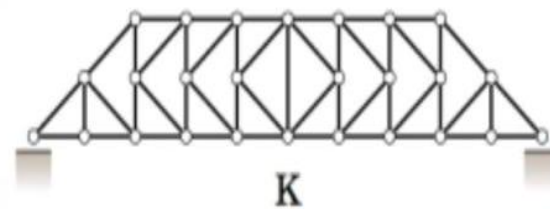
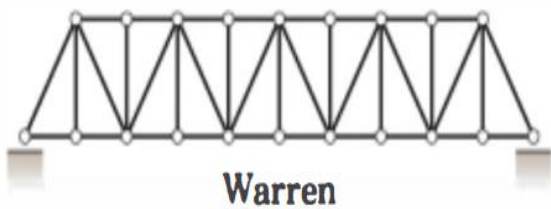
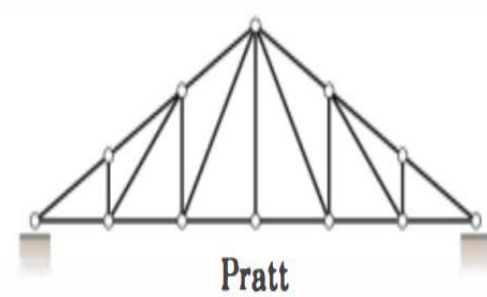
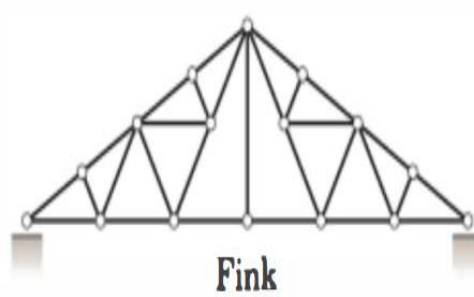
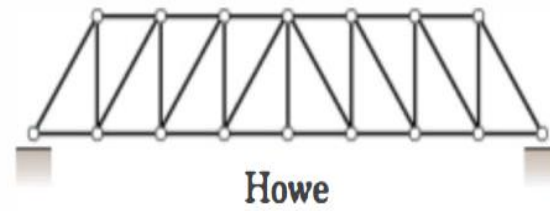
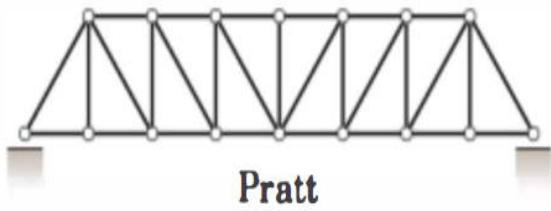


CHAPTER INTRODUCTION

- The analysis of trusses, frames and machines, and beams under concentrated loads constitutes a straightforward application of the material developed in the previous lectures. **However, only trusses will be discussed in this lecture**
- The basic procedure developed in Chapter 3 and Chapter 5 for isolating a body by constructing a correct free-body diagram is essential for the analysis of statically determinate structures.

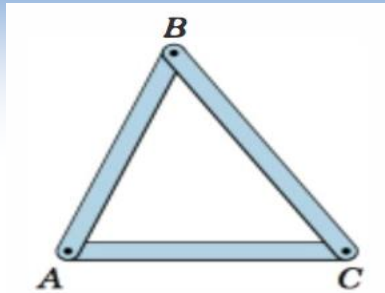


CHAPTER INTRODUCTION

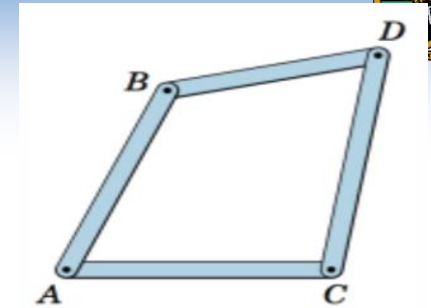


Commonly Used Bridge Trusses

Commonly Used Roof Trusses



PLANE TRUSSES



- A framework composed of members joined at their ends to form a rigid structure is called a truss. Or a truss is a structure composed of slender members joined together at their end points
- Bridges, roof supports, derricks, and other such structures common examples of trusses.
- Structural members commonly used are; beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins.
- When the members of the truss lie essentially in a single plane, it is called a plane truss.



PLANE TRUSSES

- The members commonly used in construction consist of wooden struts or metal bars.
- The truss shown in Fig. 6–1a is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss at the joints by means of a series of purlins.
- Since this loading acts in the same plane as the truss, Fig. 6–1b, the analysis of the forces developed in the truss members will be **two-dimensional**.

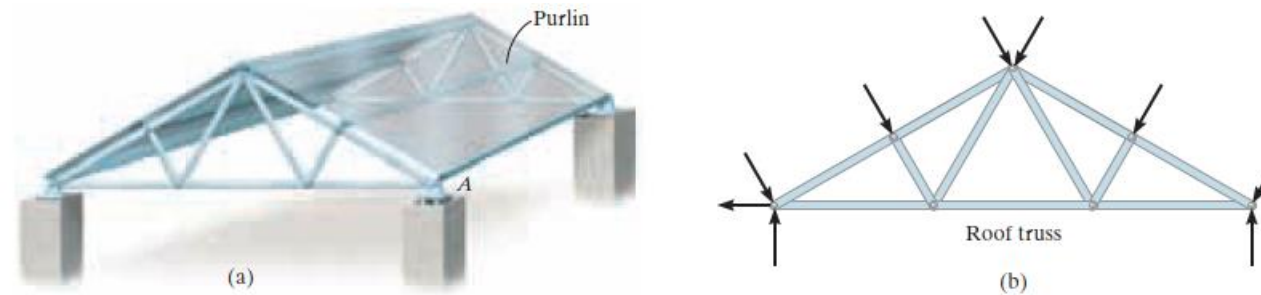
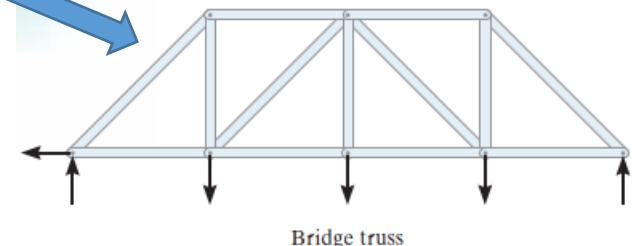
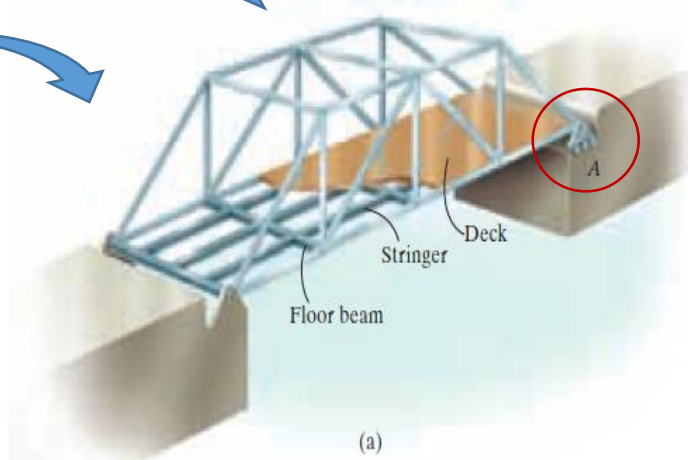
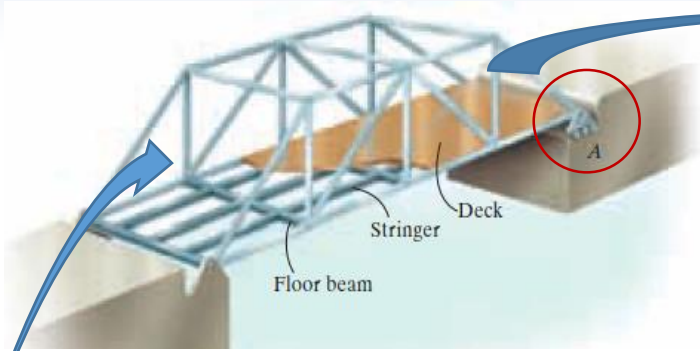


Fig. 6–1



PLANE TRUSSES



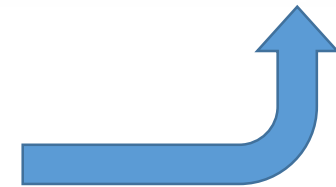
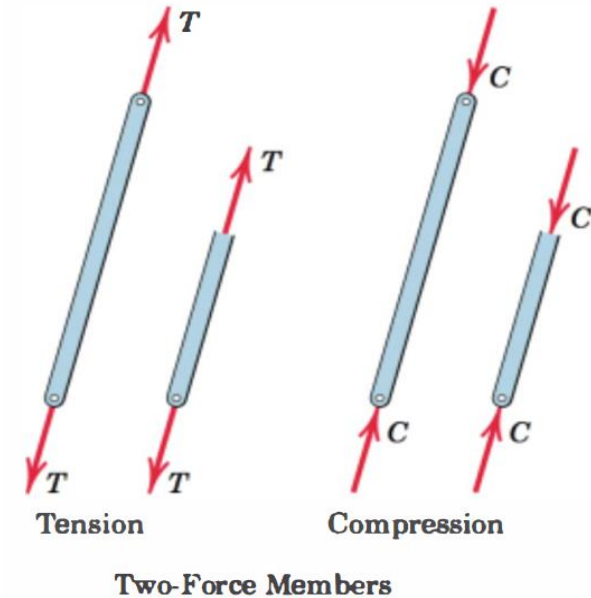
- In the case of a bridge, as shown, the load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. coplanar
- When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, shown above
- This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.



PLANE TRUSSES

Assumptions for Design

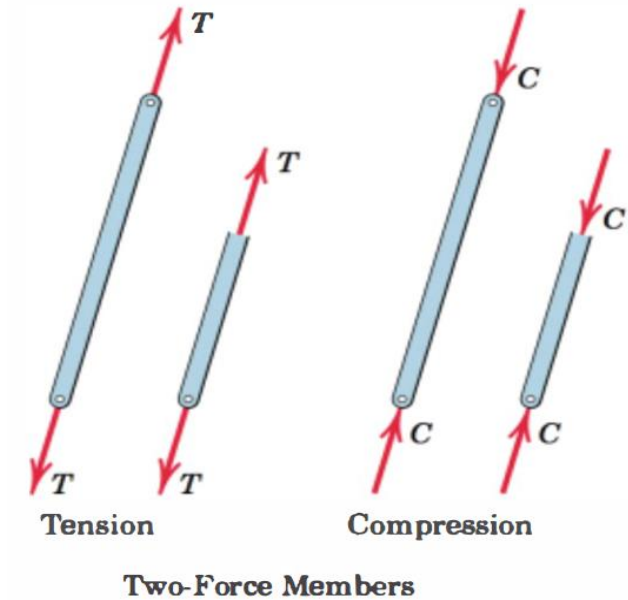
- To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading.
- To do this we will make two important assumptions;
 - ✓ The members are assumed to be connected only by frictionless pins.
 - ✓ The loads must be applied at the joints
- Because of these two assumptions, each truss member will act as a two-force member, and therefore the force acting at each end of the member will be directed along the axis of the member.



PLANE TRUSSES

Assumptions for Design

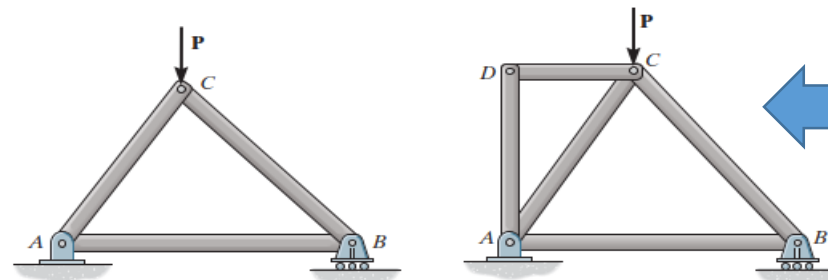
- If the force tends to elongate the member, it is a tensile force (T), whereas if it tends to shorten the member, it is a compressive force (C)
- In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive.
- Often, compression members must be made thicker than tension members because of the buckling or column effect that occurs when a member is in compression.



PLANE TRUSSES

Simple Trusses

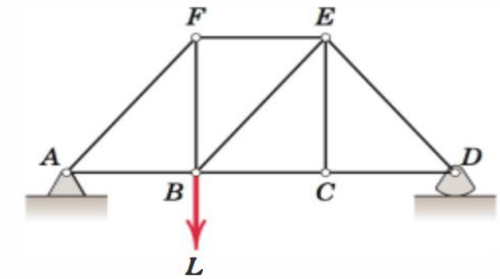
- If three members are pin connected at their ends, they form a triangular truss that will be rigid, as shown below.
- Attaching two more members and connecting these members to a new joint D forms a larger truss
- This procedure can be repeated as many times as desired to form an even larger truss.
- If a truss can be constructed by expanding the basic triangular truss in this way, it is called a **simple truss**



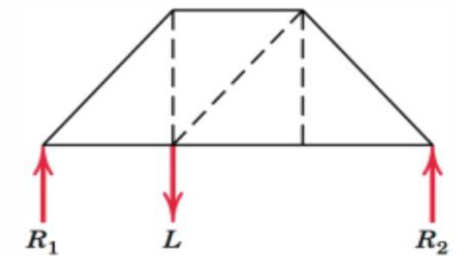
PLANE TRUSSES

Simple Trusses

- Two methods for the force analysis of simple trusses will be discussed.
- The free-body diagram of the truss as a whole is shown in Fig. 4/6b
- The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole
- Then the force analysis of the remainder of the truss is performed



(a)



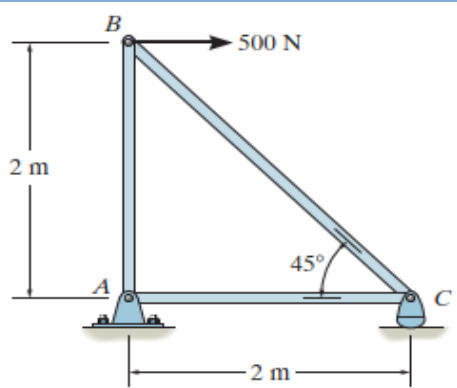
(b)

Figure 4/6

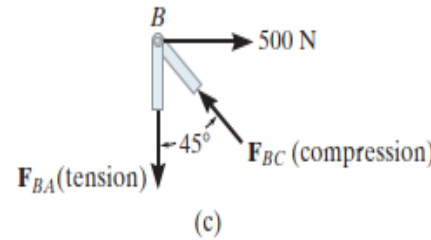
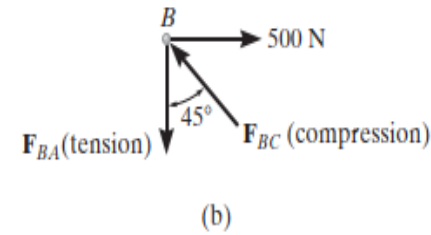


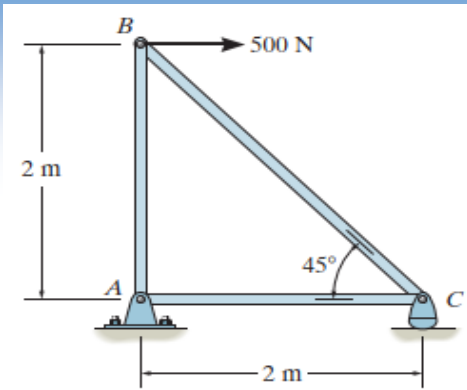
SIMPLE TRUSSES

Method of Joints



- In order to analyze or design a truss, it is necessary to determine the force in each of its members by using the method of joints.
- This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium.
- Hence if the FBD of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint.
- Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a force system that is coplanar and concurrent.
- As a result, only $\sum F_x = 0$ and $\sum F_y = 0$ need to be satisfied for equilibrium.

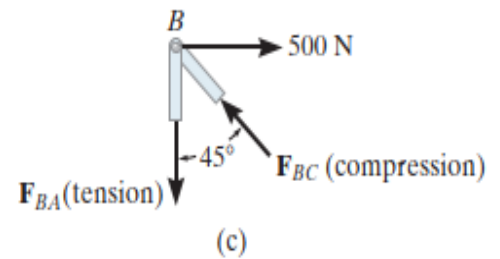
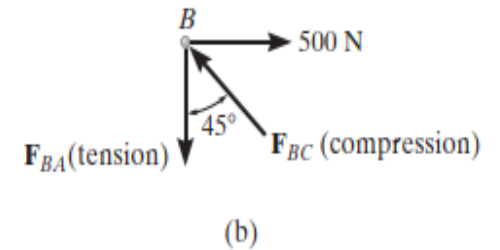


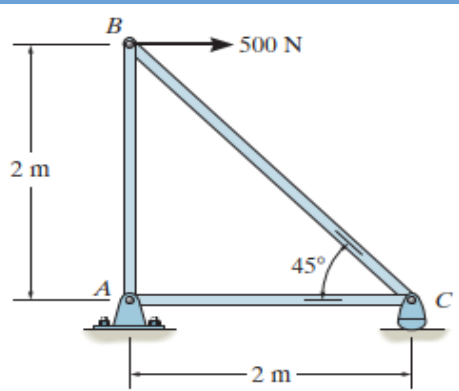


SIMPLE TRUSSES

Method of Joints

- When using the method of joints, always start at a joint having at least one known force and at most two unknown forces
- In this way, application of $\sum F_x = 0$ and $\sum F_y = 0$ yields two algebraic equations which can be solved for the two unknowns.
- When applying these equations, the correct sense of an unknown member force can be determined using one of the two possible methods:
 - 1) By inspection or 2) By assumption

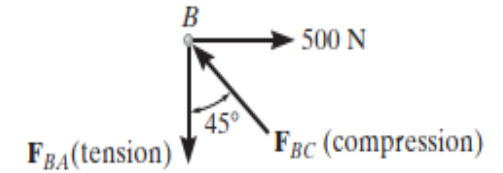




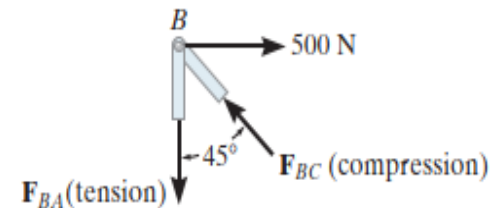
SIMPLE TRUSSES

Method of Joints

- 1) By inspection.....but in more complicated cases, the sense of an unknown member force can be assumed; then, after applying the equilibrium equations, the assumed sense can be verified from the numerical results.



(b)



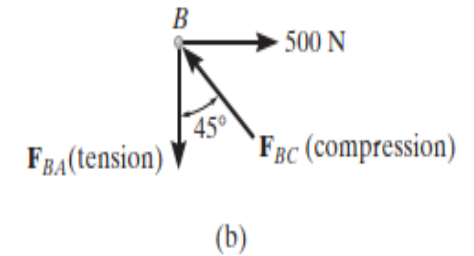
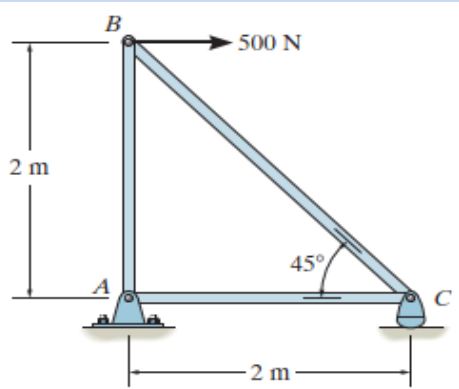
(c)

A positive answer indicates that the sense is correct, whereas a negative answer indicates that the sense shown on the FBD must be reversed.



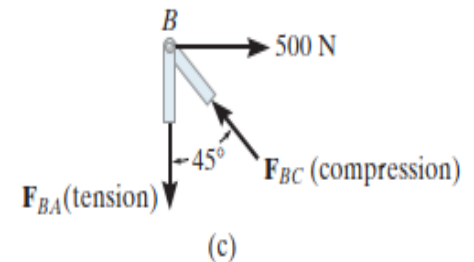
SIMPLE TRUSSES

Method of Joints



2) by assuming the unknown member forces acting on the joint's FBD to be in tension

If this is done, then numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression.



Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint FBD

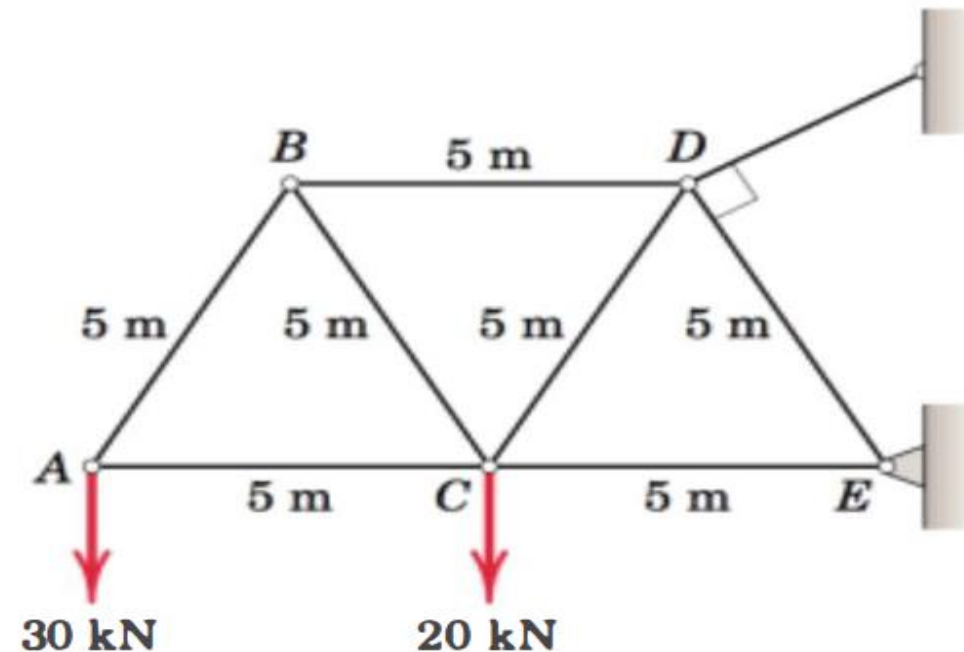


SIMPLE TRUSSES

Example 6.1

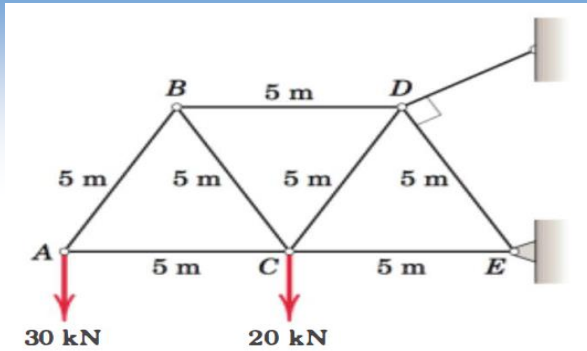
Question

- Compute the force in each member of the loaded cantilever truss by the method of joints.



SIMPLE TRUSSES

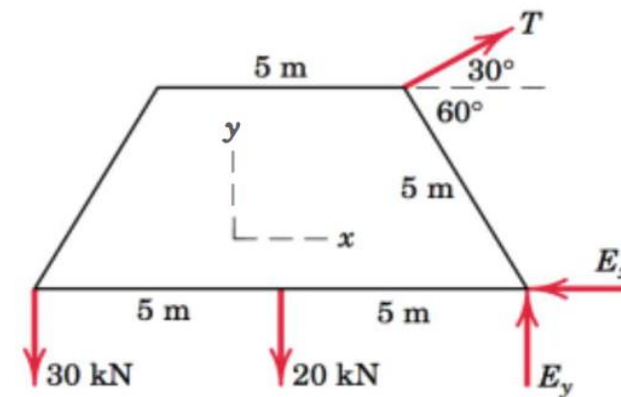
Example 6.1



Solution

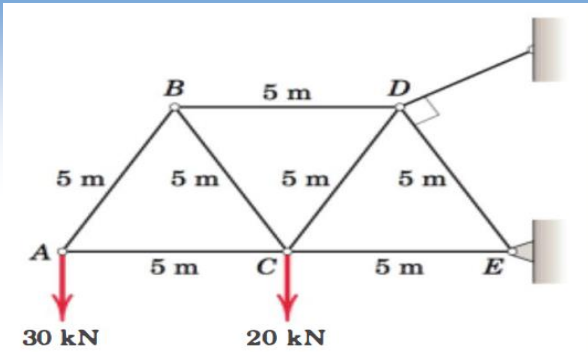
- If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the FBD of the truss as a whole. The equations of equilibrium give;

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80 \text{ kN}$
$[\Sigma F_x = 0]$	$80 \cos 30^\circ - E_x = 0$	$E_x = 69.3 \text{ kN}$
$[\Sigma F_y = 0]$	$80 \sin 30^\circ + E_y - 20 - 30 = 0$	$E_y = 10 \text{ kN}$



SIMPLE TRUSSES

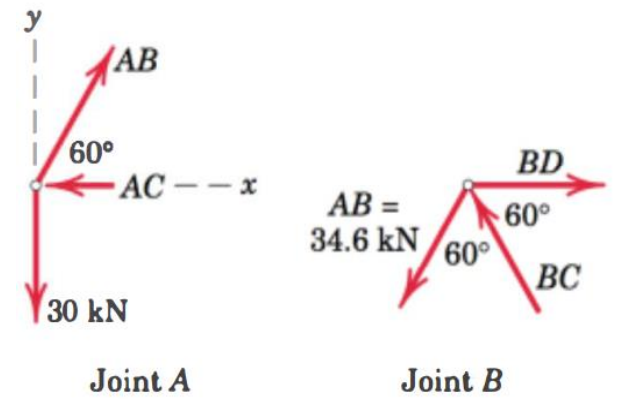
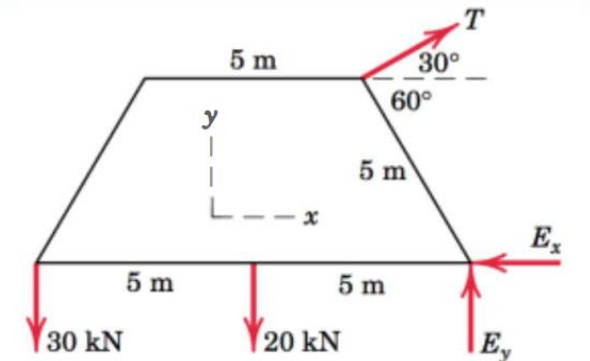
Example 6.1



Solution

- Next we draw FBD showing the forces acting on each of the connecting pins.
- The correctness of the assigned directions of the forces is verified when each joint is considered in sequence.
- There should be no question about the correct direction of the forces on joint A

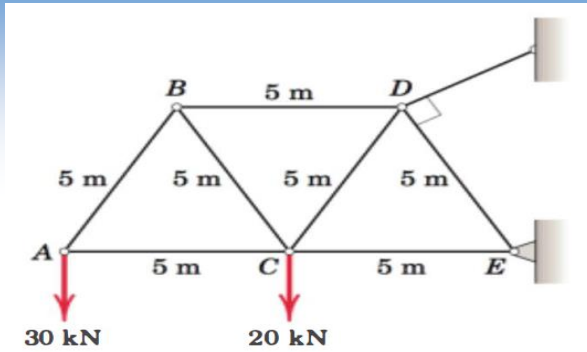
$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6 \text{ kN } T$	<i>Ans.</i>
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32 \text{ kN } C$	<i>Ans.</i>



- Where T stands for tension and C stands for compression

SIMPLE TRUSSES

Example 6.1



Solution

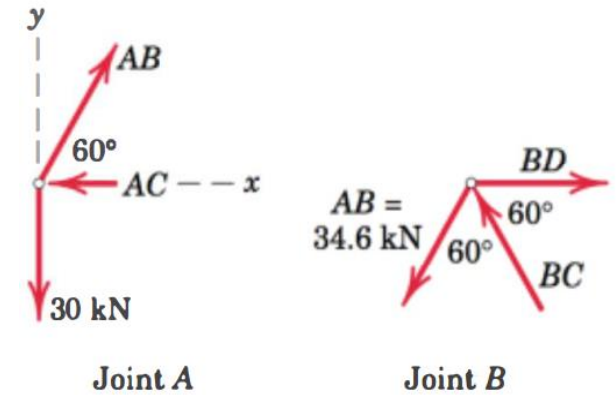
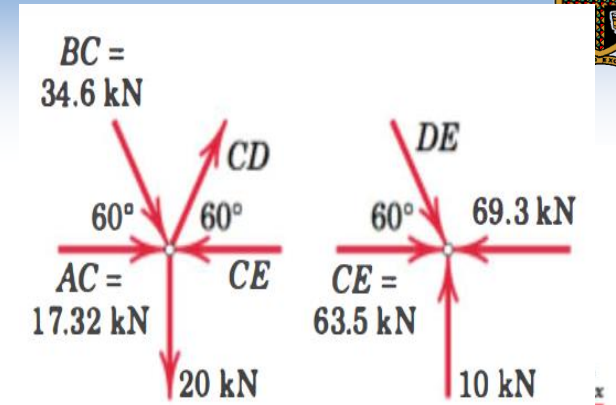
- Joint B must be analyzed next, since there are more than two unknown forces on joint C.

- The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$[\Sigma F_y = 0]$	$0.866BC - 0.866(34.6) = 0$	$BC = 34.6 \text{ kN } C$	<i>Ans.</i>
$[\Sigma F_x = 0]$	$BD - 2(0.5)(34.6) = 0$	$BD = 34.6 \text{ kN } T$	<i>Ans.</i>

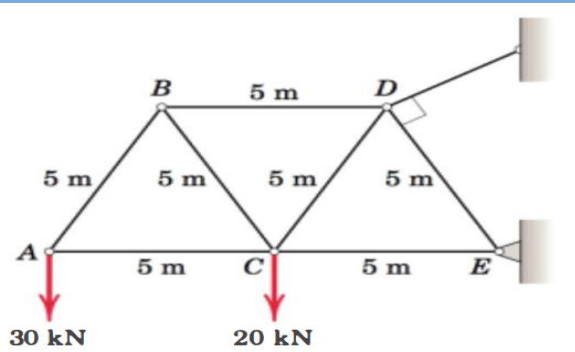
- Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0]$	$0.866CD - 0.866(34.6) - 20 = 0$	
	$CD = 57.7 \text{ kN } T$	<i>Ans.</i>



SIMPLE TRUSSES

Example 6.1



Solution

$$[\Sigma F_x = 0] \quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$$

$$CE = 63.5 \text{ kN C}$$

Ans.

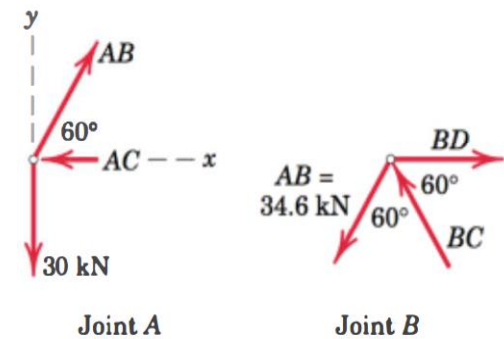
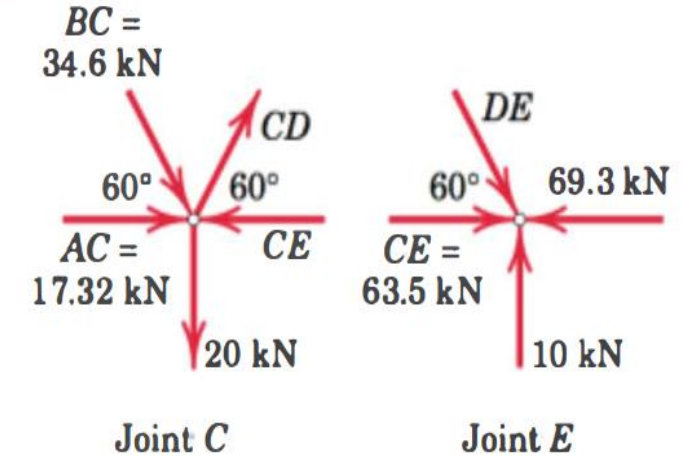
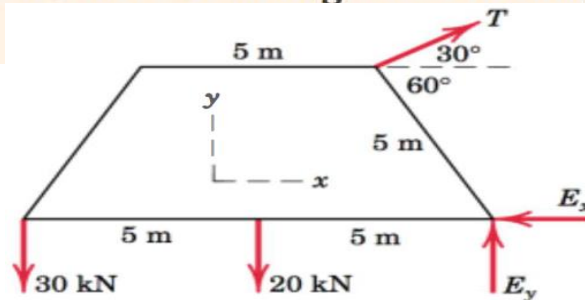
Finally, from joint E there results

$$[\Sigma F_y = 0] \quad 0.866DE = 10 \quad DE = 11.55 \text{ kN C}$$

Ans.

and the equation $\Sigma F_x = 0$ checks.

Note that the weights of the truss members have been neglected in comparison with the external loads.

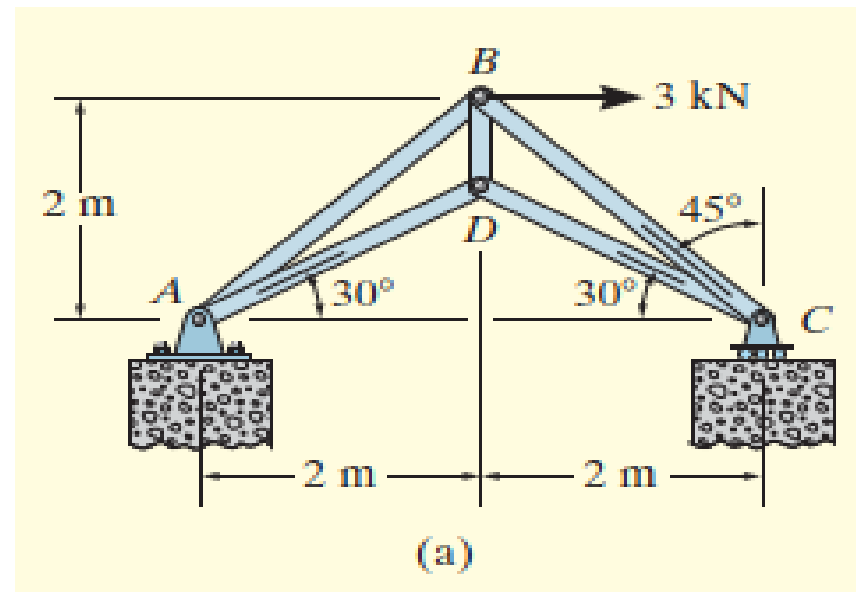


SIMPLE TRUSSES

Example 6.2

Question

- Determine the forces acting in all the members of the truss shown in Fig. below.



SIMPLE TRUSSES

Example 6.2

Solution

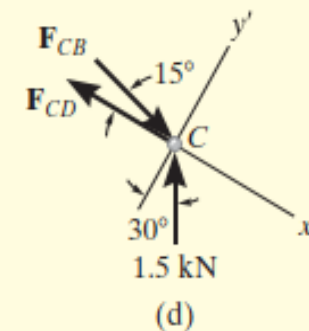
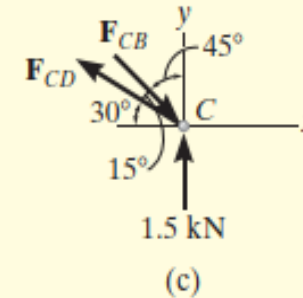
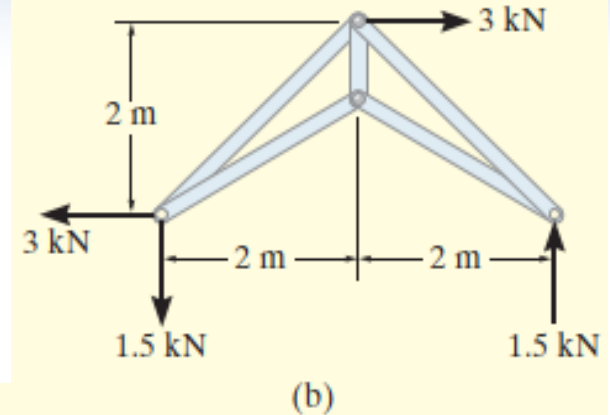
SOLUTION

By inspection, there are more than two unknowns at each joint. Consequently, the support reactions on the truss must first be determined. Show that they have been correctly calculated on the free-body diagram in Fig. 6–9*b*. We can now begin the analysis at joint *C*. Why?

Joint C. From the free-body diagram, Fig. 6–9*c*,

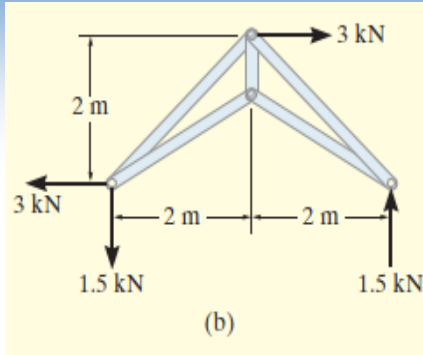
$$\begin{aligned} \pm \rightarrow \Sigma F_x &= 0; & -F_{CD} \cos 30^\circ + F_{CB} \sin 45^\circ &= 0 \\ + \uparrow \Sigma F_y &= 0; & 1.5 \text{ kN} + F_{CD} \sin 30^\circ - F_{CB} \cos 45^\circ &= 0 \end{aligned}$$

These two equations must be solved *simultaneously* for each of the two unknowns. Note, however, that a *direct solution* for one of the unknown forces may be obtained by applying a force summation along an axis that is *perpendicular* to the direction of the other unknown force. For example, summing forces along the y' axis, which is perpendicular to the direction of F_{CD} , Fig. 6–9*d*, yields a *direct solution* for F_{CB} .



SIMPLE TRUSSES

Example 6.2



Solution

$$+\nearrow \Sigma F_{y'} = 0; \quad 1.5 \cos 30^\circ \text{ kN} - F_{CB} \sin 15^\circ = 0$$

$$F_{CB} = 5.019 \text{ kN} = 5.02 \text{ kN (C)} \quad \text{Ans.}$$

Then,

$$+\searrow \Sigma F_{x'} = 0;$$

$$-F_{CD} + 5.019 \cos 15^\circ - 1.5 \sin 30^\circ = 0; \quad F_{CD} = 4.10 \text{ kN (T)} \quad \text{Ans.}$$

Joint D. We can now proceed to analyze joint D. The free-body diagram is shown in Fig. 6-9e.

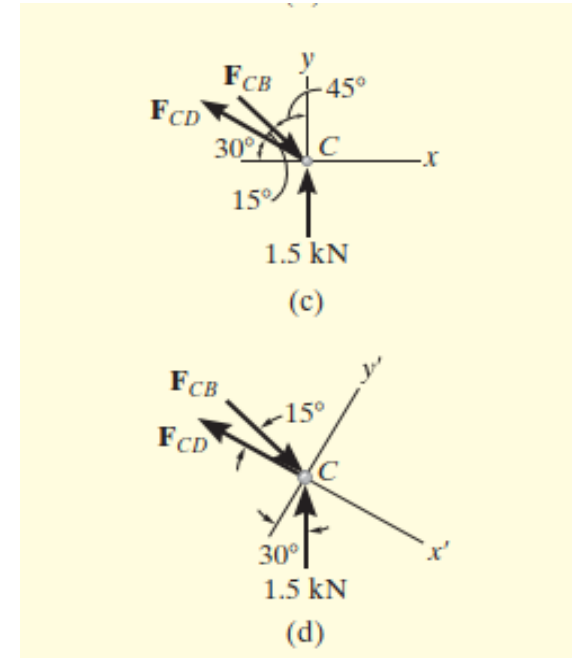
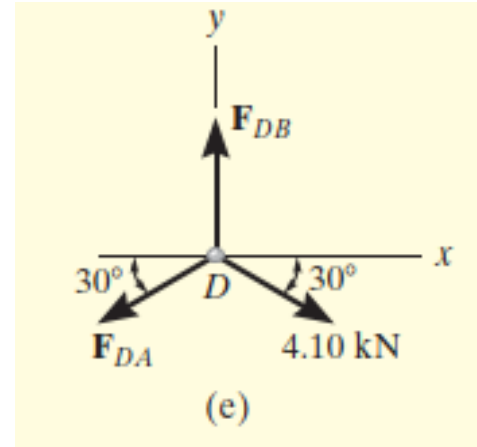
$$\pm \Sigma F_x = 0; \quad -F_{DA} \cos 30^\circ + 4.10 \cos 30^\circ \text{ kN} = 0$$

$$F_{DA} = 4.10 \text{ kN (T)} \quad \text{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{DB} - 2(4.10 \sin 30^\circ \text{ kN}) = 0$$

$$F_{DB} = 4.10 \text{ kN (T)} \quad \text{Ans.}$$

NOTE: The force in the last member, BA, can be obtained from joint B or joint A. As an exercise, draw the free-body diagram of joint B, sum the forces in the horizontal direction, and show that $F_{BA} = 0.776 \text{ kN (C)}$.

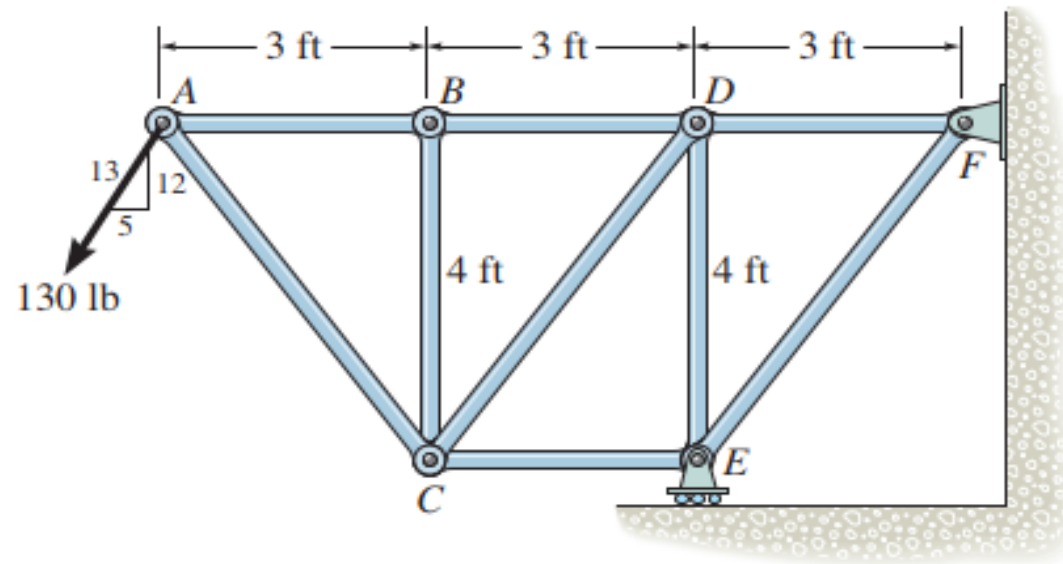


SIMPLE TRUSSES

Example 6.3

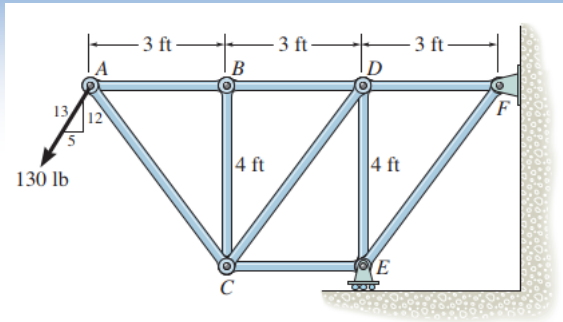
Question

- Determine the force in each member of the truss. State if the members are in tension or compression.



SIMPLE TRUSSES

Example 6.3



Solution

Joint A:

$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}(F_{AC}) - \frac{12}{13}(130) = 0$$

$$F_{AC} = 150 \text{ lb (C)}$$

$$\Rightarrow \Sigma F_x = 0; \quad F_{AB} - \frac{3}{5}(150) - \frac{5}{13}(130) = 0$$

$$F_{AB} = 140 \text{ lb (T)}$$

Joint B:

$$\Rightarrow \Sigma F_x = 0; \quad F_{BD} - 140 = 0$$

$$F_{BD} = 140 \text{ lb (T)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} = 0$$

Joint C:

$$+\uparrow \Sigma F_y = 0; \quad \left(\frac{4}{5}\right)F_{CD} - \left(\frac{4}{5}\right)150 = 0$$

$$F_{CD} = 150 \text{ lb (T)}$$

$$\Rightarrow \Sigma F_x = 0; \quad -F_{CE} + \frac{3}{5}(150) + \frac{3}{5}(150) = 0$$

$$F_{CE} = 180 \text{ lb (C)}$$

Joint D:

$$+\uparrow \Sigma F_y = 0; \quad F_{DE} - \frac{4}{5}(150) = 0$$

$$F_{DE} = 120 \text{ lb (C)}$$

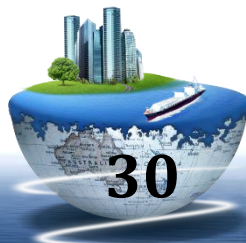
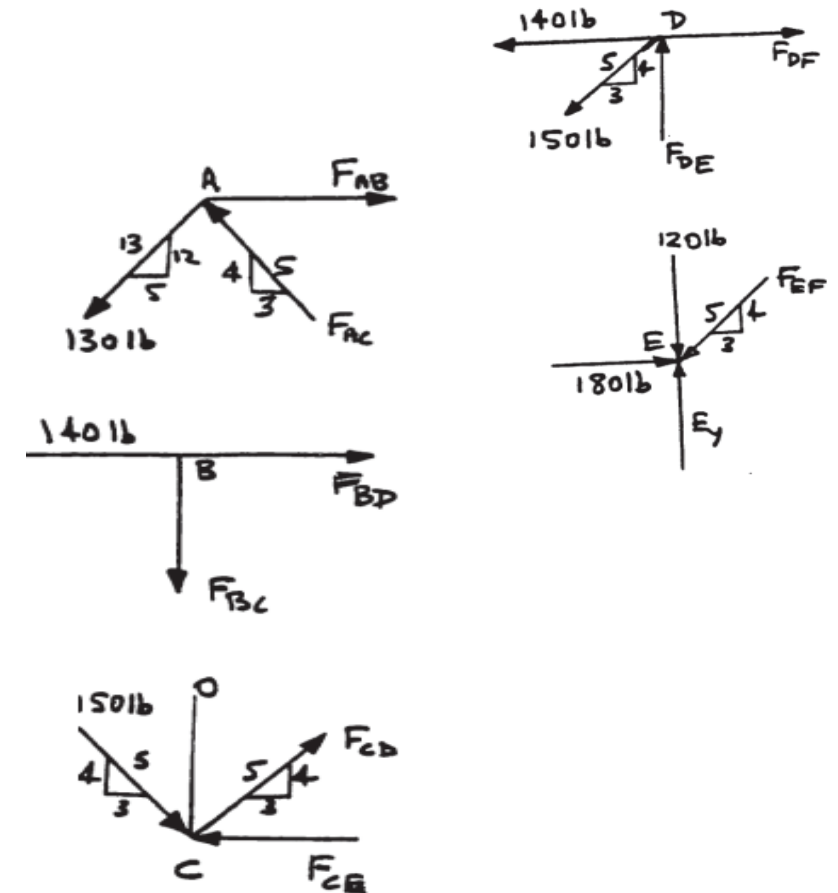
$$\Rightarrow \Sigma F_x = 0; \quad F_{DF} - 140 - \frac{3}{5}(150) = 0$$

$$F_{DF} = 230 \text{ lb (T)}$$

Joint E:

$$\Rightarrow \Sigma F_x = 0; \quad 180 - \frac{3}{5}(F_{EF}) = 0$$

$$F_{EF} = 300 \text{ lb (C)}$$

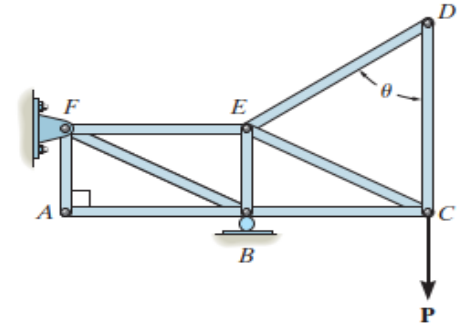




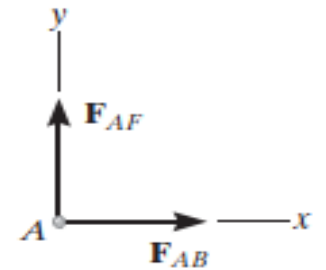
SIMPLE TRUSSES

Zero-Force Members

- Truss analysis using the method of joints is greatly simplified if we can first identify those members which support **no loading**.
- These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed.
- The zero-force members of a truss can generally be found by inspection of each of the joints.
- For example, consider the truss shown in Fig. 6–11a. If a FBD of the pin at joint A is drawn, Fig. 6–11b, it is seen that members AB and AF are zero-force members.



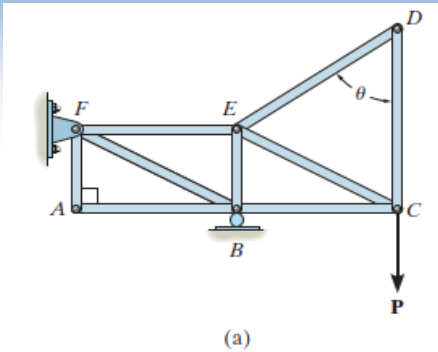
(a)



$$\begin{aligned} \rightarrow \Sigma F_x &= 0; F_{AB} = 0 \\ +\uparrow \Sigma F_y &= 0; F_{AF} = 0 \end{aligned}$$

(b)

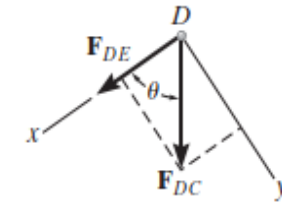




SIMPLE TRUSSES

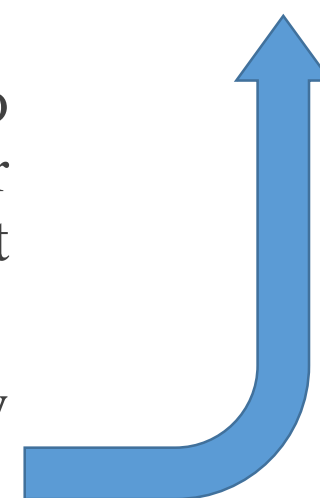
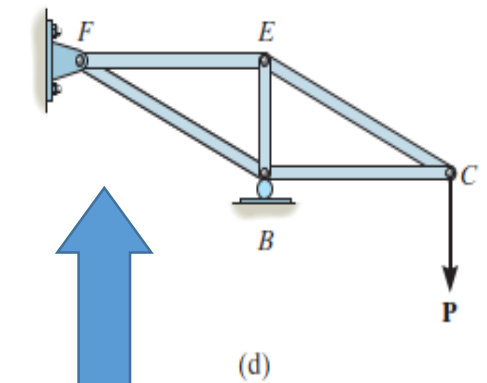
Zero-Force Members

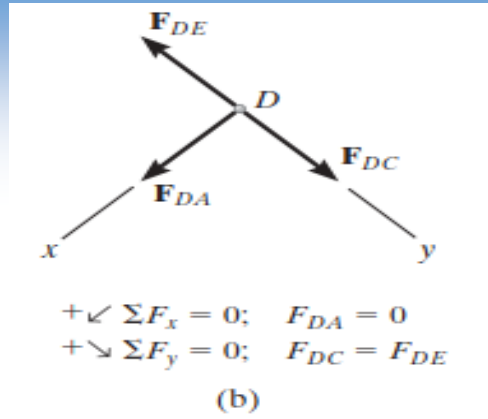
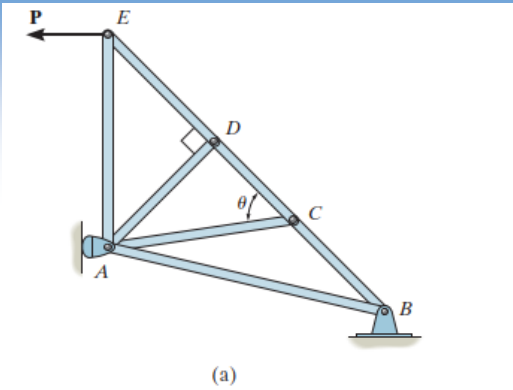
- Note that we could not have come to this conclusion if we had considered the FBDs of joints F or B simply because there are five unknowns at each of these joints.)
- In a similar manner, consider the FBD of joint D, Fig. 6–11c. Here again it is seen that DC and DE are zero-force members.
- From these observations, we can conclude that if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.
- The load on the truss in Fig. 6–11a is therefore supported by only five members as shown in Fig. 6–11d.



$$\begin{aligned}
 +\downarrow \Sigma F_y = 0; & F_{DC} \sin \theta = 0; & F_{DC} = 0 & \text{since } \sin \theta \neq 0 \\
 +\leftarrow \Sigma F_x = 0; & F_{DE} + 0 = 0; & F_{DE} = 0
 \end{aligned}$$

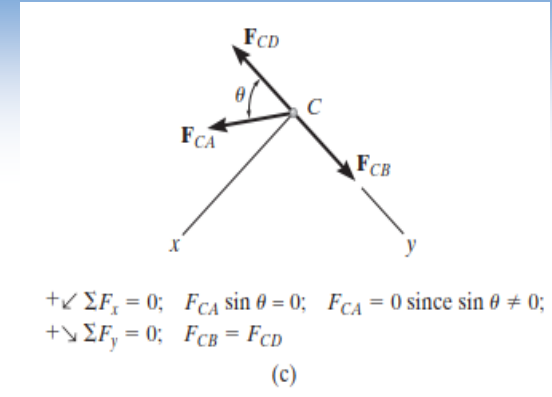
(c)



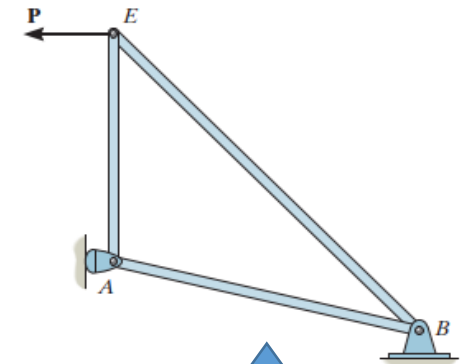


SIMPLE TRUSSES

Zero-Force Members



- Now consider the truss shown in Fig. a. The FBD of the pin at joint D is shown in Fig. b.
- By orienting the y axis along members DC and DE and the x axis along member DA, it is seen that DA is a zero-force member.
- Note that this is also the case for member CA, Fig. c.
- In general then, if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction has a component that acts along this member.
- The truss shown in Fig. 6–12d is therefore suitable for supporting the load P.





STRUCTURAL ANALYSIS - TRUSSES

BREAK FOR REVIEW





THE END

THANK YOU

