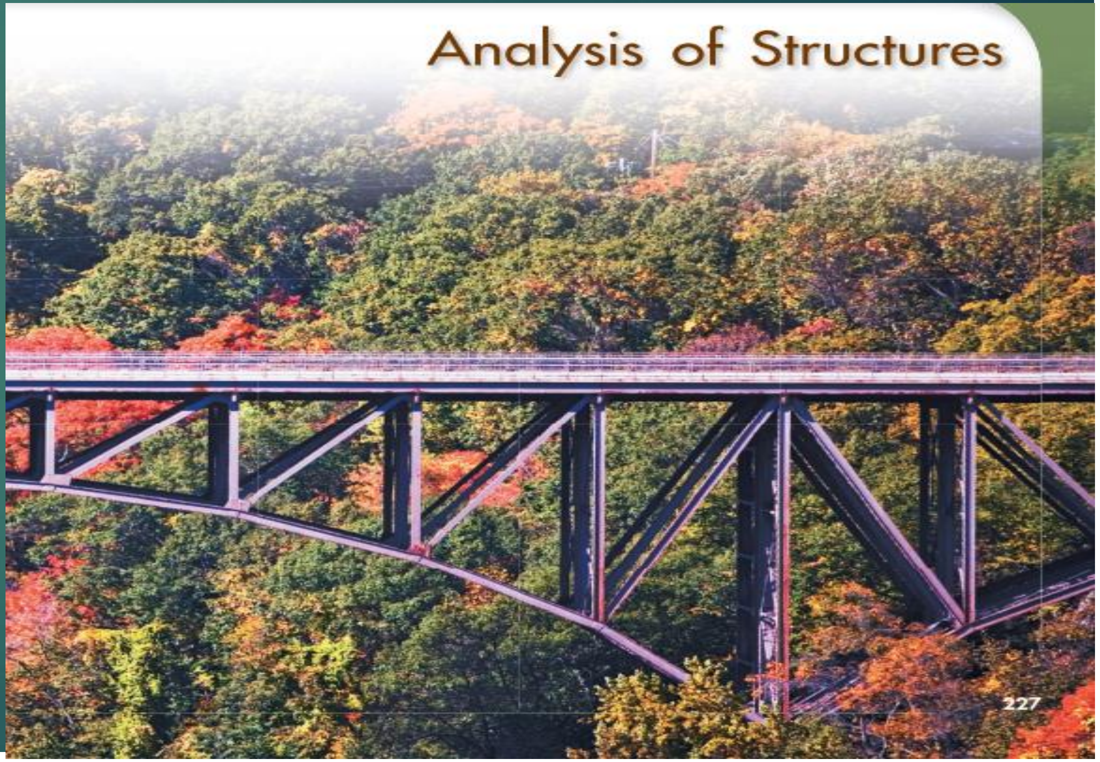
CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

JQL
2020-05-12

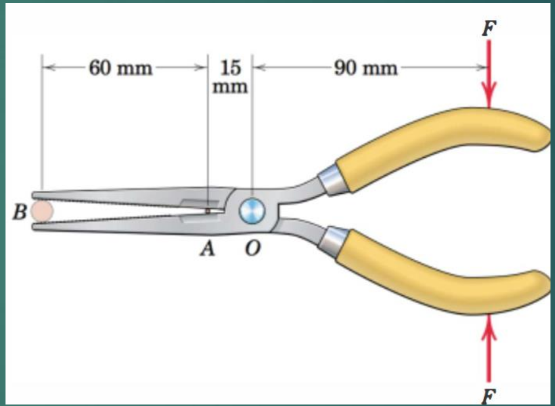
Lecture A7

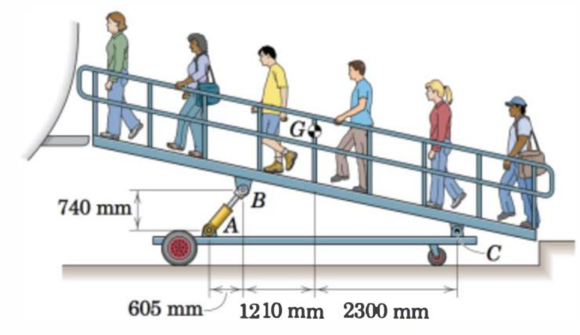
- STRUCTURAL ANALYSIS
- ✓ FRAMES
- ✓ MACHINES

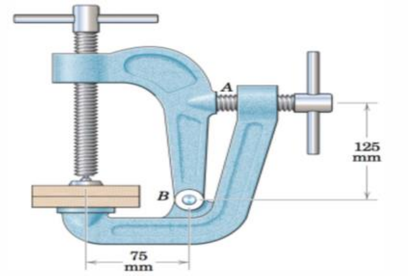


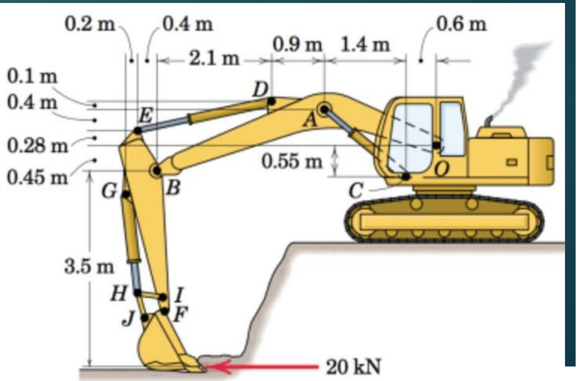
LECTURE OBJECTIVES

❖To analyze the forces acting on the members of frames and machines composed of pin-connected members.









CHAPTER INTRODUCTION

- In the previous lecture we studied how to analyse trusses by the using the method of joints and the method of sections.
- Under trusses, we have considered structures consisting entirely of pins and straight two-force members.
- The forces acting on the two-force members were known to be directed along the members themselves. For further understanding on how to analysis trusses click: https://www.youtube.com/watch?v=Hn_iozUo9m4
- We now consider structures in which at least one of the members is a multiforce member, i.e., a member acted upon by three or more forces.

CHAPTER INTRODUCTION

- These forces will generally not be directed along the members on which they act; their direction is unknown, and they should be represented therefore by two unknown components.
- After this lecture students should be able to:
  - Draw a FBD of a frame or machine and its members
  - Determine the forces acting at the joints and supports of a frame or machine
  - Differentiate a truss from frames/machines

FRAMES AND MACHINES

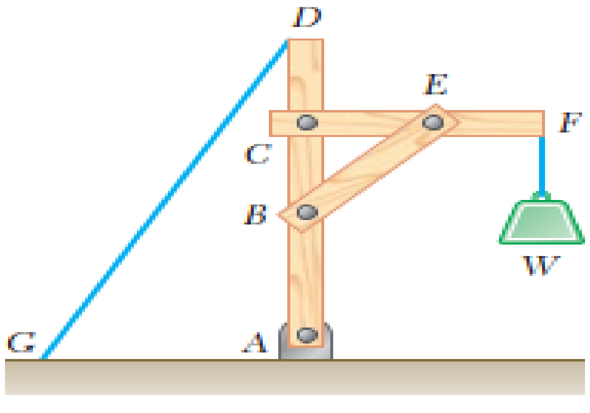
- A structure is called a frame or machine if at least one of its individual members is a multiforce member.
- A multiforce member is defined as one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it.
- Frames are structures which are designed to support applied loads and are usually fixed in position.
- Machines are structures which contain moving parts and are designed to transmit input forces or couples to output forces or couples.

FRAMES AND MACHINES

- To determine the forces internal to an engineering structure (frames and machines), we must first dismember the structure and analyze separate FBD of individual members or combinations of those members
- This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.
- In this treatment we consider only statically determinate structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration

ANALYSIS OF FRAMES

- As a first example of analysing a frame, the crane shown, which carries a given load W will be considered.
- The free-body diagram of the entire frame is shown in Fig.
- This FBD can be used to determine the external forces acting on the frame.



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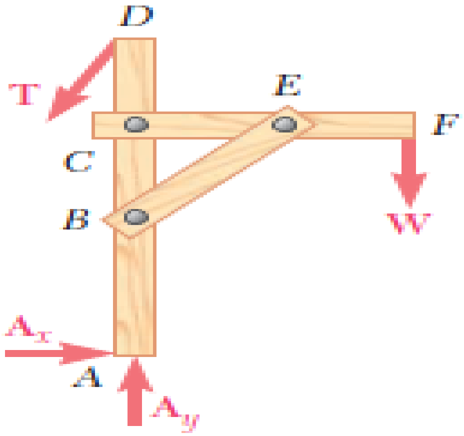

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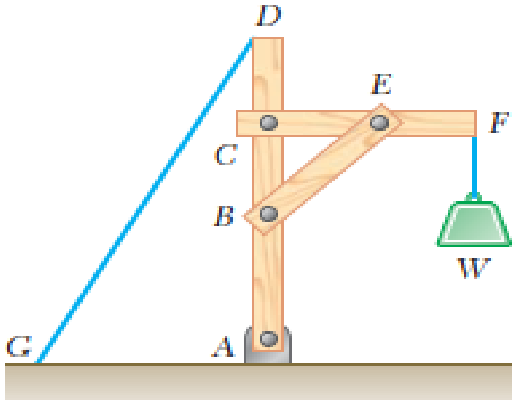
ANALYSIS OF FRAMES

- Summing moments about A, we first determine the force T exerted by the cable; summing x and y components, we then determine the components Ax and Ay of the reaction at the pin A.
- In order to determine the internal forces holding the various parts of a frame together, we must dismember the frame and draw a FBD for each of its component parts.



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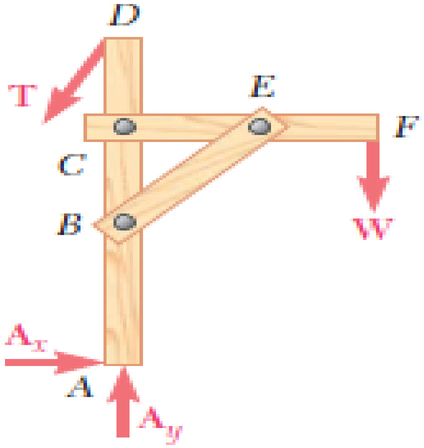

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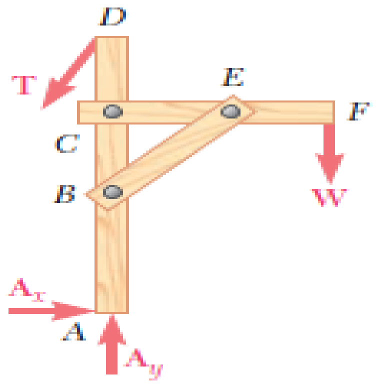
ANALYSIS OF FRAMES



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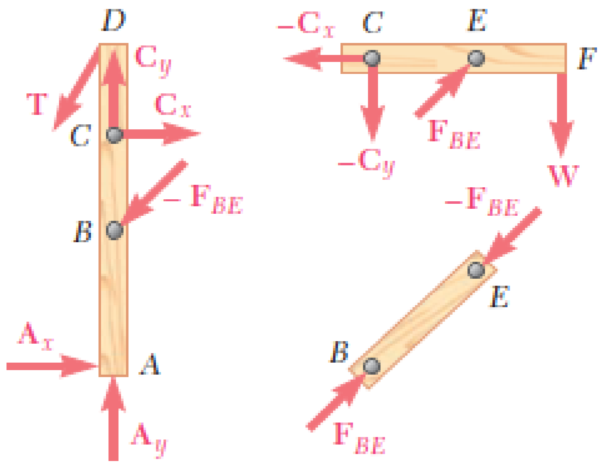
- First, the two-force members should be considered.
- In this frame, member BE is the only two-force member.
- The forces acting at each end of this member must have the same magnitude, same line of action, and opposite sense
- They are therefore directed along i.e BE and will be denoted, respectively, by F_{BE} and $-F_{BE}$.



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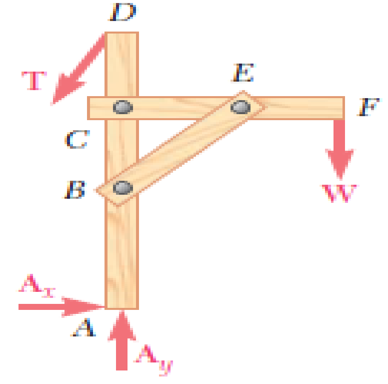
ANALYSIS OF FRAMES



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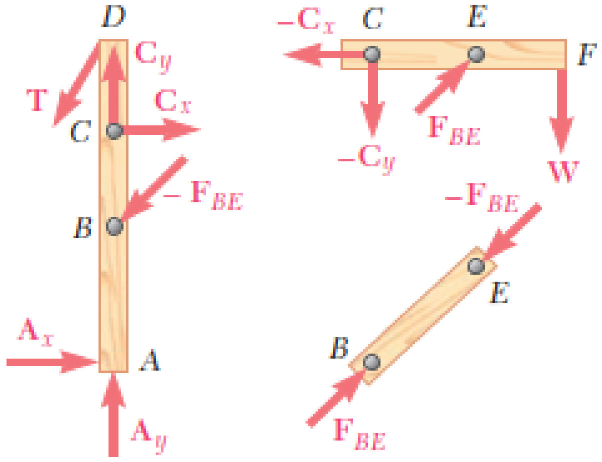

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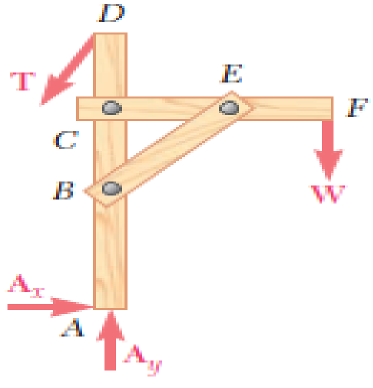
- The sense of the internal forces will be arbitrarily assumed as shown in Fig. shown; later the sign obtained for the common magnitude F_{BE} of the two forces will confirm or deny this assumption.
- Next, we consider the multiforce members, i.e., the members which are acted upon by three or more forces.
- According to Newton's third law, the force exerted at B by member BE on member AD must be equal and opposite to the force F_{BE} exerted by AD on BE.



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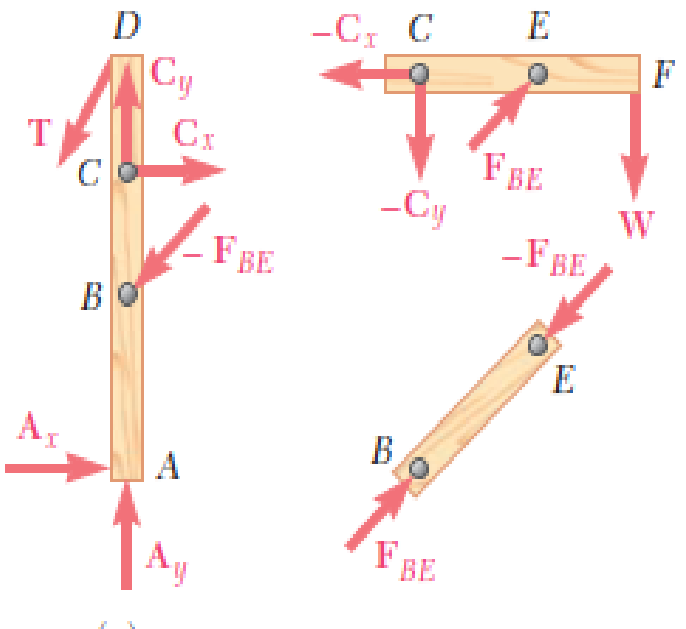
ANALYSIS OF FRAMES



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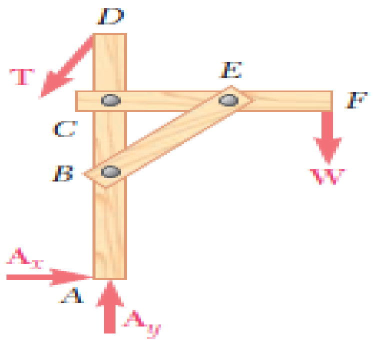
- In this similar manner, the force exerted at E by member BE on member CF must be equal and opposite to the force $-F_{BE}$ exerted by CF on BE.
- Thus the forces that the two-force member BE exerts on AD and CF are equal to $-F_{BE}$ and F_{BE}, respectively;
- They have the same magnitude F_{BE} and opposite sense and should be directed as shown in Fig.



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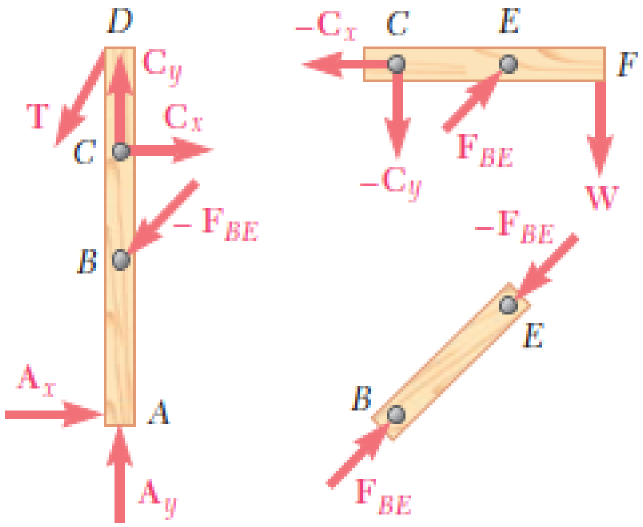
ANALYSIS OF FRAMES



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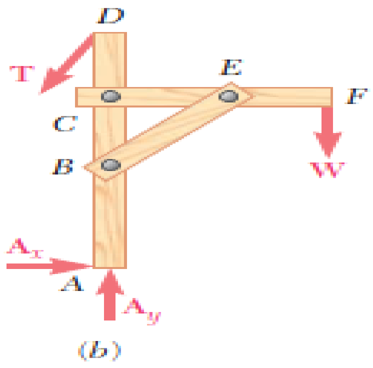
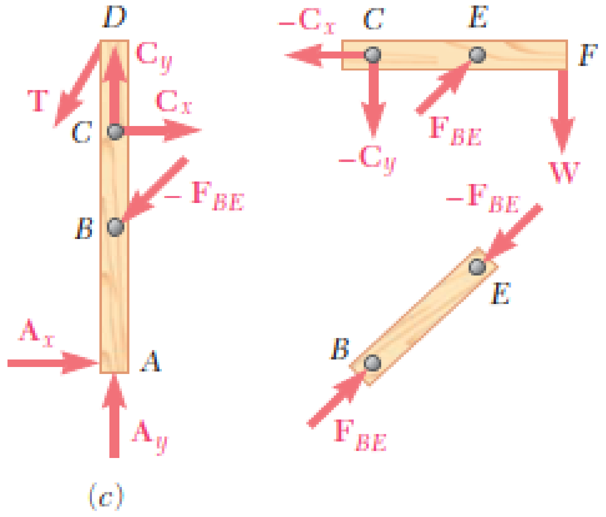
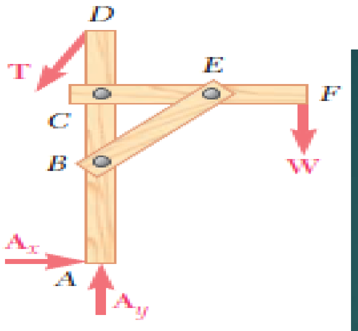
- At C two multi-force members are connected. Since neither the direction nor the magnitude of the forces acting at C is known,
- These forces will be represented by their x and y components.
- The components Cx and Cy of the force acting on member AD will be arbitrarily directed to the right and upward.



ANALYSIS OF FRAMES

- Since, according to Newton's third law, the forces exerted by member CF on AD and by member AD on CF are equal and opposite,
- The components of the force acting on member CF must be directed to the left and down-ward; they will be denoted, respectively, by -Cx and -Cy.





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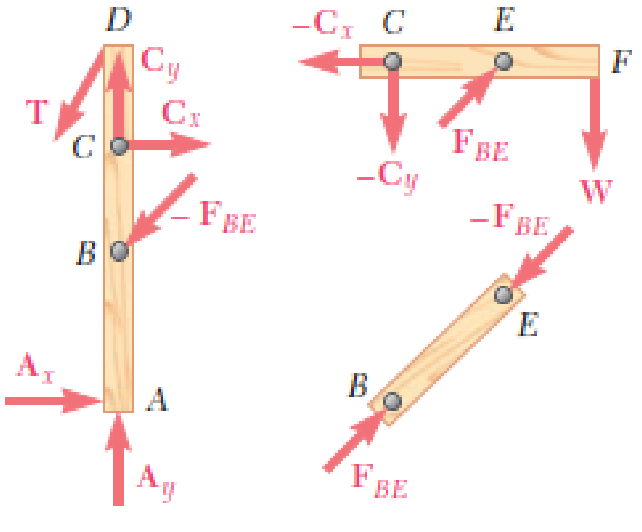
ANALYSIS OF FRAMES

- Whether the force Cx is actually directed to the right and the force -Cx is actually directed to the left will be determined later from the sign of their common magnitude Cx,
- a plus (positive +) sign indicating that the assumption made was correct and a minus sign that it was wrong.
- The free-body diagrams of the multiforce members are completed by showing the external forces acting at A, D, and F.



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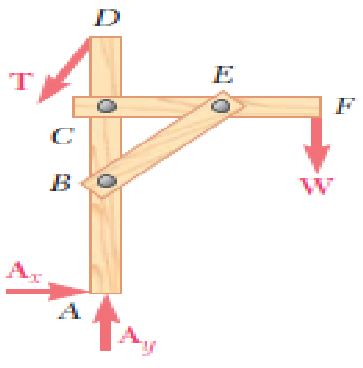

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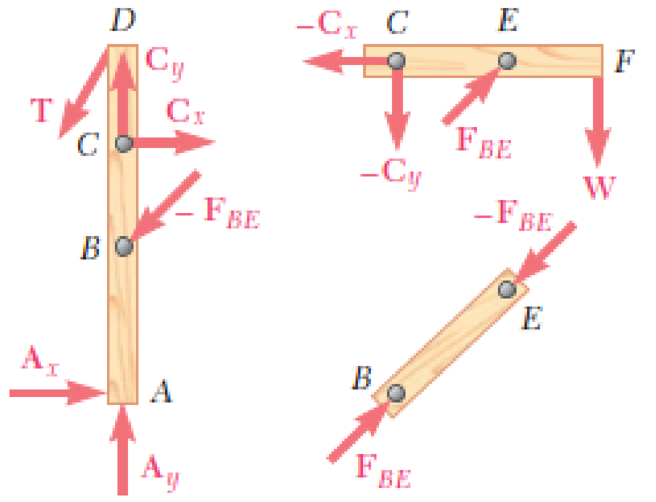
ANALYSIS OF FRAMES



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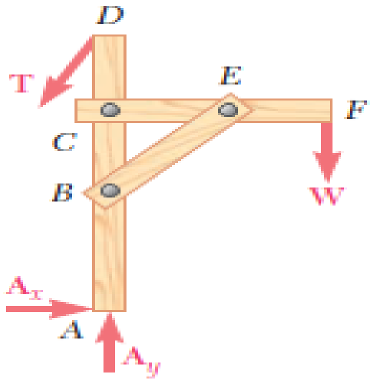
- The internal forces can now be determined by considering the FBD of either of the two multi-force members.
- Choosing the FBD of CF, for example, we write the equations $\Sigma M_C = 0$, $\Sigma M_E = 0$, and $\Sigma F_x = 0$,
- which yield the values of the magnitudes F_{BE}, Cy, and Cx, respectively.
- These values can be checked by verifying that member AD is also in equilibrium.



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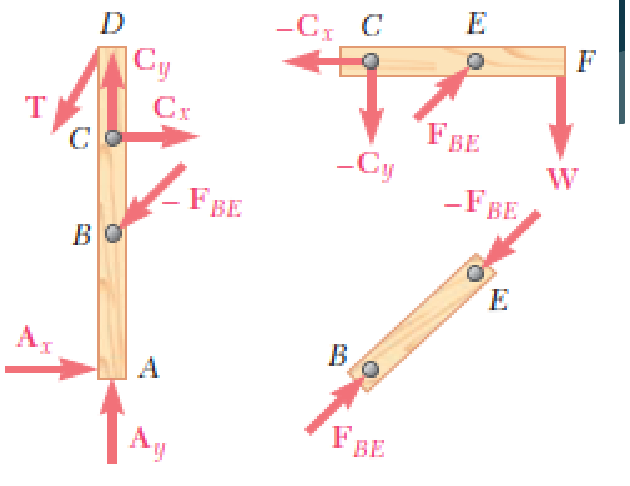
ANALYSIS OF FRAMES



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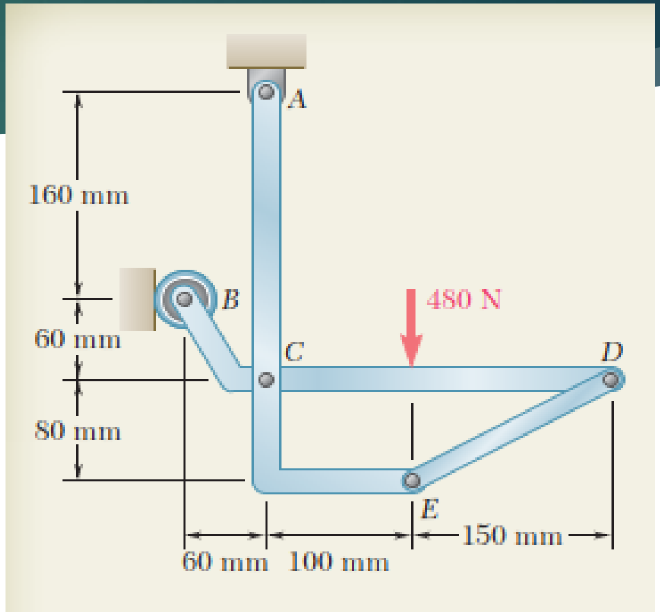
- *When a pin connects three or more members, or when a pin connects a support and two or more members, or when a load is applied to a pin, a clear decision must be made in choosing the member to which the pin will be assumed to belong.*
- If multiforce members are involved, the pin should be attached to one of these members
- The various forces exerted on the pin should then be clearly identified. This is illustrated in one example

ANALYSIS OF FRAMES
Example 7.1

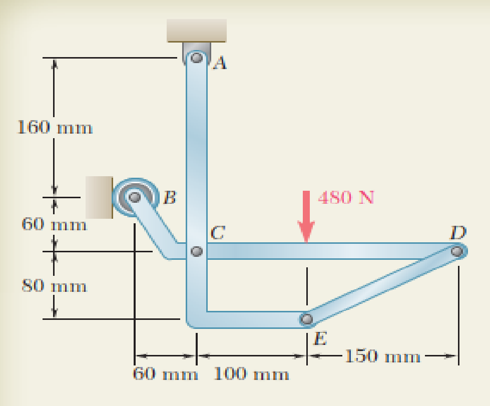
Question

- In the frame shown, members ACE and BCD are connected by a pin at C and by the link DE. For the loading shown, determine the force in link DE and the components of the force exerted at C on member BCD



ANALYSIS OF FRAMES

Example 7.1



Solutions

Free Body: Entire Frame. Since the external reactions involve only three unknowns, we compute the reactions by considering the free-body diagram of the entire frame.

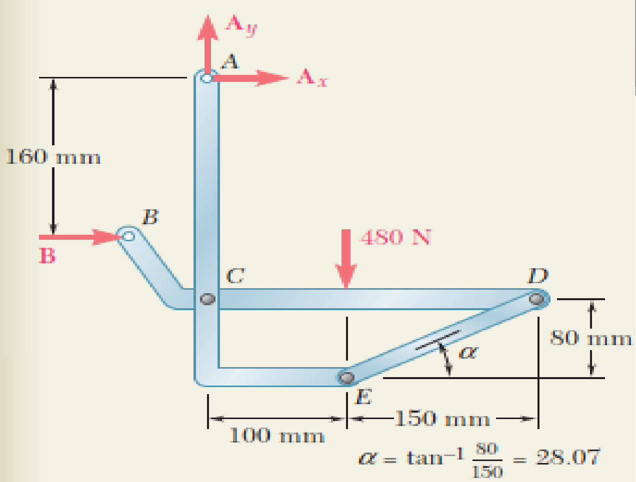
$+\uparrow\Sigma F_y = 0$: $A_y - 480\text{ N} = 0$ $\quad A_y = +480\text{ N}$ $\quad \mathbf{A}_y = 480\text{ N}\uparrow$

$+\circlearrowleft\Sigma M_A = 0$: $-(480\text{ N})(100\text{ mm}) + B(160\text{ mm}) = 0$

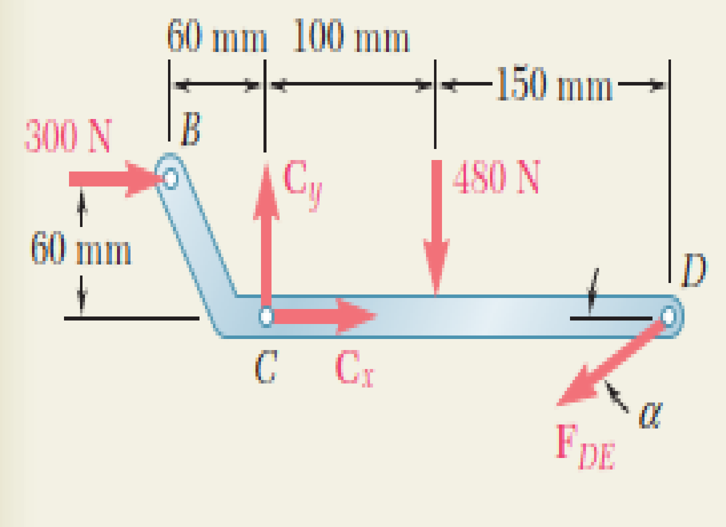
$B = +300\text{ N}$ $\quad \mathbf{B} = 300\text{ N}\rightarrow$

$\xrightarrow{+}\Sigma F_x = 0$: $B + A_x = 0$

$300\text{ N} + A_x = 0$ $\quad A_x = -300\text{ N}$ $\quad \mathbf{A}_x = 300\text{ N}\leftarrow$



Members. We now dismember the frame. Since only two members are connected at C, the components of the unknown forces acting on ACE and BCD are, respectively, equal and opposite and are assumed directed as shown. We assume that link DE is in tension and exerts equal and opposite forces at D and E, directed as shown.



ANALYSIS OF FRAMES

Example 7.1

Solutions

Free Body: Member BCD. Using the free body *BCD*, we write

$+\circlearrowright\Sigma M_C = 0$:
$$(F_{DE} \sin \alpha)(250 \text{ mm}) + (300 \text{ N})(80 \text{ mm}) + (480 \text{ N})(100 \text{ mm}) = 0$$
$$F_{DE} = -561 \text{ N} \qquad F_{DE} = 561 \text{ N } C$$

$\xrightarrow{+}\Sigma F_x = 0$:
$$C_x - F_{DE} \cos \alpha + 300 \text{ N} = 0$$
$$C_x - (-561 \text{ N}) \cos 28.07° + 300 \text{ N} = 0 \qquad C_x = -795 \text{ N}$$

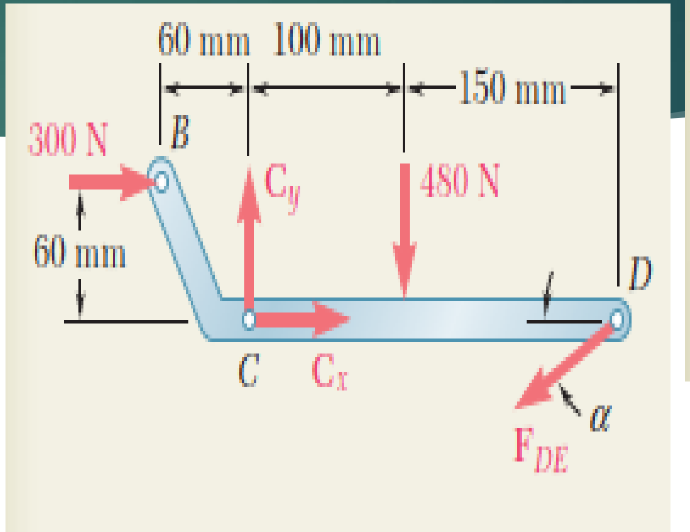
$+\uparrow\Sigma F_y = 0$:
$$C_y - F_{DE} \sin \alpha - 480 \text{ N} = 0$$
$$C_y - (-561 \text{ N}) \sin 28.07° - 480 \text{ N} = 0 \qquad C_y = +216 \text{ N}$$

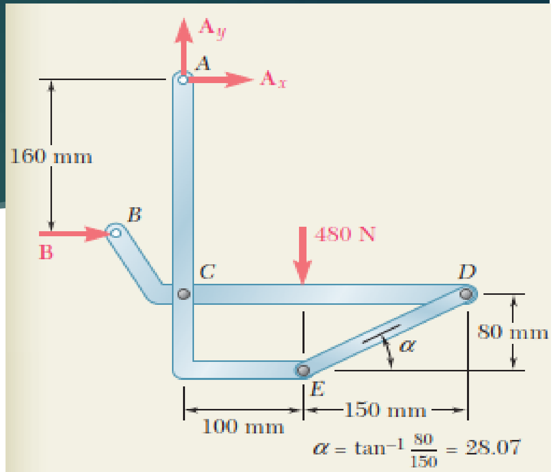
From the signs obtained for C_x and C_y we conclude that the force components $\mathbf{C}_x$ and $\mathbf{C}_y$ exerted on member *BCD* are directed, respectively, to the left and up. We have

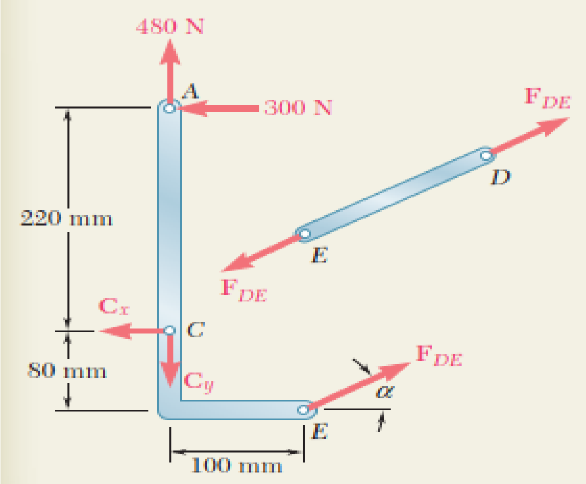
$$\mathbf{C}_x = 795 \text{ N}\leftarrow, \ \mathbf{C}_y = 216 \text{ N}\uparrow$$

Free Body: Member ACE (Check). The computations are checked by considering the free body *ACE*. For example,

$$+\circlearrowleft\Sigma M_A = (F_{DE} \cos \alpha)(300 \text{ mm}) + (F_{DE} \sin \alpha)(100 \text{ mm}) - C_x(220 \text{ mm})$$
$$= (-561 \cos \alpha)(300) + (-561 \sin \alpha)(100) - (-795)(220) = 0$$





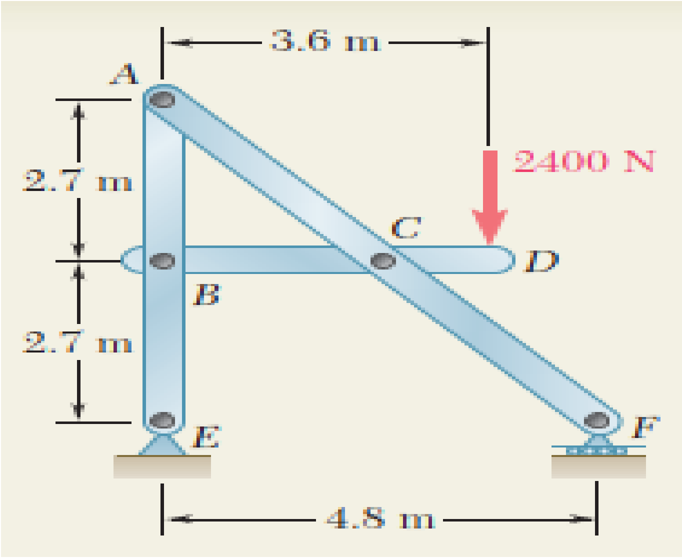


ANALYSIS OF FRAMES

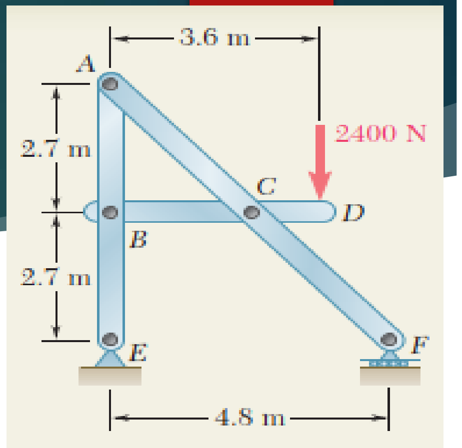
Example 7.2

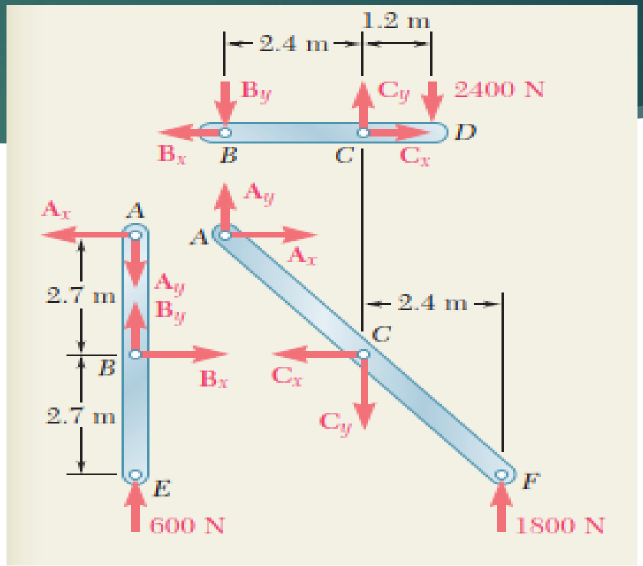
Question

- Determine the components of the forces acting on each member of the frame shown.



ANALYSIS OF FRAMES
Example 7.2





Solutions

Free Body: Entire Frame. Since the external reactions involve only three unknowns, we compute the reactions by considering the free-body diagram of the entire frame.

$+\circlearrowleft\Sigma M_E = 0$: $\quad -(2400\text{ N})(3.6\text{ m}) + F(4.8\text{ m}) = 0$

$F = +1800\text{ N}$ $\qquad$ $\mathbf{F} = 1800\text{ N}\uparrow$ ◀

$+\uparrow\Sigma F_y = 0$: $\quad -2400\text{ N} + 1800\text{ N} + E_y = 0$

$E_y = +600\text{ N}$ $\qquad$ $\mathbf{E}_y = 600\text{ N}\uparrow$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\qquad$ $\mathbf{E}_x = 0$ ◀

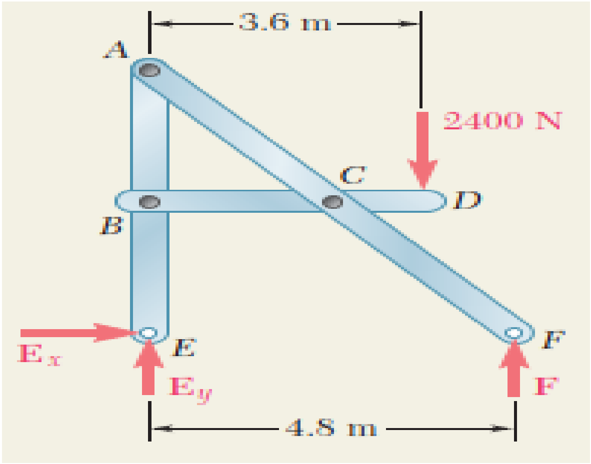
Members. The frame is now dismembered; since only two members are connected at each joint, equal and opposite components are shown on each member at each joint.

Free Body: Member BCD

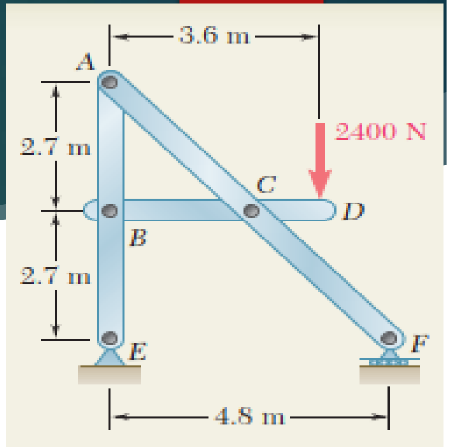
$+\circlearrowleft\Sigma M_B = 0$: $\quad -(2400\text{ N})(3.6\text{ m}) + C_y(2.4\text{ m}) = 0 \qquad C_y = +3600\text{ N}$ ◀

$+\circlearrowleft\Sigma M_C = 0$: $\quad -(2400\text{ N})(1.2\text{ m}) + B_y(2.4\text{ m}) = 0 \qquad B_y = +1200\text{ N}$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad -B_x + C_x = 0$



ANALYSIS OF FRAMES
Example 7.2



Solutions

We note that neither B_x nor C_x can be obtained by considering only member *BCD*. The positive values obtained for B_y and C_y indicate that the force components $\mathbf{B}_y$ and $\mathbf{C}_y$ are directed as assumed.

Free Body: Member ABE

$+\circlearrowleft\Sigma M_A = 0$: $\quad B_x(2.7\text{ m}) = 0 \qquad B_x = 0$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad +B_x - A_x = 0 \qquad A_x = 0$ ◀

$+\uparrow\Sigma F_y = 0$: $\quad -A_y + B_y + 600\text{ N} = 0$

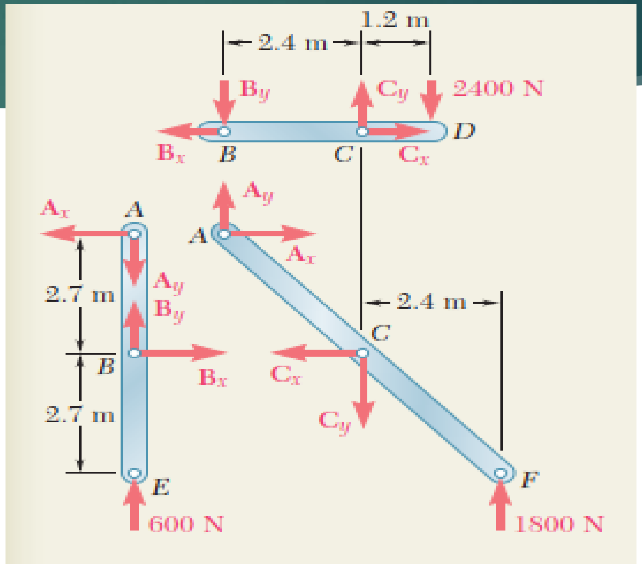
$-A_y + 1200\text{ N} + 600\text{ N} = 0 \qquad A_y = +1800\text{ N}$ ◀

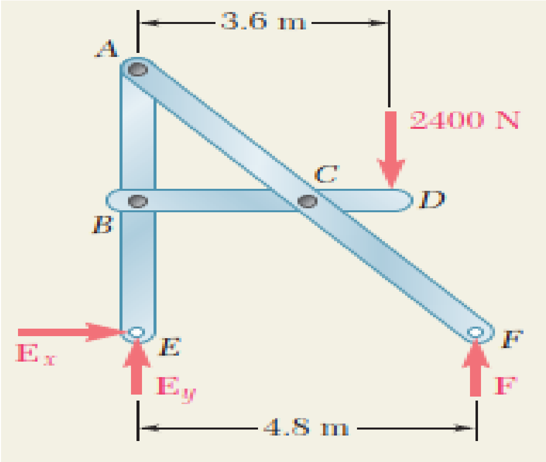
Free Body: Member BCD. Returning now to member *BCD*, we write

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad -B_x + C_x = 0 \qquad 0 + C_x = 0 \qquad C_x = 0$ ◀

Free Body: Member ACF (Check). All unknown components have now been found; to check the results, we verify that member *ACF* is in equilibrium.

$$+\circlearrowleft\Sigma M_C = (1800\text{ N})(2.4\text{ m}) - A_y(2.4\text{ m}) - A_x(2.7\text{ m})$$
$$= (1800\text{ N})(2.4\text{ m}) - (1800\text{ N})(2.4\text{ m}) - 0 = 0 \qquad \text{(checks)}$$

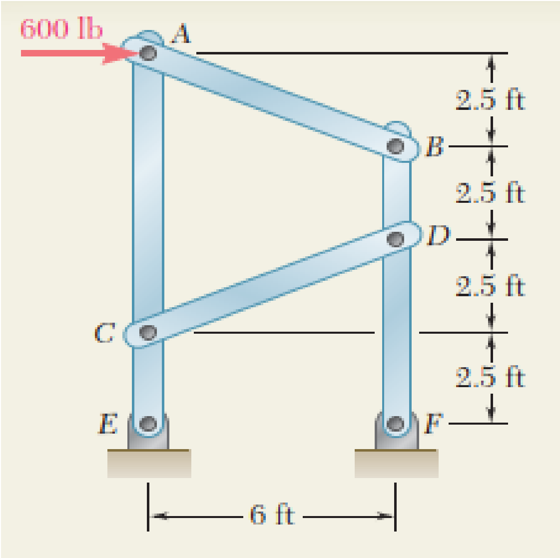




ANALYSIS OF FRAMES
Example 7.3

Question

- A 600-lb horizontal force is applied to pin A of the frame shown. Determine the forces acting on the two vertical members of the frame.



ANALYSIS OF FRAMES
Example 7.3

Solutions

Free Body: Entire Frame. The entire frame is chosen as a free body; although the reactions involve four unknowns, $\mathbf{E}_y$ and $\mathbf{F}_y$ may be determined by writing

$+\circlearrowleft\Sigma M_E = 0$: $\quad -(600\text{ lb})(10\text{ ft}) + F_y(6\text{ ft}) = 0$

$F_y = +1000\text{ lb}$ $\qquad \mathbf{F}_y = 1000\text{ lb}\uparrow$ ◀

$+\uparrow\Sigma F_y = 0$: $\quad E_y + F_y = 0$

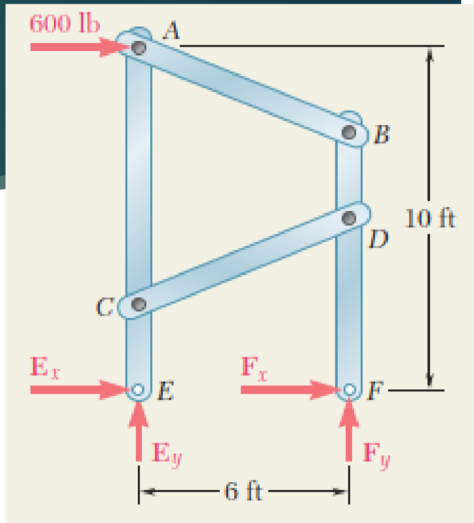
$E_y = -1000\text{ lb}$ $\qquad \mathbf{E}_y = 1000\text{ lb}\downarrow$ ◀

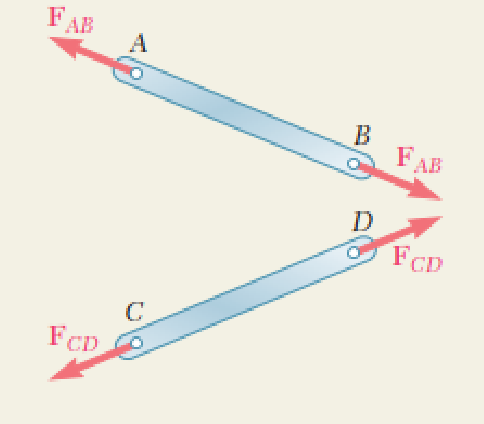
Members. The equations of equilibrium of the entire frame are not sufficient to determine $\mathbf{E}_x$ and $\mathbf{F}_x$. The free-body diagrams of the various members must now be considered in order to proceed with the solution. In dismembering the frame, we will assume that pin *A* is attached to the multiforce member *ACE* and, thus, that the 600-lb force is applied to that member. We also note that *AB* and *CD* are two-force members.

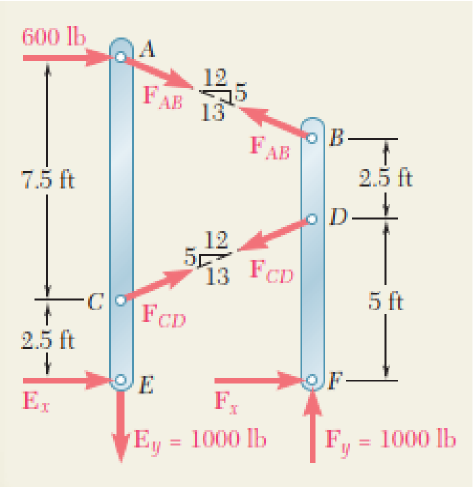
Free Body: Member ACE

$+\uparrow\Sigma F_y = 0$: $\quad -\frac{5}{13}F_{AB} + \frac{5}{13}F_{CD} - 1000\text{ lb} = 0$

$+\circlearrowleft\Sigma M_E = 0$: $\quad -(600\text{ lb})(10\text{ ft}) - (\frac{12}{13}F_{AB})(10\text{ ft}) - (\frac{12}{13}F_{CD})(2.5\text{ ft}) = 0$







Example 7.3

Solutions

Solving these equations simultaneously, we find

$$F_{AB} = -1040 \text{ lb} \qquad F_{CD} = +1560 \text{ lb} \quad \blacktriangleleft$$

The signs obtained indicate that the sense assumed for F_{CD} was correct and the sense for F_{AB} incorrect. Summing now x components,

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad 600 \text{ lb} + \tfrac{12}{13}(-1040 \text{ lb}) + \tfrac{12}{13}(+1560 \text{ lb}) + E_x = 0$$

$$E_x = -1080 \text{ lb} \qquad \mathbf{E}_x = 1080 \text{ lb} \leftarrow \quad \blacktriangleleft$$

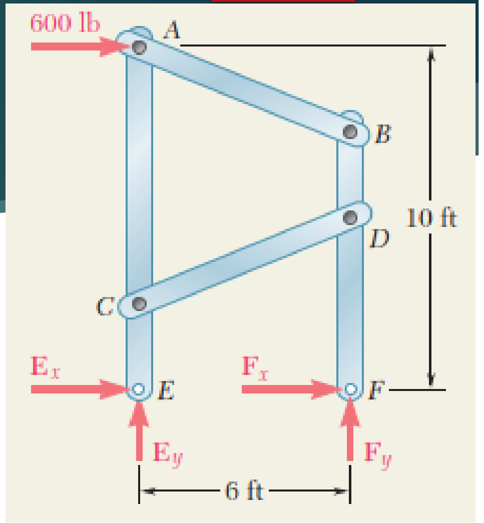
Free Body: Entire Frame. Since $\mathbf{E}_x$ has been determined, we can return to the free-body diagram of the entire frame and write

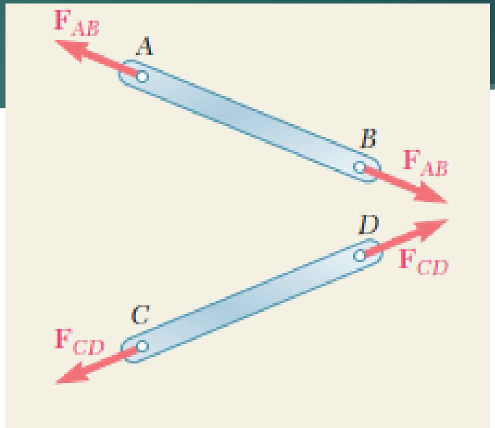
$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad 600 \text{ lb} - 1080 \text{ lb} + F_x = 0$$

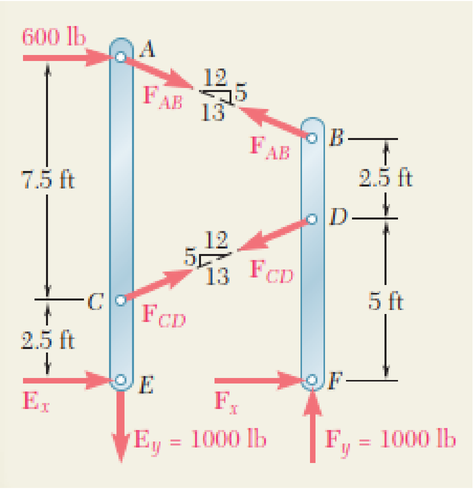
$$F_x = +480 \text{ lb} \qquad \mathbf{F}_x = 480 \text{ lb} \rightarrow \quad \blacktriangleleft$$

Free Body: Member BDF (Check). We can check our computations by verifying that the equation $\Sigma M_B = 0$ is satisfied by the forces acting on member BDF.

$$\begin{aligned} +\circlearrowleft\Sigma M_B &= -\left(\tfrac{12}{13}F_{CD}\right)(2.5 \text{ ft}) + (F_x)(7.5 \text{ ft}) \\ &= -\tfrac{12}{13}(1560 \text{ lb})(2.5 \text{ ft}) + (480 \text{ lb})(7.5 \text{ ft}) \\ &= -3600 \text{ lb}\cdot\text{ft} + 3600 \text{ lb}\cdot\text{ft} = 0 \qquad \text{(checks)} \end{aligned}$$



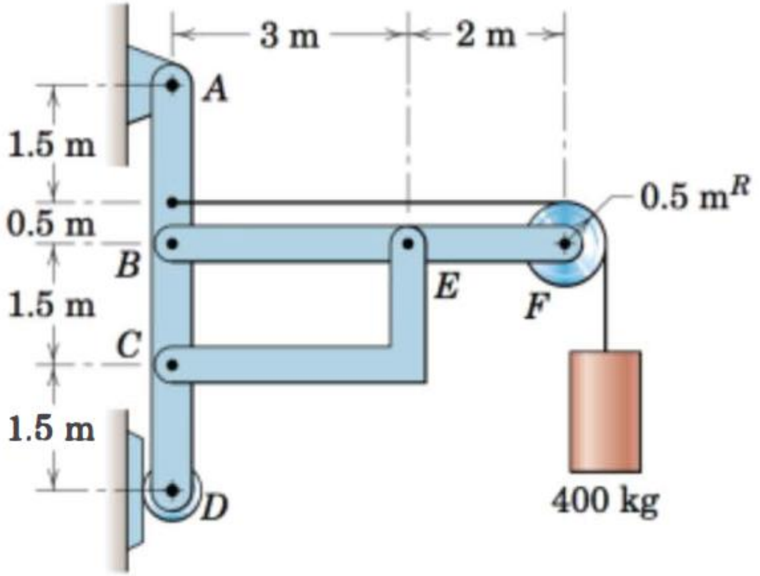




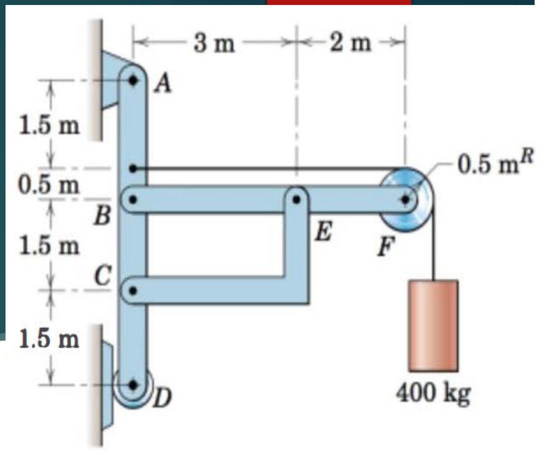
ANALYSIS OF FRAMES
Example 7.4

Question

- The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.



ANALYSIS OF FRAMES
Example 7.4



Solutions

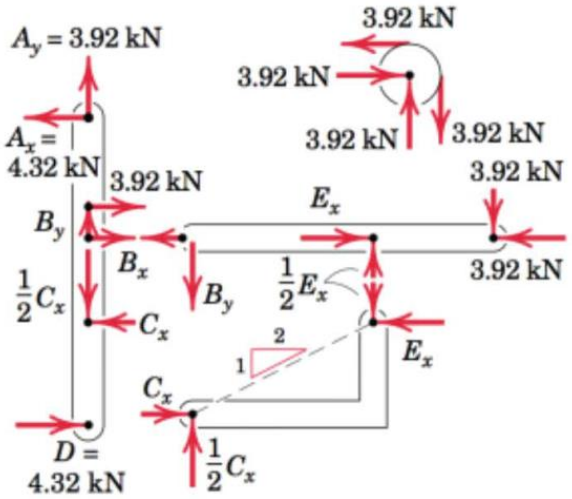
Solution. We observe first that the three supporting members which constitute the frame form a rigid assembly that can be analyzed as a single unit. We also observe that the arrangement of the external supports makes the frame statically determinate.

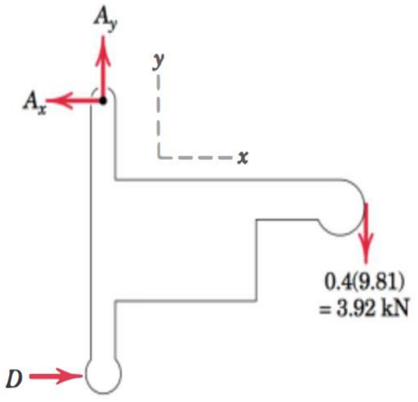
From the free-body diagram of the entire frame we determine the external reactions. Thus,

$[\Sigma M_A = 0]$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32$ kN
$[\Sigma F_x = 0]$	$A_x - 4.32 = 0$	$A_x = 4.32$ kN
$[\Sigma F_y = 0]$	$A_y - 3.92 = 0$	$A_y = 3.92$ kN

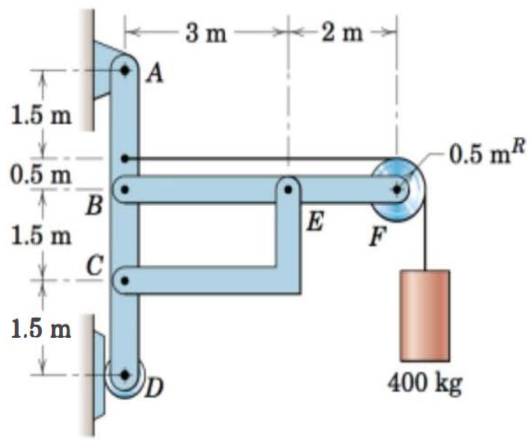
Next we dismember the frame and draw a separate free-body diagram of each member. The diagrams are arranged in their approximate relative positions to aid in keeping track of the common forces of interaction. The external reactions just obtained are entered onto the diagram for *AD*. Other known forces are the 3.92-kN forces exerted by the shaft of the pulley on the member *BF*, as obtained from the free-body diagram of the pulley. The cable tension of 3.92 kN is also shown acting on *AD* at its attachment point.

Next, the components of all unknown forces are shown on the diagrams. Here we observe that *CE* is a two-force member. The force components on *CE* have equal and opposite reactions, which are shown on *BF* at *E* and on *AD* at *C*. We may not recognize the actual sense of the components at *B* at first glance, so they may be arbitrarily but consistently assigned.





ANALYSIS OF FRAMES
Example 7.4



Solutions

The solution may proceed by use of a moment equation about B or E for member BF, followed by the two force equations. Thus,

$[\Sigma M_B = 0]$ $\quad 3.92(5) - \frac{1}{2}E_x(3) = 0 \quad E_x = 13.08 \text{ kN}$ *Ans.*

$[\Sigma F_y = 0]$ $\quad B_y + 3.92 - 13.08/2 = 0 \quad B_y = 2.62 \text{ kN}$ *Ans.*

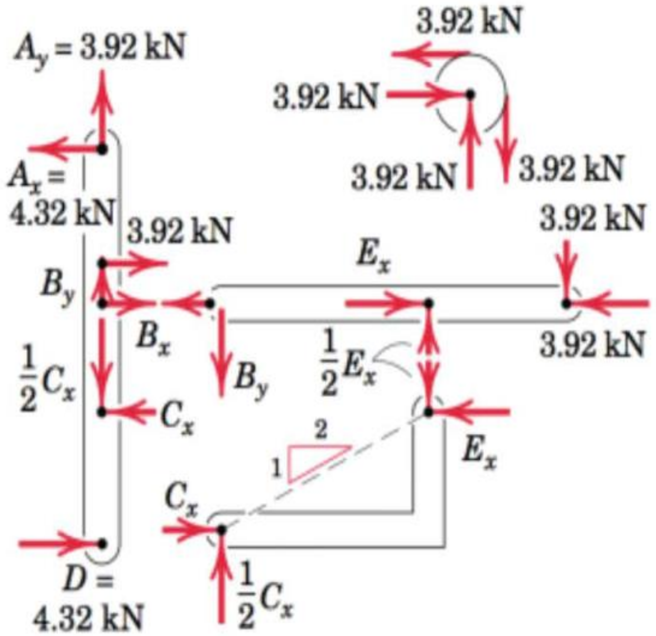
$[\Sigma F_x = 0]$ $\quad B_x + 3.92 - 13.08 = 0 \quad B_x = 9.15 \text{ kN}$ *Ans.*

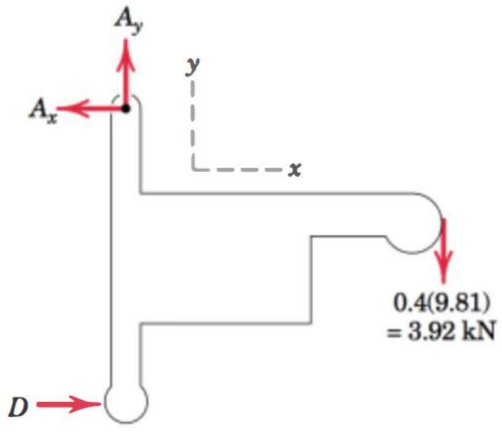
Positive numerical values of the unknowns mean that we assumed their directions correctly on the free-body diagrams. The value of $C_x = E_x = 13.08$ kN obtained by inspection of the free-body diagram of CE is now entered onto the diagram for AD, along with the values of B_x and B_y just determined. The equations of equilibrium may now be applied to member AD as a check, since all the forces acting on it have already been computed. The equations give

$[\Sigma M_C = 0]$ $\quad 4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$

$[\Sigma F_x = 0]$ $\quad 4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$

$[\Sigma F_y = 0]$ $\quad -13.08/2 + 2.62 + 3.92 = 0$

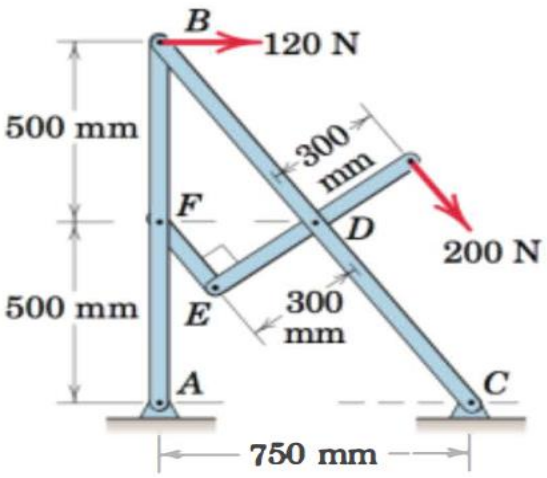




ANALYSIS OF FRAMES
Example 7.5

Question

- Neglect the weight of the frame and compute the forces acting on all of its members.



ANALYSIS OF FRAMES
Example 7.5

Solutions

Solution. We note first that the frame is not a rigid unit when removed from its supports since *BDEF* is a movable quadrilateral and not a rigid triangle. Consequently, the external reactions cannot be completely determined until the individual members are analyzed. However, we can determine the vertical components of the reactions at *A* and *C* from the free-body diagram of the frame as a whole. Thus,

$[\Sigma M_C = 0] \quad 200(0.3) + 120(0.1) - 0.75A_y = 0 \quad A_y = 240\text{ N}$ *Ans.*

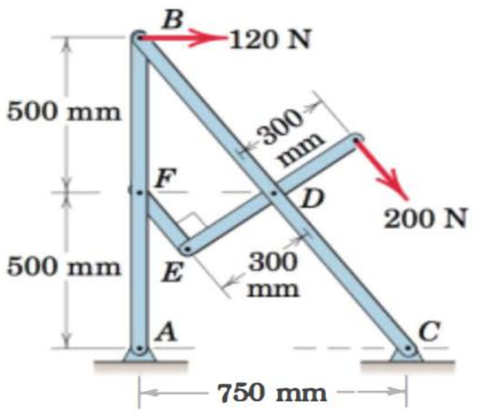
$[\Sigma F_y = 0] \quad C_y - 200(4/5) - 240 = 0 \quad C_y = 400\text{ N}$ *Ans.*

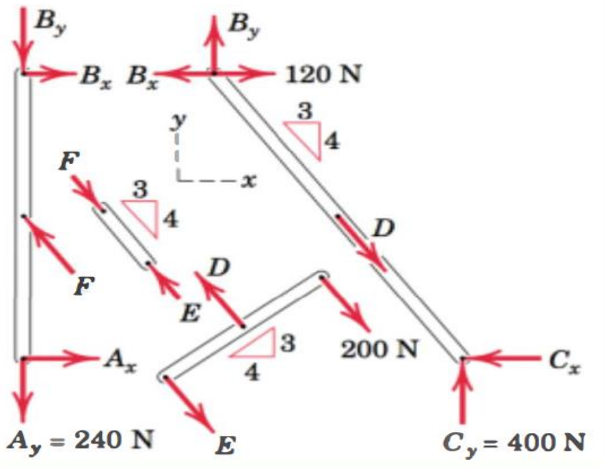
Next we dismember the frame and draw the free-body diagram of each part. Since *EF* is a two-force member, the direction of the force at *E* on *ED* and at *F* on *AB* is known. We assume that the 120-N force is applied to the pin as a part of member *BC*. There should be no difficulty in assigning the correct directions for forces *E*, *F*, *D*, and B_x. The direction of B_y, however, may not be assigned by inspection and therefore is arbitrarily shown as downward on *AB* and upward on *BC*.

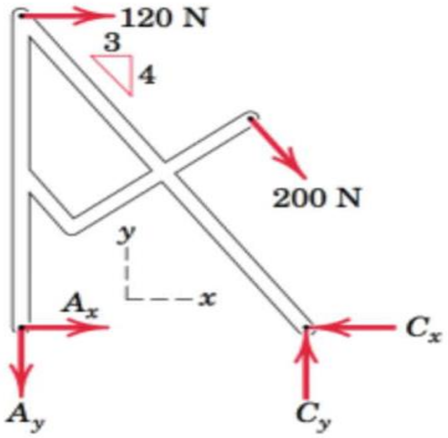
Member ED. The two unknowns are easily obtained by

$[\Sigma M_D = 0] \quad 200(0.3) - 0.3E = 0 \quad E = 200\text{ N}$ *Ans.*

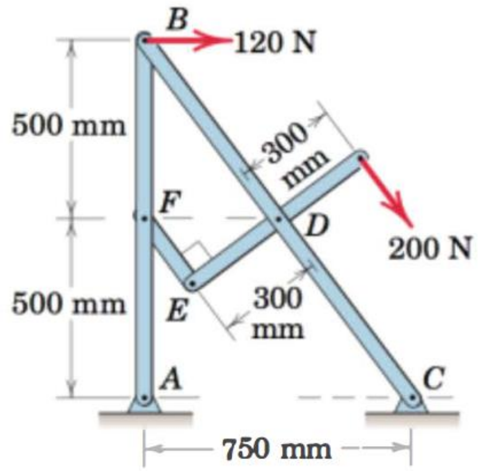
$[\Sigma F = 0] \quad D - 200 - 200 = 0 \quad D = 400\text{ N}$ *Ans.*







ANALYSIS OF FRAMES
Example 7.5



Solutions

Member EF. Clearly F is equal and opposite to E with the magnitude of 200 N.

Member AB. Since F is now known, we solve for B_x, A_x, and B_y from

$[\Sigma M_A = 0]$ $\quad 200(3/5)(0.5) - B_x(1.0) = 0 \quad B_x = 60$ N $\quad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad A_x + 60 - 200(3/5) = 0 \quad A_x = 60$ N $\quad$ *Ans.*

$[\Sigma F_y = 0]$ $\quad 200(4/5) - 240 - B_y = 0 \quad B_y = -80$ N $\quad$ *Ans.*

The minus sign shows that we assigned B_y in the wrong direction.

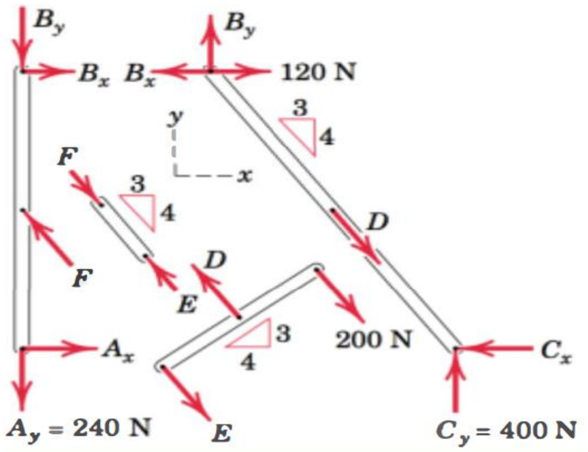
Member BC. The results for B_x, B_y, and D are now transferred to BC, and the remaining unknown C_x is found from

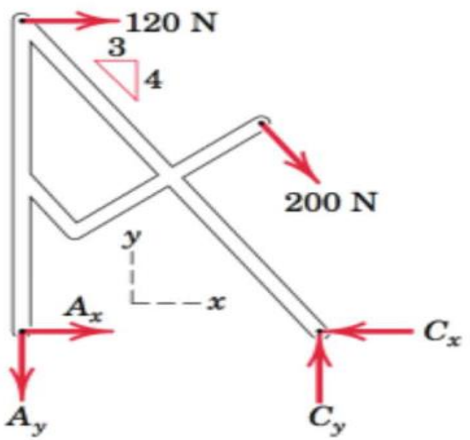
$[\Sigma F_x = 0]$ $\quad 120 + 400(3/5) - 60 - C_x = 0 \quad C_x = 300$ N $\quad$ *Ans.*

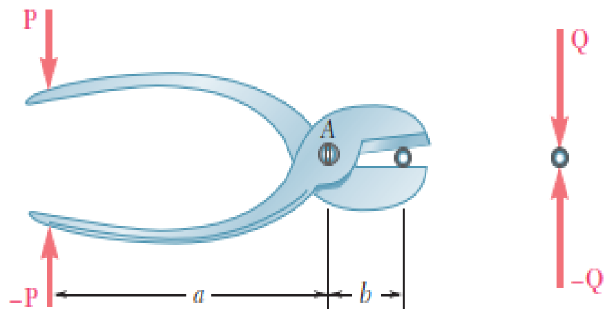
We may apply the remaining two equilibrium equations as a check. Thus,

$[\Sigma F_y = 0]$ $\quad 400 + (-80) - 400(4/5) = 0$

$[\Sigma M_C = 0]$ $\quad (120 - 60)(1.0) + (-80)(0.75) = 0$





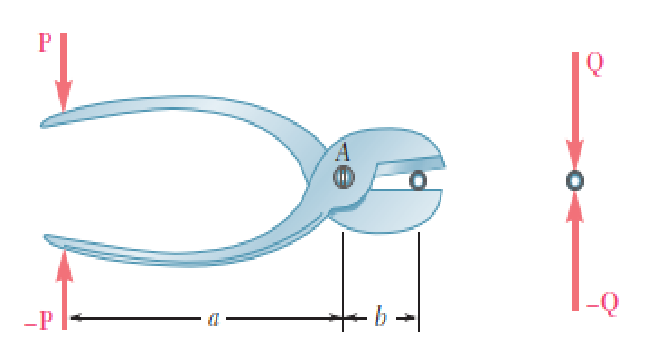


(a) (b)

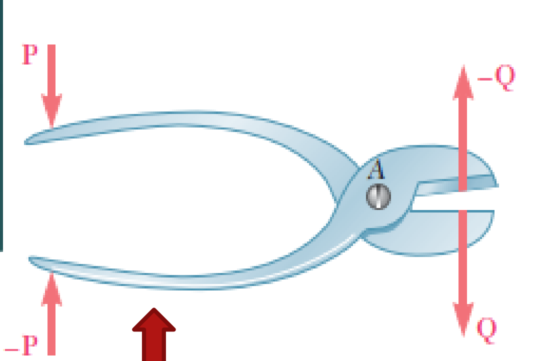
ANALYSIS OF A MACHINE

- Machines are structures designed to transmit and modify forces.
- Whether they are simple tools or include complicated mechanisms, their main purpose is to transform input forces into output forces.
- Consider, for example, a pair of cutting pliers used to cut a wire in Fig. shown.
- If we apply two equal and opposite forces P and -P on their handles, they will exert two equal and opposite forces Q and -Q on the wire (that is a couple).

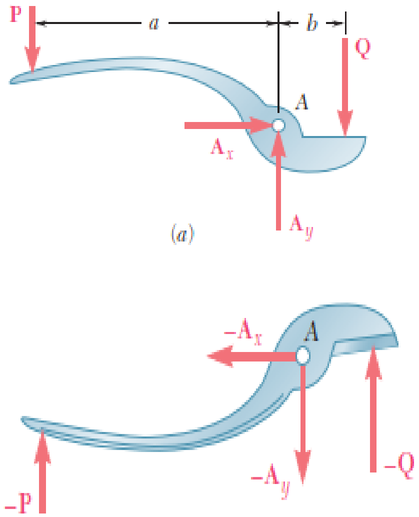
ANALYSIS OF A MACHINE



(a) (b)

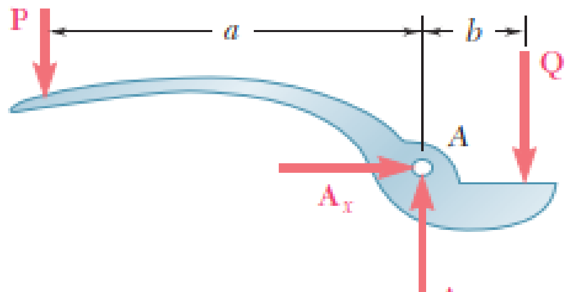


- To determine the magnitude Q of the output forces when the magnitude P of the input forces is known (or, conversely, to determine P when Q is known), we draw a free-body diagram of the pliers alone, showing the input forces P and -P and the reactions -Q and Q that the wire exerts on the pliers (Fig. shown).



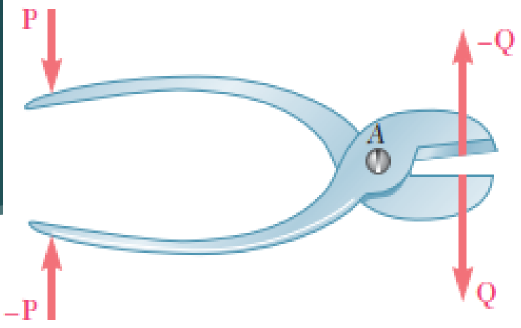
(a)
(b)

- However, since a pair of pliers forms a nonrigid structure, we must use one of the component parts as a free body in order to determine the unknown forces.

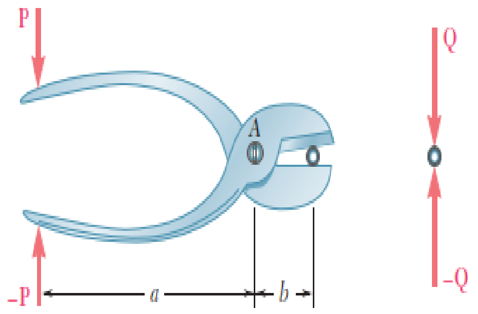


(a)

ANALYSIS OF A MACHINE



- Considering Fig. above, for example, and taking moments about A, we obtain the relation Pa = Qb, which defines the magnitude Q in terms of P or P in terms of Q.
- The same free-body diagram can be used to determine the components of the internal force at A; we find Ax = 0 and Ay = P + Q.
- *In the case of more complicated machines, it generally will be necessary to use several FBD and, possibly, to solve simultaneous equations involving various internal forces.*



(a) (b)

ANALYSIS OF A MACHINE

- *The free bodies should be chosen to include the input forces and the reactions to the output forces,*
- And the total number of unknown force components involved should not exceed the number of available independent equations.
- *It is advisable, before attempting to solve a problem, to determine whether the structure considered is determinate or not.*
- *There is no point in discussing the rigidity of a machine, since a machine includes moving parts and thus must be nonrigid.*

ANALYSIS OF A MACHINE
Example 7.6

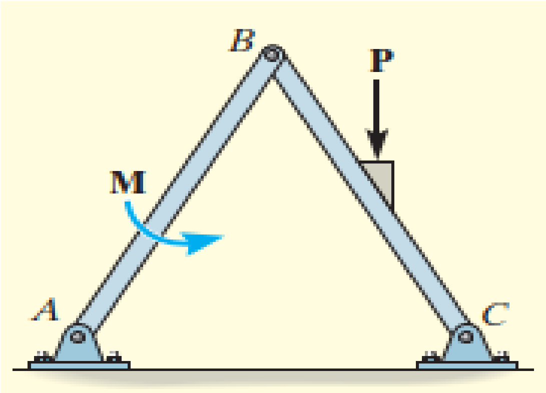
Question

- For the frame shown in Fig. below, draw the free-body diagram of the following:

(a) each member,

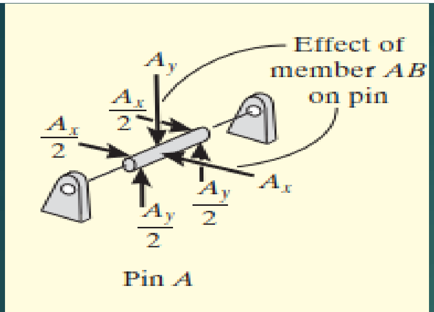
(b) the pins at B and A, and

(c) the two members connected together..



ANALYSIS OF A MACHINE

Example 7.6



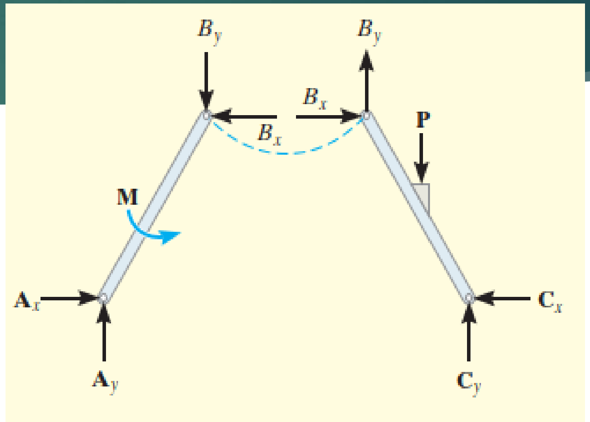
(d)

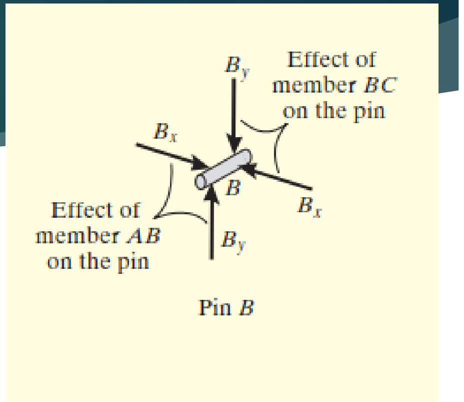
Solutions

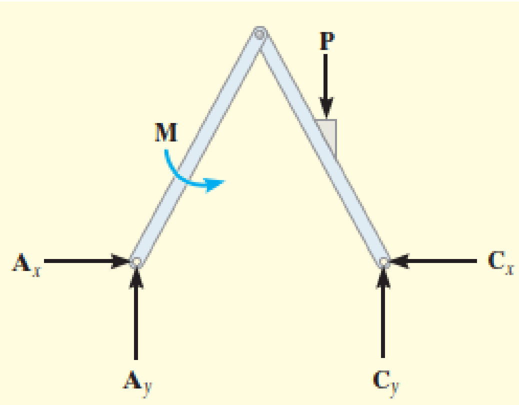
Part (a). By inspection, members BA and BC are *not* two-force members. Instead, as shown on the free-body diagrams, Fig. 6–21*b*, BC is subjected to a force from each of the pins at B and C and the external force $\mathbf{P}$. Likewise, AB is subjected to a force from each of the pins at A and B and the external couple moment $\mathbf{M}$. The pin forces are represented by their x and y components.

Part (b). The pin at B is subjected to only *two forces*, i.e., the force of member BC and the force of member AB. For *equilibrium* these forces (or their respective components) must be equal but opposite, Fig. 6–21*c*. Realize that Newton's third law is applied between the pin and its connected members, i.e., the effect of the pin on the two members, Fig. 6–21*b*, and the equal but opposite effect of the two members on the pin, Fig. 6–21*c*. In the same manner, there are three forces on pin A, Fig. 6–21*d*, caused by the force components of member AB and each of the two pin leafs.

Part (c). The free-body diagram of both members connected together, yet removed from the supporting pins at A and C, is shown in Fig. 6–21*e*. The force components $\mathbf{B}_x$ and $\mathbf{B}_y$ are *not shown* on this diagram since they are *internal* forces (Fig. 6–21*b*) and therefore cancel out. Also, to be consistent when later applying the equilibrium equations, the unknown force components at A and C must act in the *same sense* as those shown in Fig. 6–21*b*.







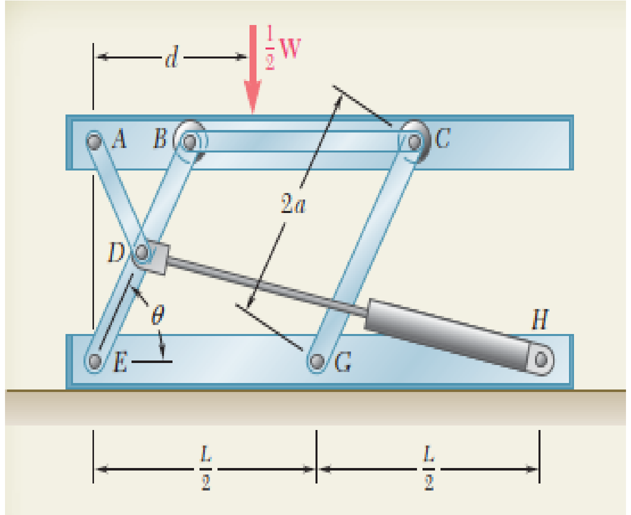
(e)

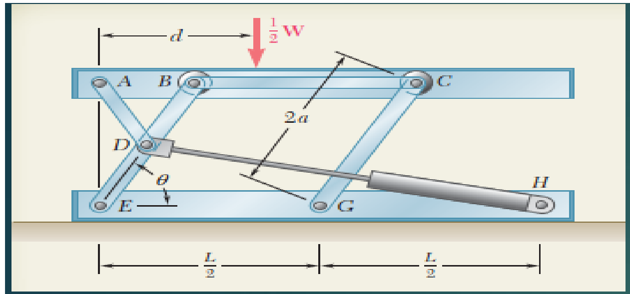
ANALYSIS OF A MACHINE

Example 7.7

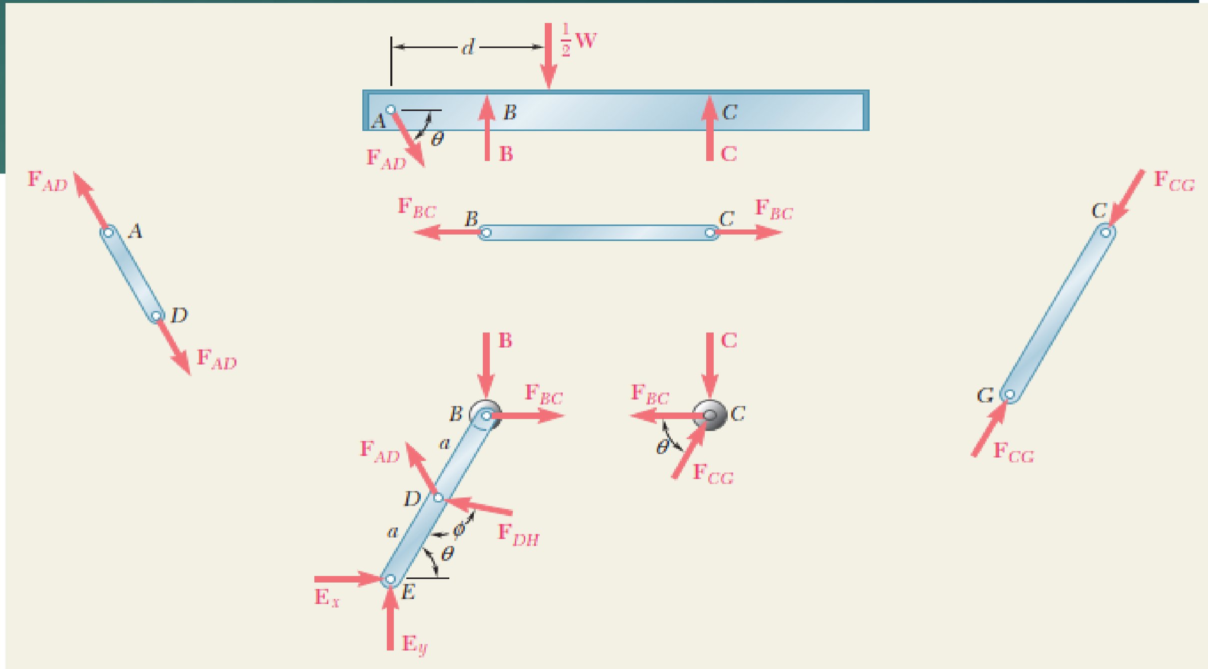
Question

- A hydraulic-lift table is used to raise a 1000-kg crate. It consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown) Members EDB and CG are each of length 2a, and member AD is pinned to the midpoint of EDB. If the crate is placed on the table, so that half of its weight is supported by the system shown, determine the force exerted by each cylinder in raising the crate for θ 60°, a = 0.70 m, and L = 3.20 m. Show that the result obtained is independent of the distance d.



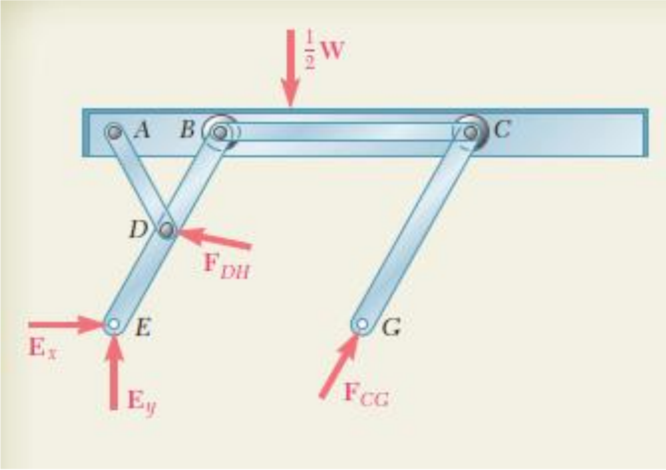


ANALYSIS OF A MACHINE
Example 7.7

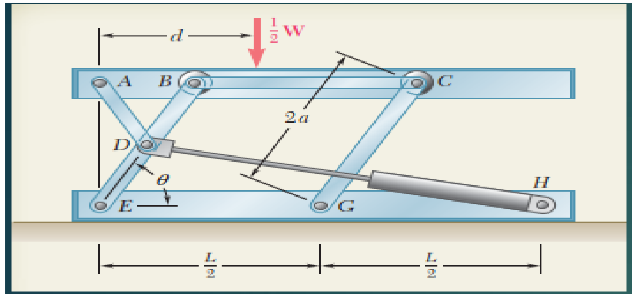


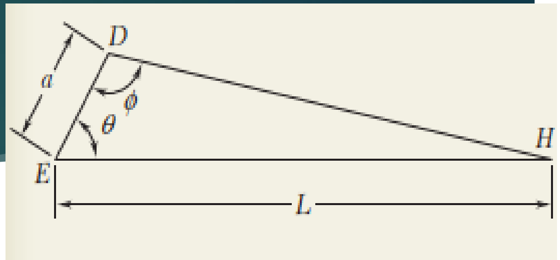
Solutions

The machine considered consists of the platform and of the linkage. Its free-body diagram includes an input force $\mathbf{F}_{DH}$ exerted by the cylinder, the weight $\frac{1}{2}\mathbf{W}$, equal and opposite to the output force, and reactions at E and G that we assume to be directed as shown. Since more than three unknowns are involved, this diagram will not be used. The mechanism is dismembered and a free-body diagram is drawn for each of its component parts. We note that AD, BC, and CG are two-force members. We already assumed member CG to be in compression; we now assume that AD and BC are in tension and direct as shown the forces exerted on them. Equal and opposite vectors will be used to represent the forces exerted by the two-force members on the platform, on member BDE, and on roller C.



ANALYSIS OF A MACHINE
Example 7.7



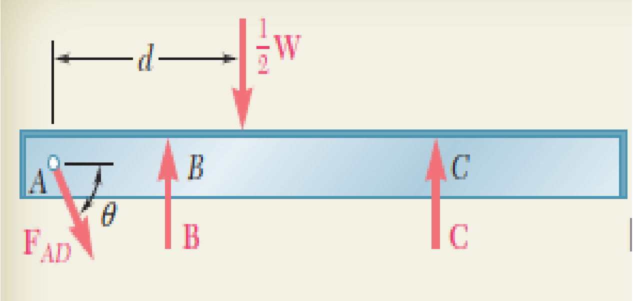


Solutions

Free Body: Platform *ABC*.

$\xrightarrow{+}\Sigma F_x = 0$: $\quad F_{AD} \cos\theta = 0 \quad F_{AD} = 0$

$+\uparrow\Sigma F_y = 0$: $\quad B + C - \frac{1}{2}W = 0 \quad B + C = \frac{1}{2}W \quad (1)$



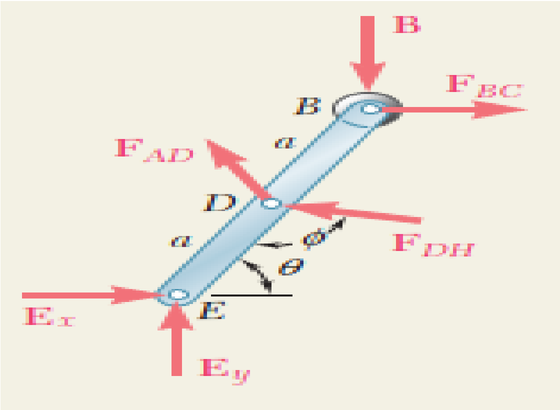
Free Body: Roller *C*. We draw a force triangle and obtain $F_{BC} = C \cot\theta$.

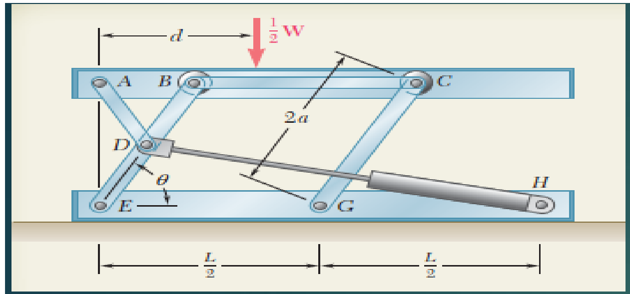
Free Body: Member *BDE*. Recalling that $F_{AD} = 0$,

$+\circlearrowleft\Sigma M_E = 0$: $\quad F_{DH} \cos(\phi - 90°)a - B(2a\cos\theta) - F_{BC}(2a\sin\theta) = 0$

$F_{DH} a \sin\phi - B(2a\cos\theta) - (C\cot\theta)(2a\sin\theta) = 0$

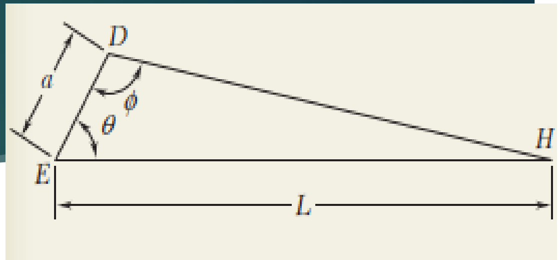
$F_{DH} \sin\phi - 2(B + C)\cos\theta = 0$





ANALYSIS OF A MACHINE

Example 7.7



Solutions

Recalling Eq. (1), we have

$$F_{DH} = W\frac{\cos\theta}{\sin\phi} \quad (2)$$

and we observe that the result obtained is independent of d. ◀

Applying first the law of sines to triangle EDH, we write

$$\frac{\sin\phi}{EH} = \frac{\sin\theta}{DH} \qquad \sin\phi = \frac{EH}{DH}\sin\theta \quad (3)$$

Using now the law of cosines, we have

$$(DH)^2 = a^2 + L^2 - 2aL\cos\theta$$
$$= (0.70)^2 + (3.20)^2 - 2(0.70)(3.20)\cos 60°$$
$$(DH)^2 = 8.49 \qquad DH = 2.91 \text{ m}$$

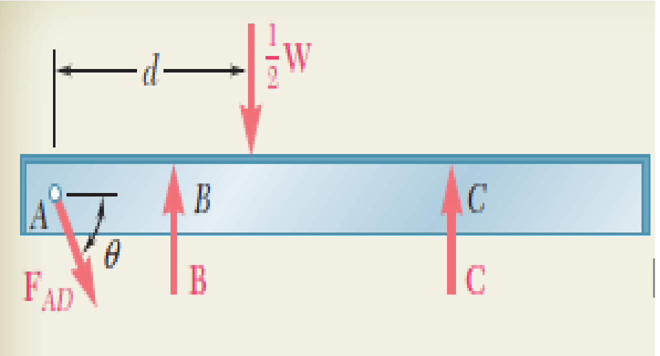
We also note that

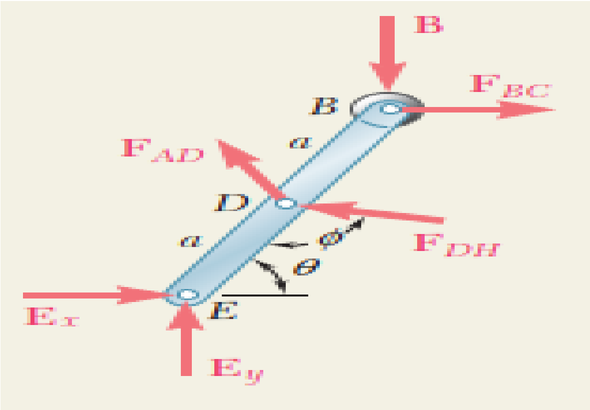
$$W = mg = (1000 \text{ kg})(9.81 \text{ m/s}^2) = 9810 \text{ N} = 9.81 \text{ kN}$$

Substituting for $\sin\phi$ from (3) into (2) and using the numerical data, we write

$$F_{DH} = W\frac{DH}{EH}\cot\theta = (9.81 \text{ kN})\frac{2.91 \text{ m}}{3.20 \text{ m}}\cot 60°$$

$$F_{DH} = 5.15 \text{ kN}$$ ◀

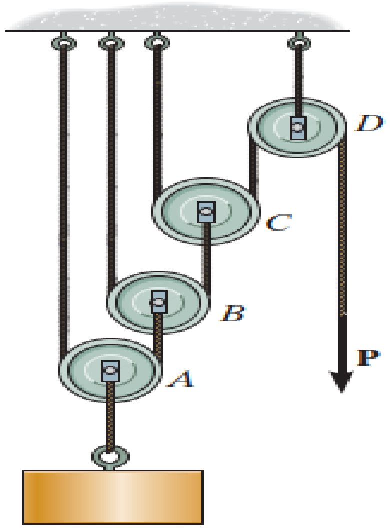




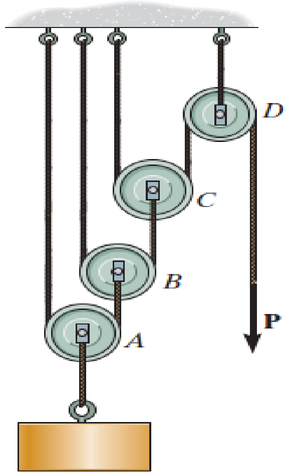
ANALYSIS OF A MACHINE
Example 7.8

Question

- Determine the force required to hold the 100-lb weight in equilibrium



ANALYSIS OF A MACHINE
Example 7.8



Solutions

Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of pulley A on the free - body diagram, Fig. a,

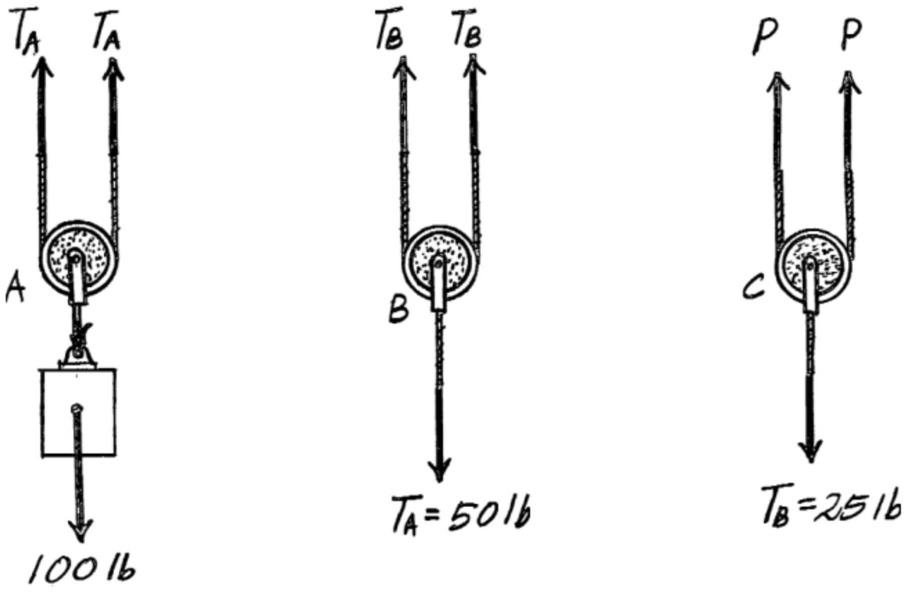
$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 100 = 0 \qquad T_A = 50 \text{ lb}$

Applying $\Sigma F_y = 0$ to the free - body diagram of pulley B, Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad 2T_B - 50 = 0 \qquad T_B = 25 \text{ lb}$

From the free - body diagram of pulley C, Fig. c,

$+\uparrow \Sigma F_y = 0; \qquad 2P - 25 = 0 \qquad P = 12.5 \text{ lb}$ **Ans.**

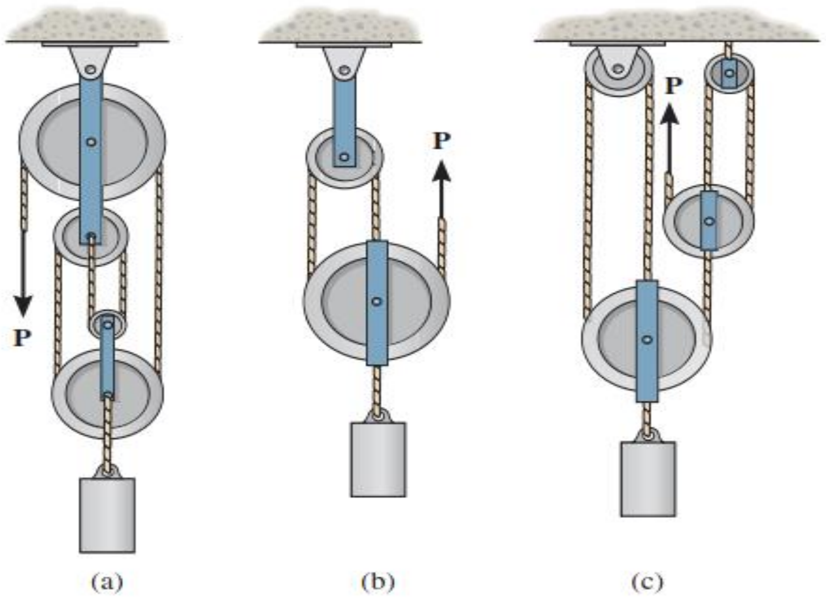


(a) (b) (c)

ANALYSIS OF A MACHINE
Example 7.9

Question

- In each case, determine the force P required to maintain equilibrium. The block weighs 100 lb.



ANALYSIS OF A MACHINE
Example 7.9

Solutions

Equations of Equilibrium:

a) $+\uparrow \Sigma F_y = 0; \qquad 4P - 100 = 0$

$$P = 25.0 \text{ lb}$$

b) $+\uparrow \Sigma F_y = 0; \qquad 3P - 100 = 0$

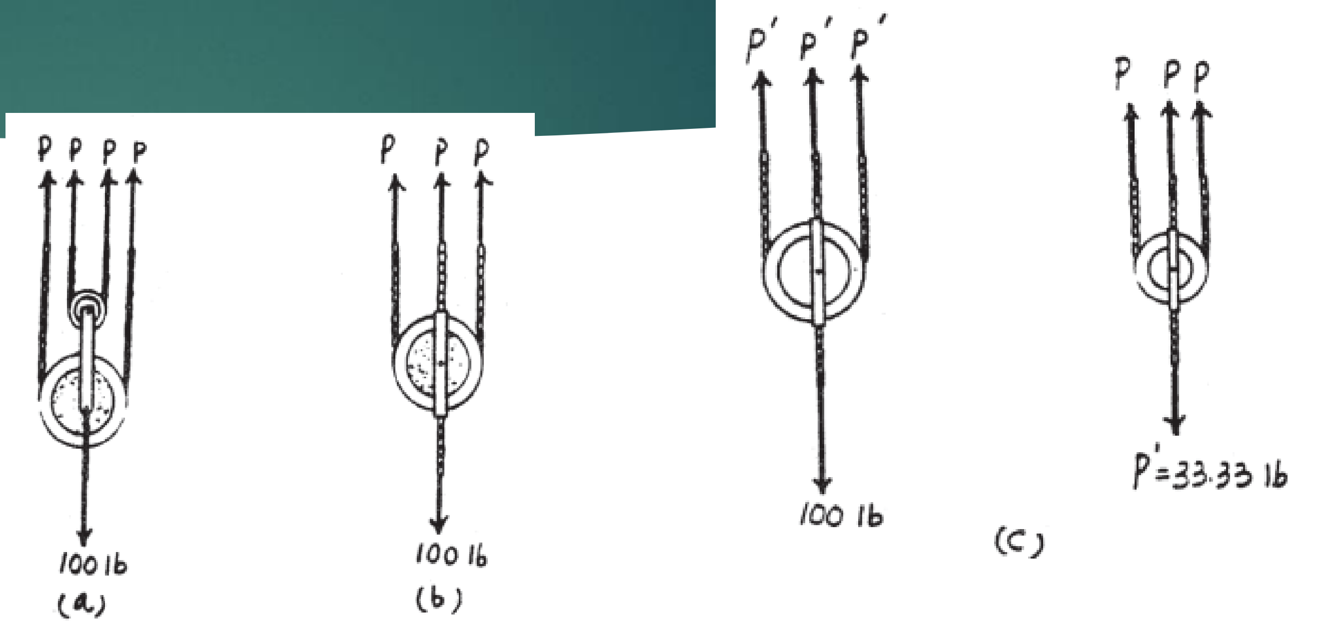
$$P = 33.3 \text{ lb}$$

c) $+\uparrow \Sigma F_y = 0; \qquad 3P' - 100 = 0$

$$P' = 33.33 \text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad 3P - 33.33 = 0$

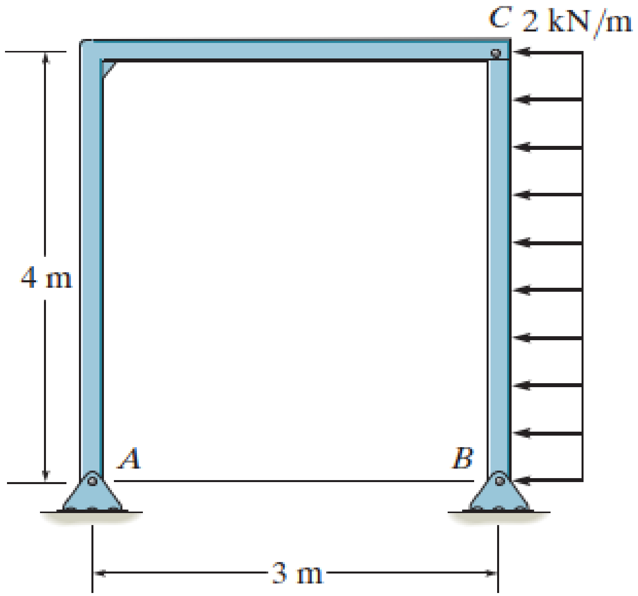
$$P = 11.1 \text{ lb}$$



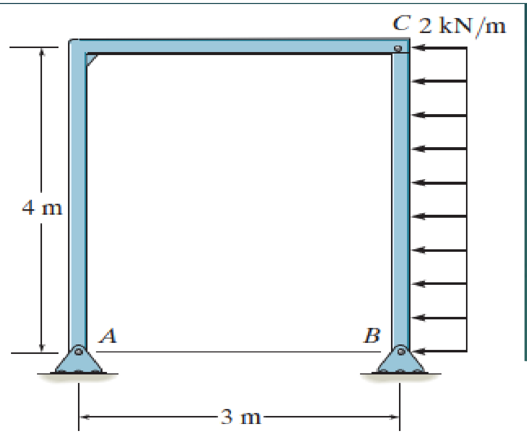
ANALYSIS OF A MACHINE
Example 7.10

Question

- Determine the horizontal and vertical components of force that pins A and B exert on the frame.



ANALYSIS OF A MACHINE
Example 7.10



Solutions

Free Body Diagram. The frame will be dismembered into members *BC* and *AC*. The solution will be very much simplified if one recognizes that member *AC* is a two force member. The *FBDs* of member *BC* and pin *A* are shown in Figs. *a* and *b*, respectively.

Equations of Equilibrium. Consider the equilibrium of member *BC*, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0; \quad 2(4)(2) - F_{AC}\left(\frac{3}{5}\right)(4) = 0 \quad F_{AC} = 6.6667 \text{ kN}$

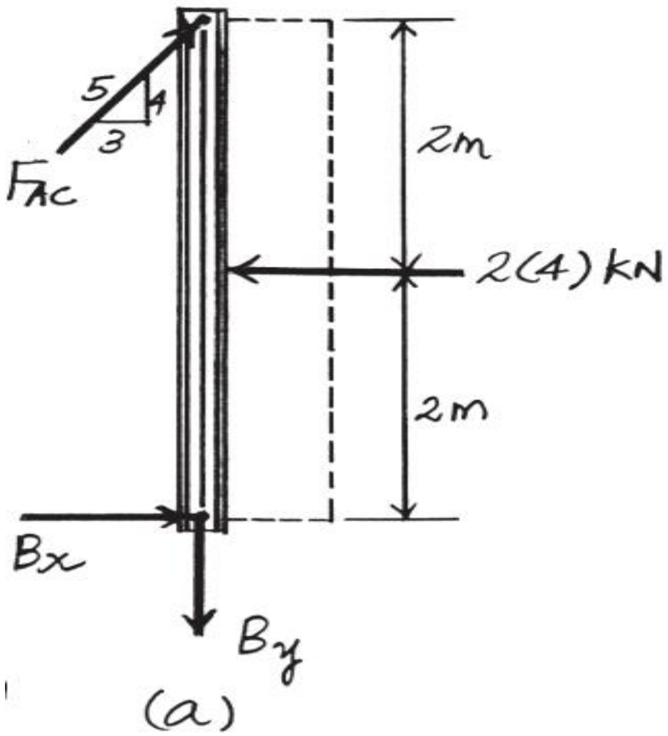
$\circlearrowleft + \Sigma M_C = 0; \quad B_x(4) - 2(4)(2) = 0 \quad B_x = 4.00 \text{ kN}$ **Ans.**

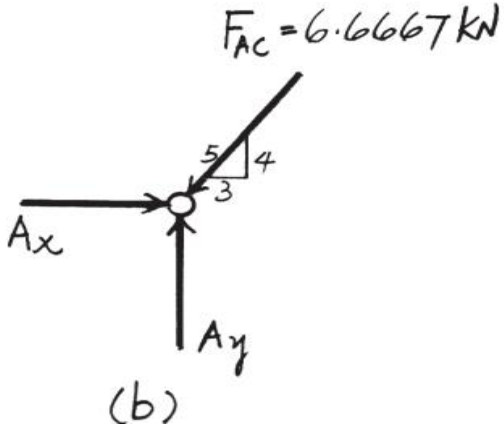
$+\uparrow \Sigma F_y = 0; \quad 6.6667\left(\frac{4}{5}\right) - B_y = 0 \quad B_y = 5.333 \text{ kN} = 5.33 \text{ kN}$ **Ans.**

Then, the equilibrium of pin *A* gives

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 6.6667\left(\frac{3}{5}\right) = 0 \quad A_x = 4.00 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - 6.6667\left(\frac{4}{5}\right) = 0 \quad A_y = 5.333 \text{ kN} = 5.33 \text{ kN}$ **Ans.**





HOME WORK EXERCISE

6-66, 6-8, 6-73, 6-75, 6-79, 6-81, 6-86, 6-88, 6-91, 6-99, 6-101, 6-108, 6-112 ,6-115, 6-117

THE END – THANK YOU