CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

Lecture A4 (part I)

- ❖ INTRODUCTION TO MOMENTS, COUPLES & RESULTANT FORCES
- ✓ CROSS PRODUCT
- ✓ TRIPLE SCALAR PRODUCT
- ✓ INTERNAL & EXTERNAL FORCES

LECTURE OBJECTIVES

- **To discuss the concept of the moment of a force and show how to calculate it in 2D & 3D.**
- **To provide a method for finding the moment of a force about a specified axis.**
- **To define the moment of a couple.**
- **To show how to find the resultant effect of a non-concurrent force system.**

CHAPTER INTRODUCTION

- *Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered*
- *Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body*
- *In this chapter you will study the effect of forces exerted on a rigid body, and you will learn how to replace a given system of forces by a* ***simpler equivalent system****.*

CHAPTER INTRODUCTION

- In this lecture you will learn how the forces can be replaced with a simpler equivalent system:
  - moment of a force about a point
  - moment of a force about an axis
  - moment due to a couple
- *Note that the determination of these quantities (M) involves the computation of* ***vector products*** *and [****scalar products****] of two vectors,*

CHAPTER INTRODUCTION

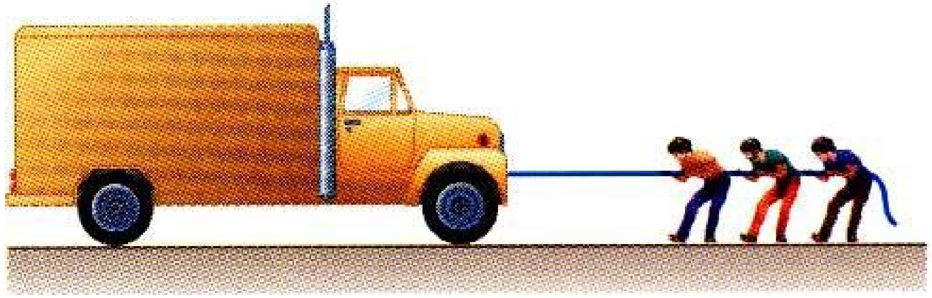
- *Another concept introduced in this lecture is that of a couple, any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple*
- *This basic system is called a* ***force-couple system***
- *In the case of concurrent, coplanar, or parallel forces, the equivalent force-couple system can be further reduced to a single force, called the resultant of the system, or to a single couple, called the* ***resultant couple of the system.***

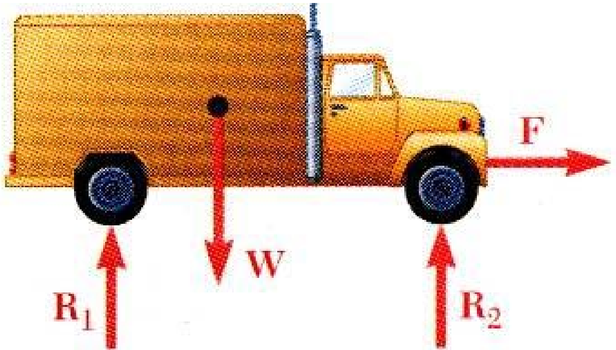
INTERNAL AND EXTERNAL FORCES

- Forces acting on rigid bodies are divided into two groups: External & Internal forces
- The internal forces are the forces which hold together the particles forming the rigid body.
- If the rigid body is structurally composed of several parts, the forces holding the component parts together are also defined as internal forces.
- Internal forces will be considered in other upcoming lectures.

INTERNAL AND EXTERNAL FORCES

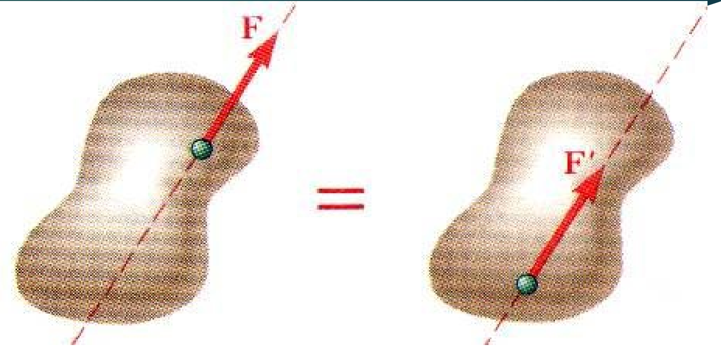
- The external forces represent the action of other bodies on the rigid body under consideration as shown using a FBD.
- They are entirely responsible for the external behavior of the rigid body. They will either cause it to move or ensure that it remains at rest.
- If unopposed, each external force can impart a motion of translation or rotation, or both
- We shall be concerned only with external forces in this lecture

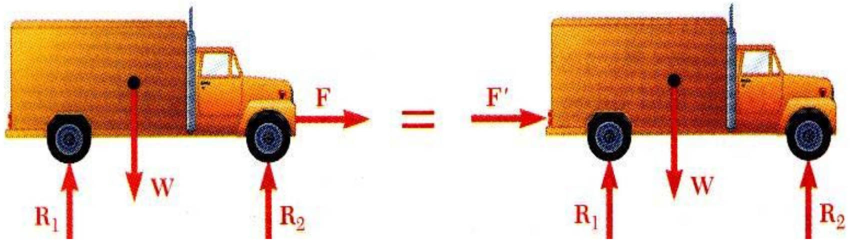


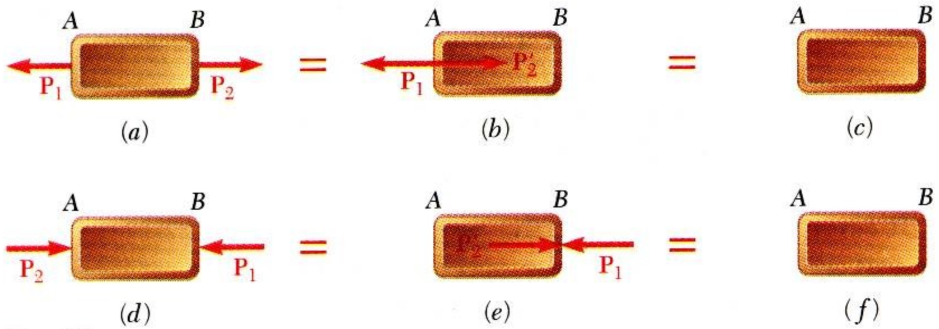


PRINCIPLE OF TRANSMISSIBILITY

- The principle states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force F acting at a given point of the rigid body is replaced by a force F' of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action
- Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck
- Principle of transmissibility may not always apply in determining internal forces and deformations

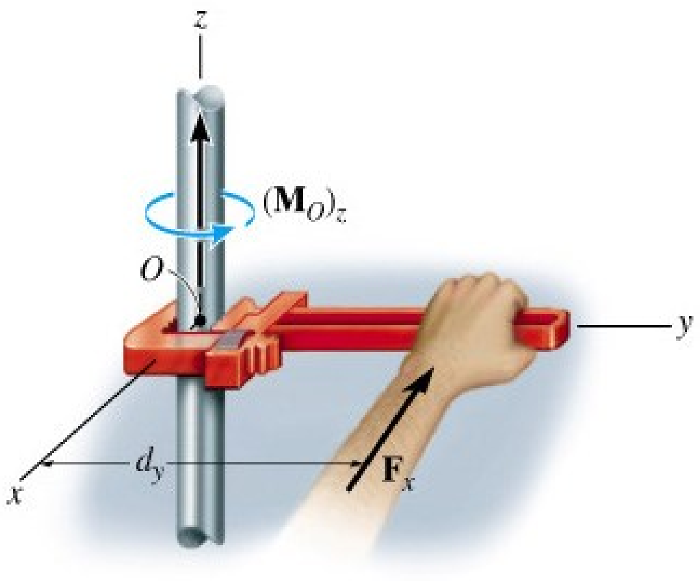






MOMENT OF A FORCE
Scalar Formulation

- The tendency of a force to rotate a rigid body about any defined axis is called the Moment of the force about the axis
- The moment, M, of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).
- The Moment of Force (**F**) about an axis through Point O, the Moment of **F** about O, is the product of the magnitude of the force and the perpendicular distance between O and the line of action of Force (**F**)

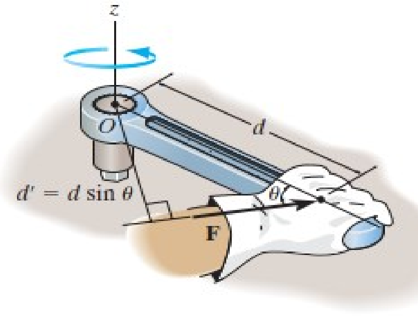


(a)

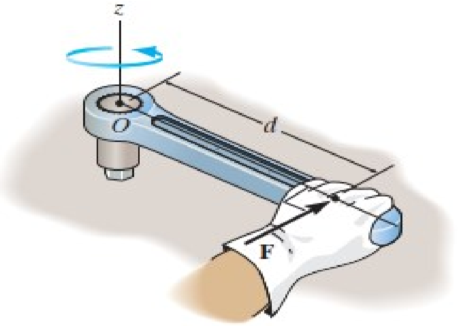
$$M = Fx * dy$$

MOMENT OF A FORCE
Scalar Formulation

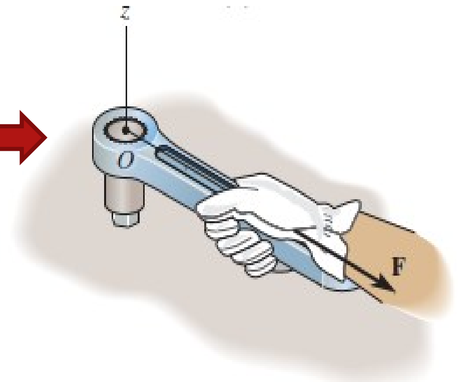
- Consider a wrench used to unscrew the bolt in (a) If a force is applied to the handle of the wrench it will tend to turn the bolt about point O.
- The magnitude of the moment is directly proportional to the magnitude of F and the perpendicular distance or moment arm d.
- Max. moment or turning effect occurs at an angle 90° in (a)
- If F is applied along the wrench, Fig. c, its moment arm will be zero since the line of action of F will intersect point O (the z axis).



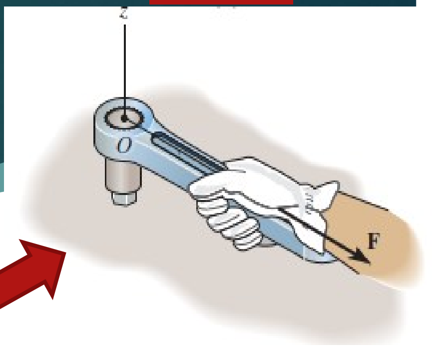
(b)



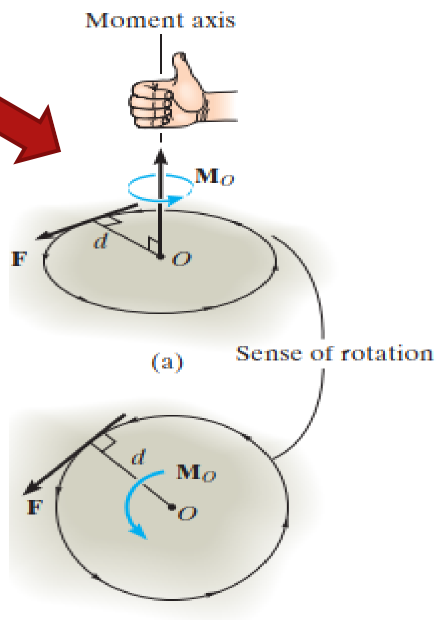
(a)



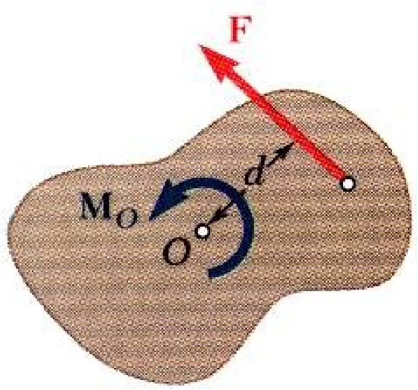
MOMENT OF A FORCE
Scalar Formulation



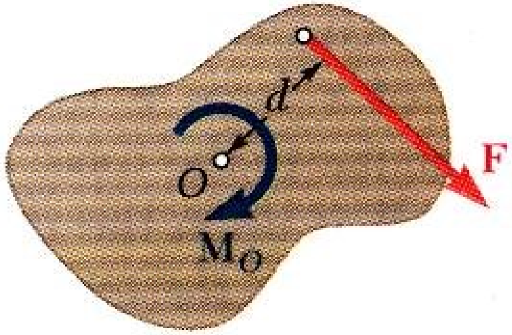
- As a result, the moment of F about O is also zero and no turning can occur.
- We can generalize the above discussion and consider the force F and point O which lie in the shaded plane as shown in Fig. a. The moment Mo about point O, or about an axis passing through O and perpendicular to the plane, is a vector quantity since it has a specified **magnitude and direction**.
- **The magnitude of Mo = Fd**



MOMENT OF A FORCE
Scalar Formulation

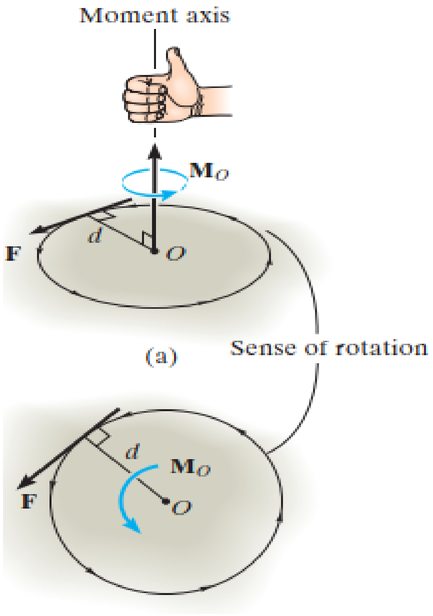


(a) $M_O = +Fd$



(b) $M_O = -Fd$

- **The direction of Mo** is defined by its moment axis, which is perpendicular to the plane that contains the force F and its moment arm d.
- The right-hand rule is used to establish the sense of direction of Mo.
- counterclockwise rotationmeans the moment vector is positive.



MOMENT OF A FORCE

Scalar Formulation

- Resultant Moment. For two-dimensional problems, where all the forces lie within the x–y plane, Fig. 4–3, the resultant moment (M_R) about point O can be determined by finding the algebraic sum of the moments caused by all the forces in the system.
- As a convention, we will generally consider positive moments as counterclockwise & clockwise moments will be negative.
- Doing this, the directional sense of each moment can be represented by a plus or minus sign.

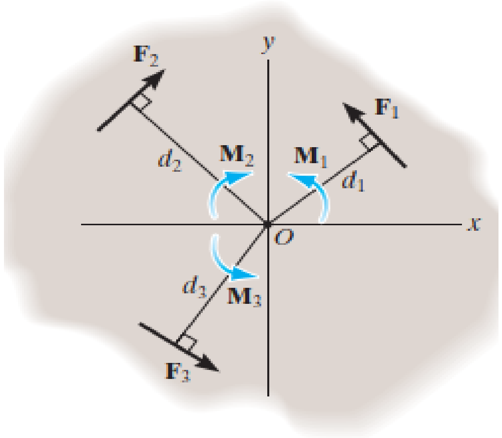
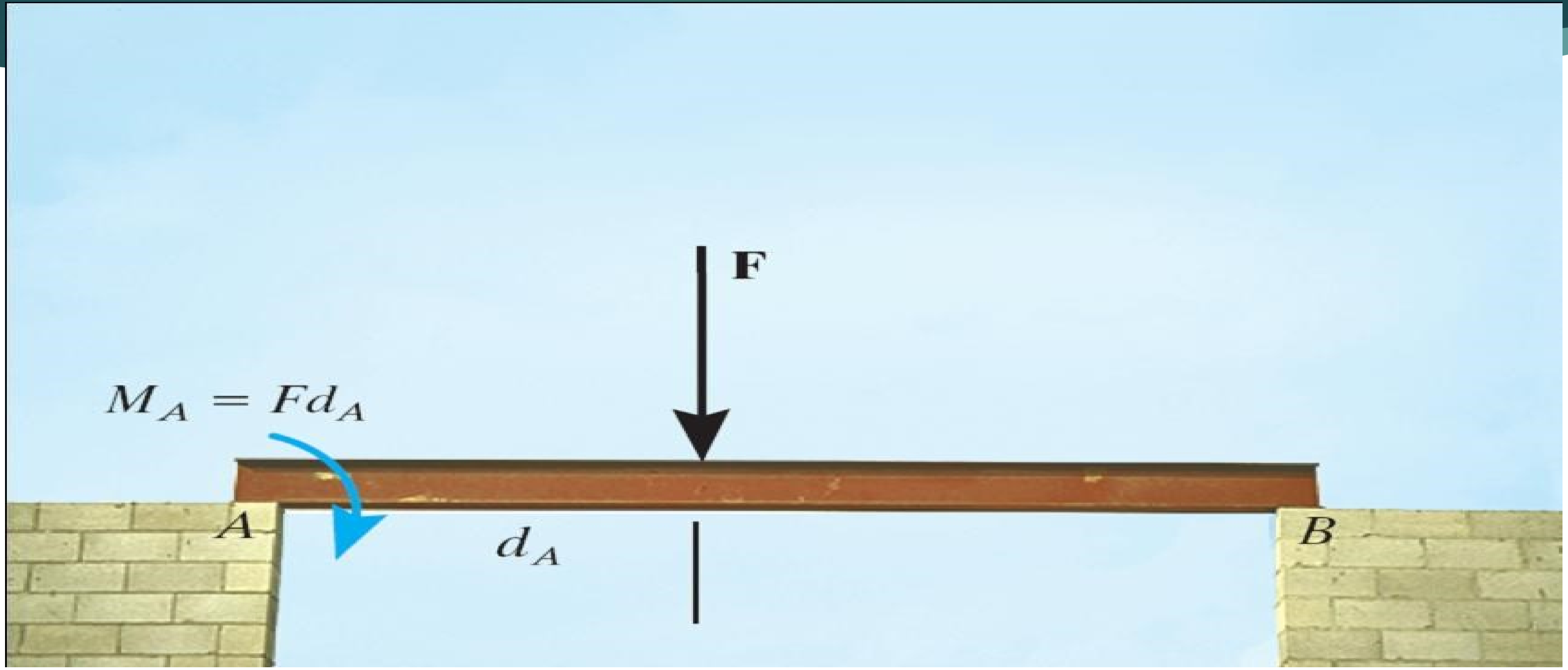


Fig. 4–3

$$\circlearrowleft +(M_R)_O = \Sigma Fd; \quad (M_R)_O = F_1 d_1 - F_2 d_2 + F_3 d_3$$

MOMENT OF A FORCE

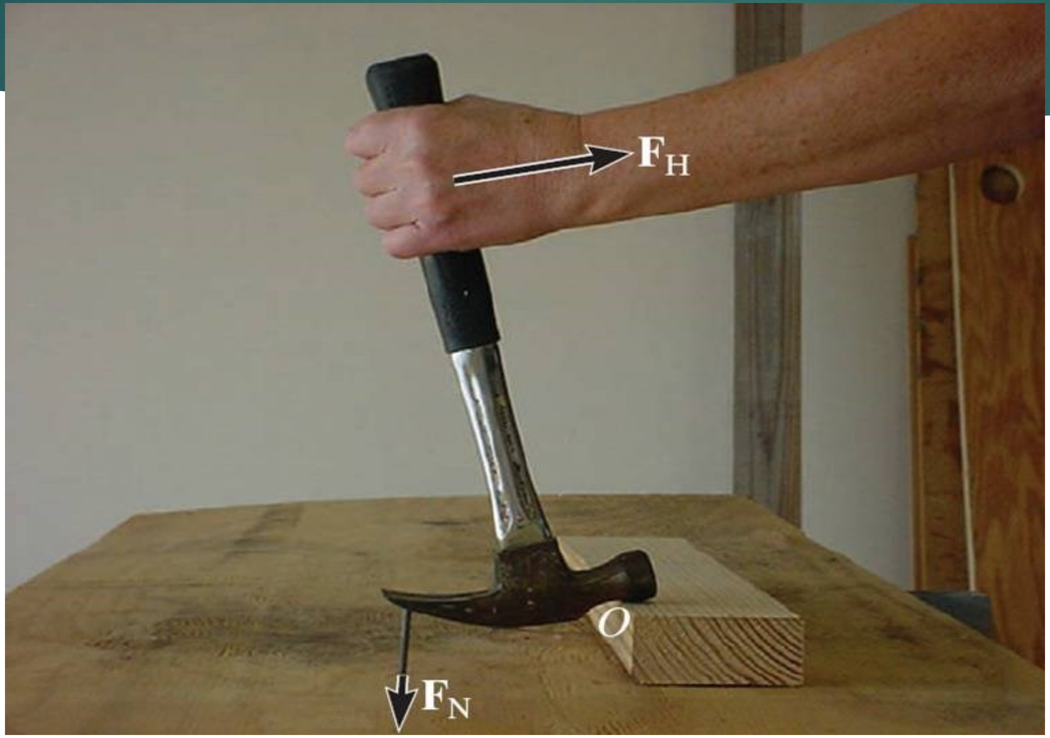
Application



- Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the beam supports.
- What do you think those impacts are at points A and B?

MOMENT OF A FORCE

Application



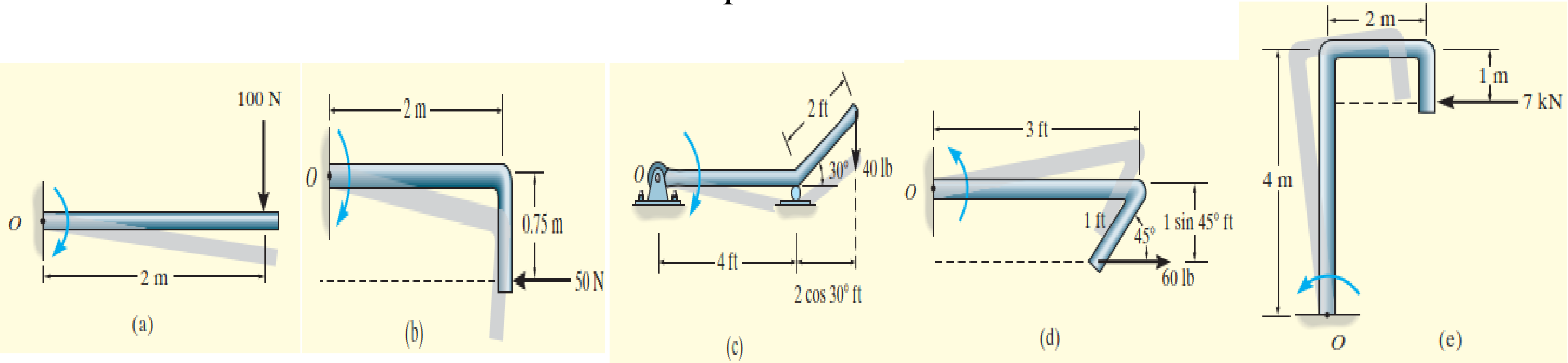
Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

MOMENT OF A FORCE

Example 4.1

Question

Determine the moment of the force about point O



MOMENT OF A FORCE

Example 4.1

Solution

- The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus

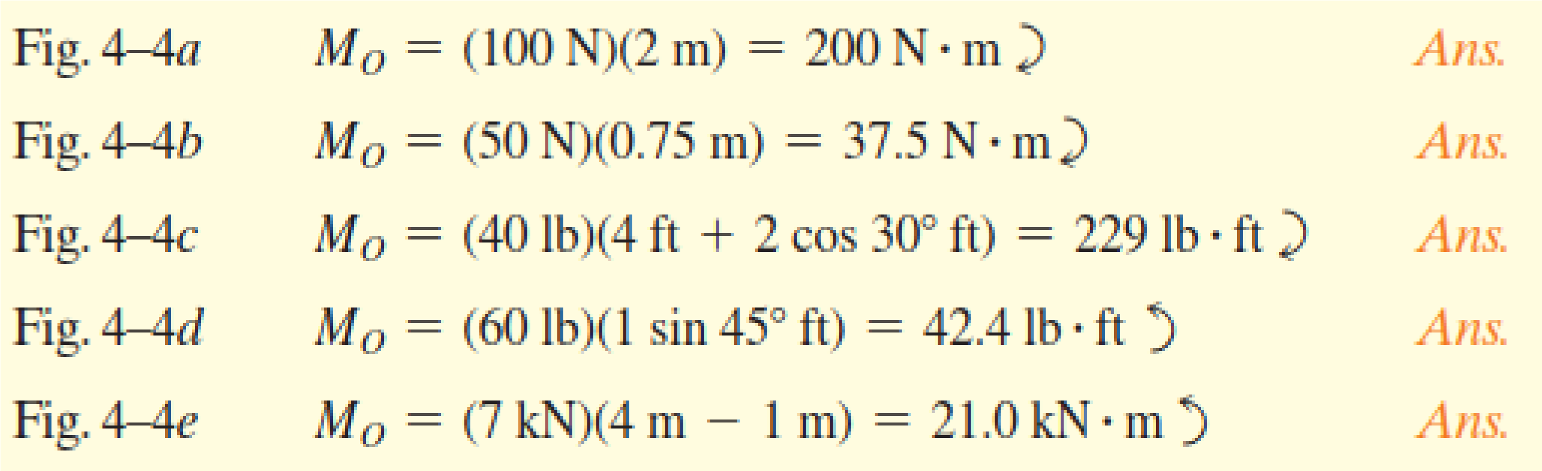


Fig. 4–4*a* $M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m}$ ↻ *Ans.*

Fig. 4–4*b* $M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m}$ ↻ *Ans.*

Fig. 4–4*c* $M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft}$ ↻ *Ans.*

Fig. 4–4*d* $M_O = (60\text{ lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{ lb}\cdot\text{ft}$ ↺ *Ans.*

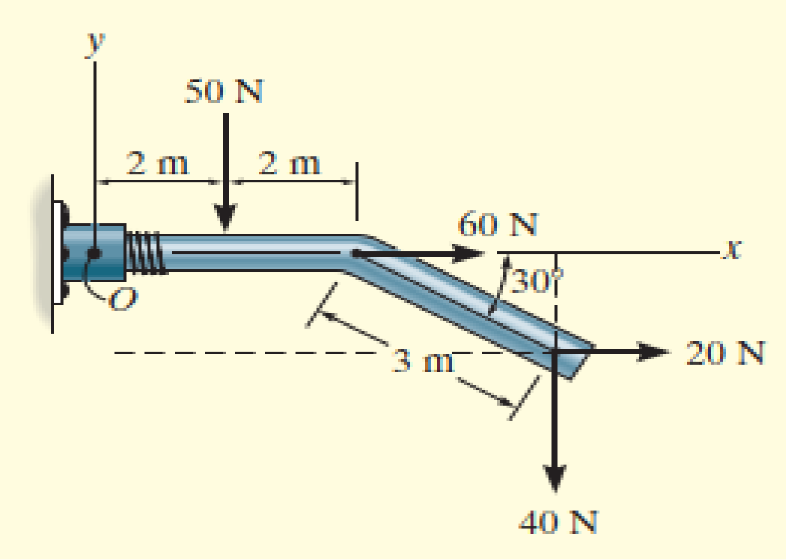
Fig. 4–4*e* $M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m}$ ↺ *Ans.*

MOMENT OF A FORCE

Example 4.2

Question

Determine the resultant moment of the four forces acting on the rod shown about point O.

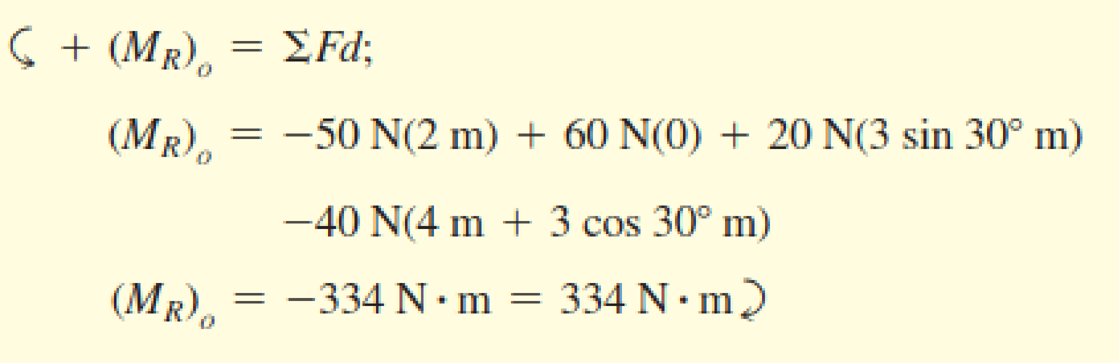


MOMENT OF A FORCE

Example 4.2

Solution

- Assuming that positive moments act in the + k direction, i.e.,
- counterclockwise, we have

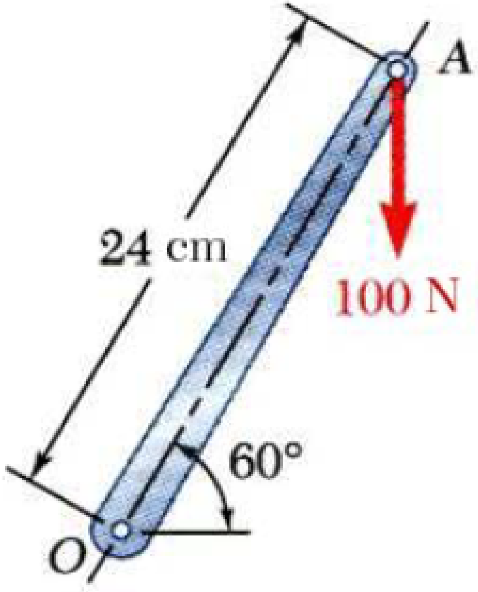


$$
\begin{aligned}
\circlearrowleft + (M_R)_o &= \Sigma Fd; \\
(M_R)_o &= -50\ \text{N}(2\ \text{m}) + 60\ \text{N}(0) + 20\ \text{N}(3 \sin 30^\circ\ \text{m}) \\
&\quad -40\ \text{N}(4\ \text{m} + 3 \cos 30^\circ\ \text{m}) \\
(M_R)_o &= -334\ \text{N}\cdot\text{m} = 334\ \text{N}\cdot\text{m} \circlearrowright
\end{aligned}
$$

- For this calculation, note how the moment-arm distances for the 20-N
- and 40-N forces are established from the extended (dashed) lines of
- action of each of these forces.

MOMENT OF A FORCE

Example 4.3



A 100-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

a) moment about *O*,

b) horizontal force at *A* which creates the same moment,

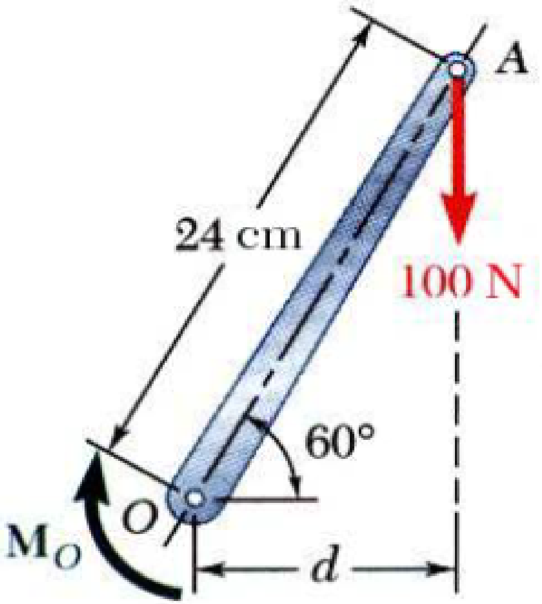
c) smallest force at A which produces the same moment,

d) location for a 240-N vertical force to produce the same moment,

e) whether any of the forces from b, c, and d is equivalent to the original force.

MOMENT OF A FORCE
Example 4.3

Solution



a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

$$M_O = Fd$$

$$d = (24\,\text{cm})\cos 60° = 12\,\text{cm}$$

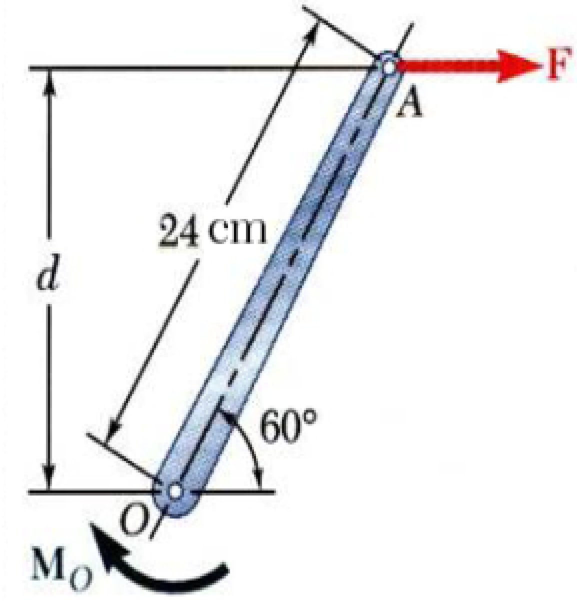
$$M_O = (100\,\text{N})(12\,\text{cm})$$

$$\boxed{M_O = 1200\,\text{N}\cdot\text{cm}}$$

MOMENT OF A FORCE

Example 4.3

Solution



c) Horizontal force at *A* that produces the same moment,

$$d = (24\text{ cm})\sin 60^\circ = 20.8\text{ cm}$$

$$M_O = Fd$$

$$1200\text{ N}\cdot\text{cm} = F(20.8\text{ cm})$$

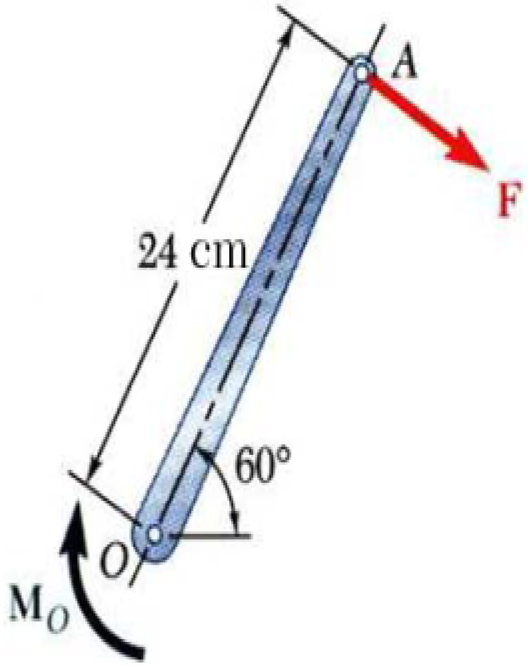
$$F = \frac{1200\text{ N}\cdot\text{cm}}{20.8\text{ cm}}$$

$$\boxed{F = 57.7\text{ N}}$$

MOMENT OF A FORCE

Example 4.3

Solution



c) The smallest force *A* to produce the same moment occurs when the perpendicular distance is a maximum or when *F* is perpendicular to *OA*.

$$M_O = Fd$$

$$1200\ \text{N}\cdot\text{cm} = F(24\ \text{cm})$$

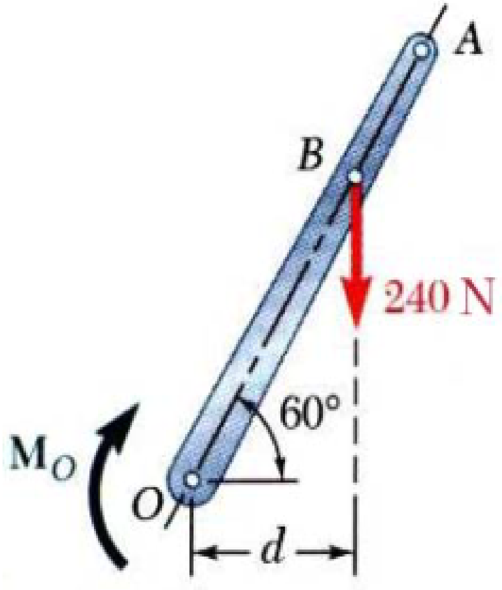
$$F = \frac{1200\ \text{N}\cdot\text{cm}}{24\ \text{cm}}$$

$$\boxed{F = 50\ \text{N}}$$

MOMENT OF A FORCE

Example 4.3

Solution



d) To determine the point of application of a 240 lb force to produce the same moment,

$$M_O = Fd$$

$$1200\ \text{N}\cdot\text{cm} = (240\ \text{N})d$$

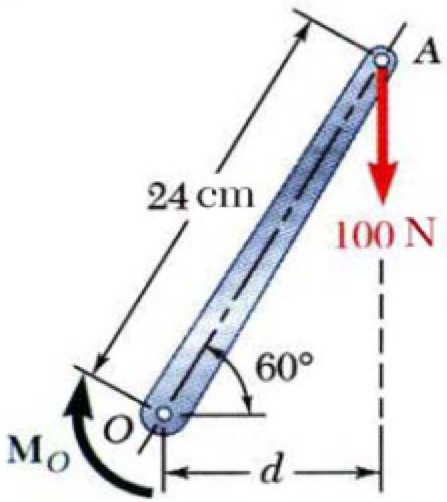
$$d = \frac{1200\ \text{N}\cdot\text{cm}}{240\ \text{N}} = 5\ \text{cm}$$

$$OB\cos 60^\circ = 5\ \text{cm} \qquad \boxed{OB = 10\ \text{cm}}$$

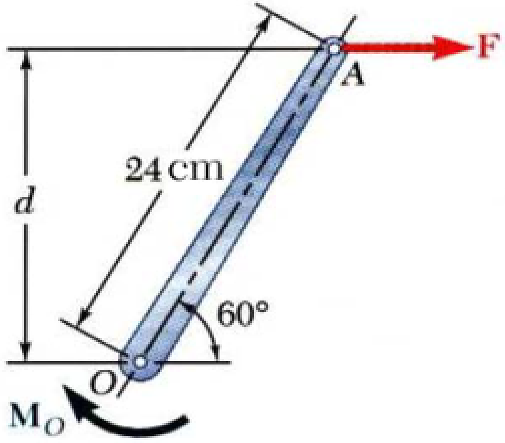
MOMENT OF A FORCE

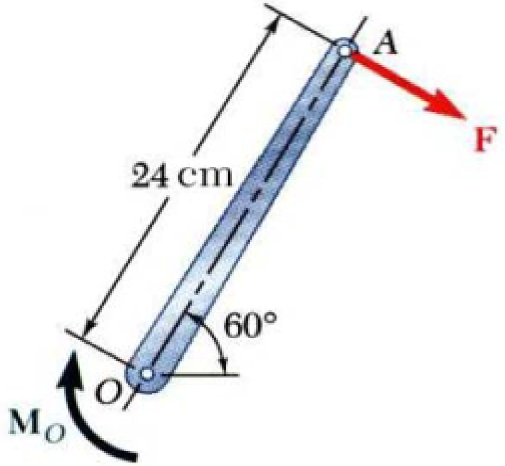
Example 4.3

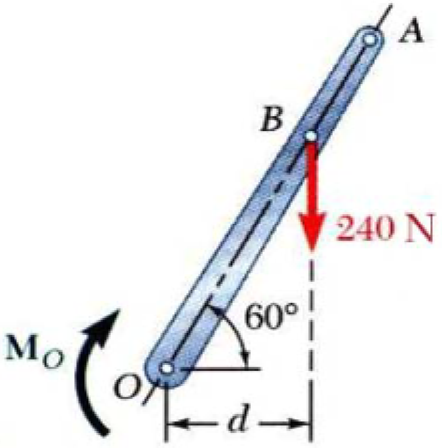
Solution



e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 N force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 N force.







THE VECTOR PRODUCT OR CROSS PRODUCT

- Before formulating vector moments, it is first necessary to expand our knowledge of vector algebra and introduce the cross-product method of vector multiplication
- Concept of moments of a force about a point is more easily understood through application of **cross product**
- The **cross product** of two vectors **P** and **Q** yields the vector **V**, which is written

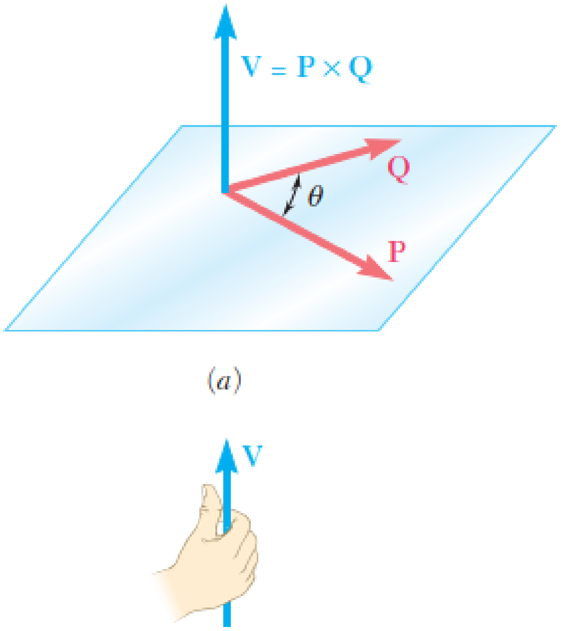
$$\mathbf{V} = \mathbf{P} \times \mathbf{Q}$$

- and is read "**V equals P cross Q**"

THE VECTOR PRODUCT OR CROSS PRODUCT

- Vector product of two vectors is defined as the vector ***V*** which satisfies the following:

1. Line of action of *V* is perpendicular to plane
2. Magnitude of *V* = P x Q = *PQ* sin θ
3. Direction of *V* is obtained from the right-hand rule

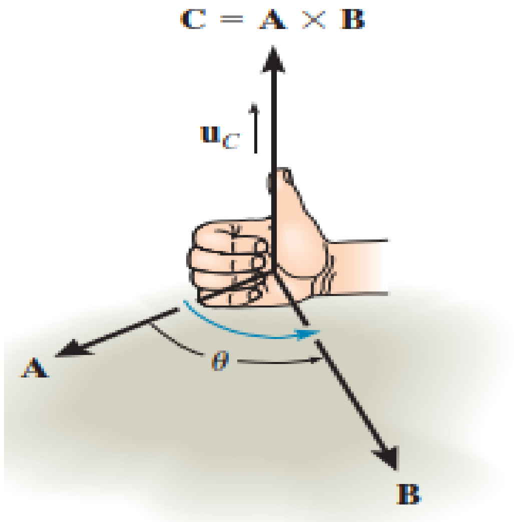


(*a*)

(*b*)

THE VECTOR PRODUCT OR CROSS PRODUCT

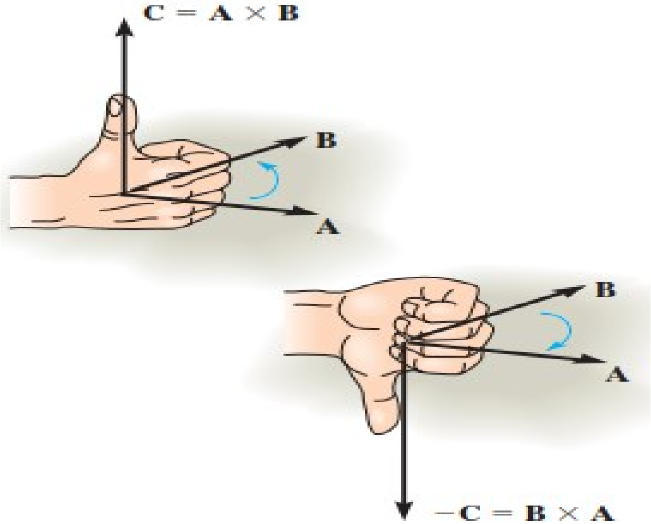
- The magnitude of Vector product **C** can be defined as the product of the magnitudes of **A** and **B** and the sine of the angle θ between their tails ($180° \geq \theta \leq 0°$) Thus, **C** = AB sin θ
- Vector product has a direction that is perpendicular to the plane containing **A** and **B** such that **C** is specified by the right-hand rule
- Knowing both the magnitude and direction of **C**, we can write **C** = **A** x **B** = (ABsinθ)**u**C
- where the scalar ABsinθ defines the magnitude of **C** and the unit vector **u**C defines the direction of **C**.



THE VECTOR PRODUCT OR CROSS PRODUCT

- Laws of Operation.
  - Are not commutative $A \times B = -(B \times A)$
  - Are distributive $A \times (B_1 + B_2) = A \times B_1 + A \times B_2$
  - Are not associative (only by a scalar) $(A \times B) \times D \neq A \times (B \times D)$

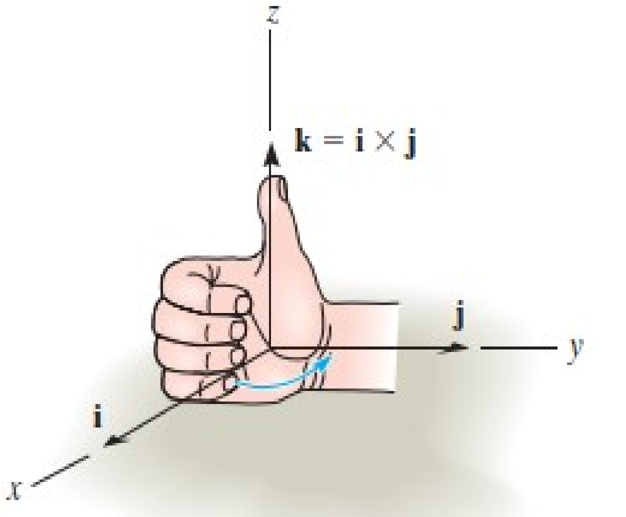
$$a(A x B) = (aA) x B = A x (a B) = (A x B) a$$



- *It is important to note that proper order of the cross products must be maintained, since they are not commutative & not associative for cross product.*

THE VECTOR PRODUCT OR CROSS PRODUCT

Cartesian Vector Formulation



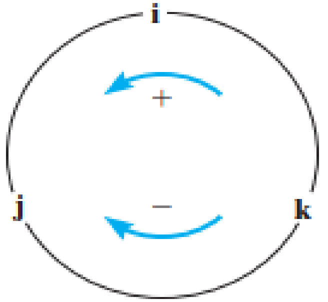
- Eqn. shown may be used to find the cross product of any pair Cartesian unit vectors. $\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB\sin\theta)\mathbf{u}_C$
- For example, to find i x j, the magnitude of the resultant vect is (i)(j)(sin 90°) = (1)(1)(1) = 1, and its direction is determined using the right-hand rule.
- Note that the resultant vector points in the +k direction.
- Thus, ixj = (1)k. In a similar manner

$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = 0$

$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = 0$

$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = 0$

- **These results should not be memorized; rather, just understood the right-hand rule and the definition of the cross product**



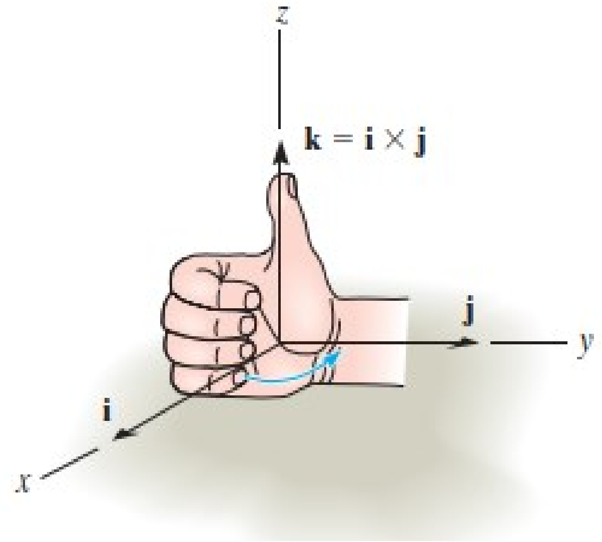
THE VECTOR PRODUCT OR CROSS PRODUCT

Cartesian Vector Formulation

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin\theta)\mathbf{u}_C$$

- Let us now consider the cross product of two general vectors A and B which are expressed in Cartesian vector form.

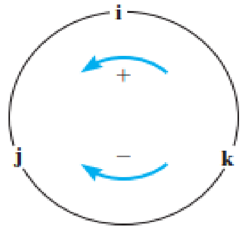
$$\begin{aligned}
\mathbf{A} \times \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\
&= A_xB_x(\mathbf{i} \times \mathbf{i}) + A_xB_y(\mathbf{i} \times \mathbf{j}) + A_xB_z(\mathbf{i} \times \mathbf{k}) \\
&\quad + A_yB_x(\mathbf{j} \times \mathbf{i}) + A_yB_y(\mathbf{j} \times \mathbf{j}) + A_yB_z(\mathbf{j} \times \mathbf{k}) \\
&\quad + A_zB_x(\mathbf{k} \times \mathbf{i}) + A_zB_y(\mathbf{k} \times \mathbf{j}) + A_zB_z(\mathbf{k} \times \mathbf{k})
\end{aligned}$$



- Carrying out the cross-product operations and combining terms yields

$$A \times B = (A_yB_z - A_zB_y)i - (A_xB_z - A_zB_x)j + (A_xB_y - A_yB_x)k$$

- The rectangular components of the vector product **C** are thus found to be



- $C_x = A_yB_z - A_zB_y$ $C_y = -(A_xB_z - A_zB_x)$ $C_z = A_xB_y - A_yB_x$

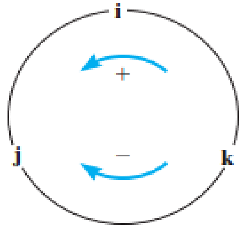
THE VECTOR PRODUCT OR CROSS PRODUCT

Cartesian Vector Formulation

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin\theta)\mathbf{u}_C$$

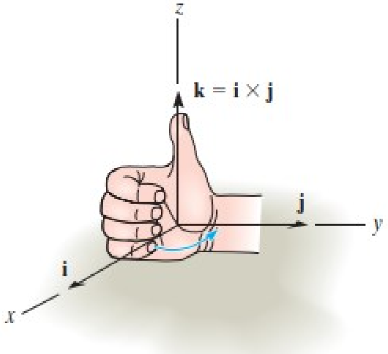
$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

- This equation may also be written in a more compact determinant form which is more easily memorized



$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- Note that expanded form of AxB shows the first row of elements consisting of the unit vectors i, j, and k and whose second and third rows represent the x, y, z components of the two vectors A and B, respectively



THE VECTOR PRODUCT OR CROSS PRODUCT

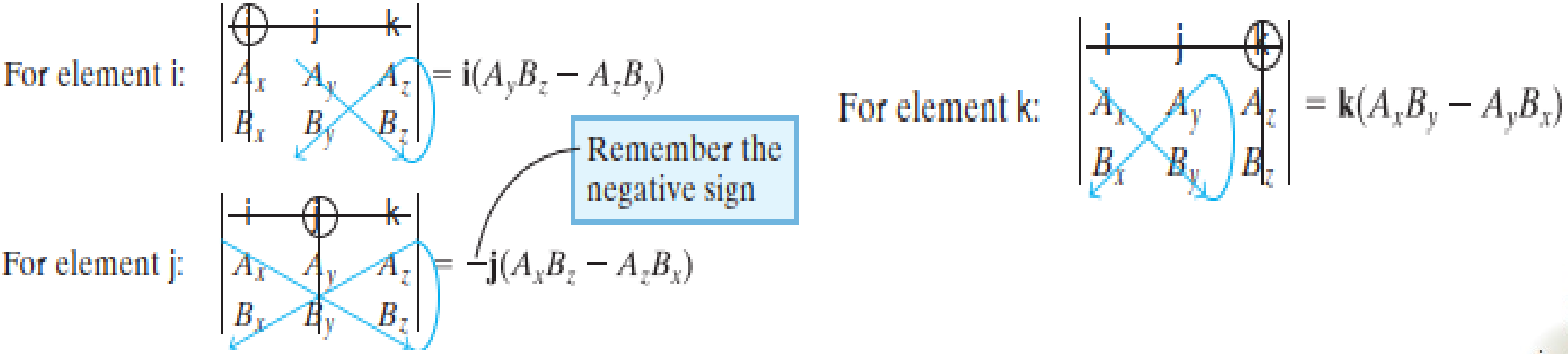
Cartesian Vector Formulation

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin\theta)\mathbf{u}_C$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- **For a 3 x 3 determinant**, such as the one shown, the three minors can be generated in accordance with the following scheme:

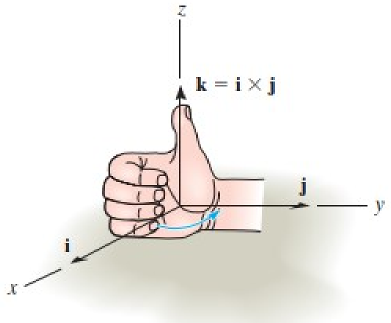


For element i: $= \mathbf{i}(A_yB_z - A_zB_y)$

For element j: $= -\mathbf{j}(A_xB_z - A_zB_x)$

Remember the negative sign

For element k: $= \mathbf{k}(A_xB_y - A_yB_x)$



- C**i** = **i**(AyBz-AzBy) C**j** = -**j**(AxBz-AzBx) C**k** = **k**(AxBy-AyBx)
- Or for vector product PxQ

$$\overline{V} = \overline{P} * \overline{Q} = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} = i\begin{vmatrix} P_y & P_z \\ Q_y & Q_z \end{vmatrix} - j\begin{vmatrix} P_x & P_z \\ Q_x & Q_z \end{vmatrix} + k\begin{vmatrix} P_x & P_y \\ Q_x & Q_y \end{vmatrix}$$

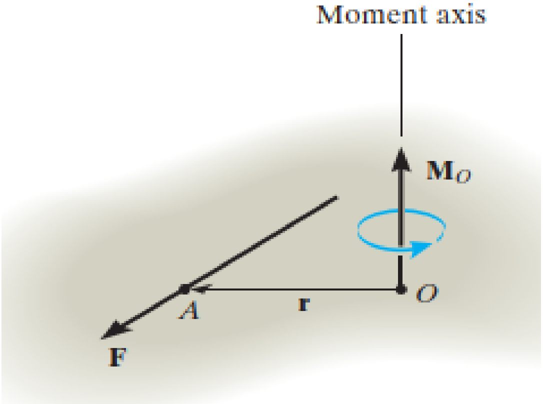
MOMENT OF A FORCE

Vector Formulation

- The moment of a force F about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and F, Fig. (a) can be expressed using the vector cross product, namely,

$$M_O = r \times F$$

- **Here r represents a position vector directed from O to any point on the line of action of F.**



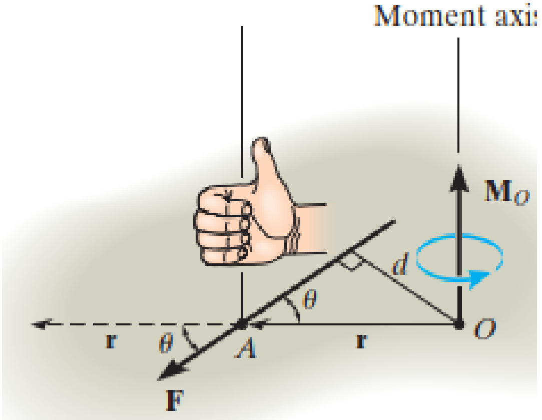
(a)

MOMENT OF A FORCE
Vector Formulation

- The magnitude of the cross product is defined as

$$rxF = M_o = rF\sin\theta$$

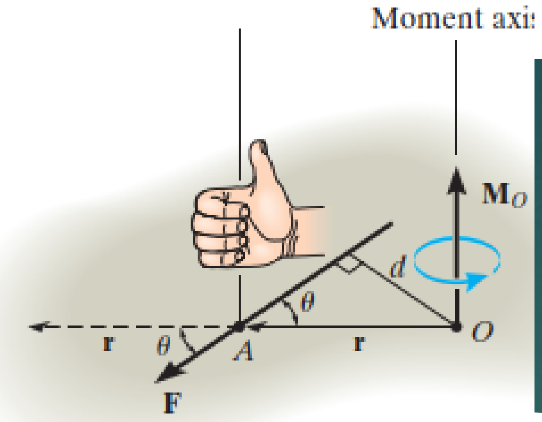
- <u>**where the angle θ is measured between the tails of r and F.**</u>



- Since the moment arm $d = r\sin\theta$, then

$$M_o = rF\sin\theta = F(r\sin\theta) = Fd$$

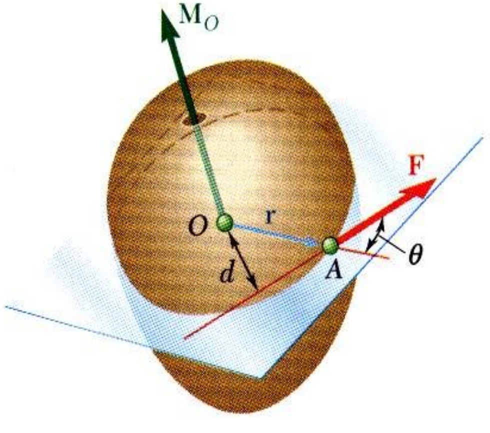
- which agrees with $M_o = Fd$



MOMENT OF A FORCE

Vector Formulation

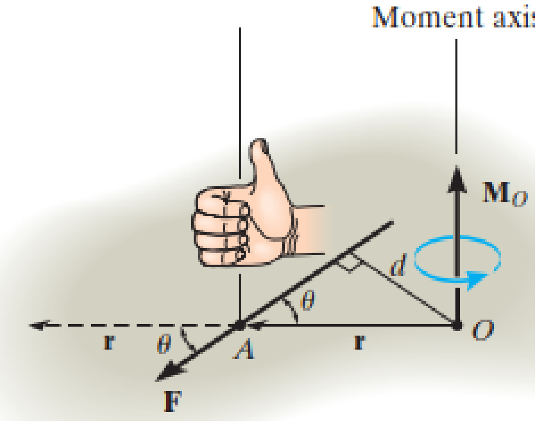
- The direction and sense of **M**o are determined by the right-hand rule.
- Since the cross product does not obey the commutative law, the order of rxF must be maintained to produce the correct sense of direction for **M**o.
- The moment vector M_O is perpendicular to the plane containing *O* and the force ***F***.



(*a*)

(*b*)

MOMENT OF A FORCE
Vector Formulation

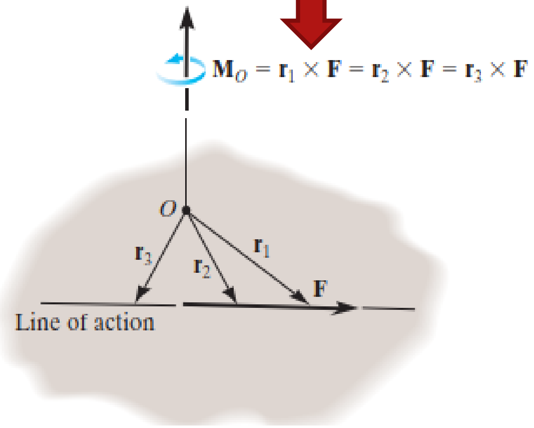


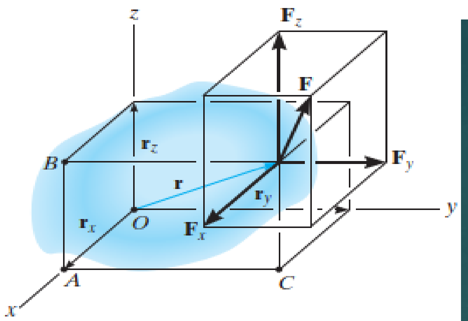
- The cross product operation is often used in three dimensions since the perpendicular distance (d) or moment arm from point O to the line of action of the force is not needed.

- We can use any position vector r measured from point O to any point on the line of action of the force F. Thus, $\mathbf{M}_O = \mathbf{r}_1 x F = \mathbf{r}_2 x F \ldots$

- Since F can be applied at any point along its line of action and still create this same moment about point O, then F can be considered a sliding vector.

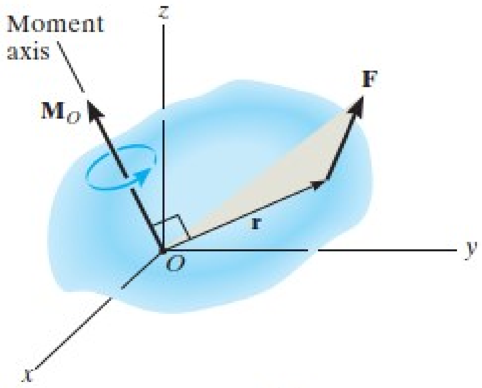
- This property is called the **principle of transmissibility** of a force.





(b)

MOMENT OF A FORCE
Vector Formulation



(a)

- The position vector r and force F can be expressed in cartesian vector formulation.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- Where $\mathbf{r}_x$, $\mathbf{r}_y$, $\mathbf{r}_z$ represent the x, y, z components of the position vector drawn from point O to any point on the line of action of the force & Fx, Fy, Fz represent the x, y, z components of the force vector

- If the determinant is expanded, then we have

$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

MOMENT OF A FORCE

Vector Formulation

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

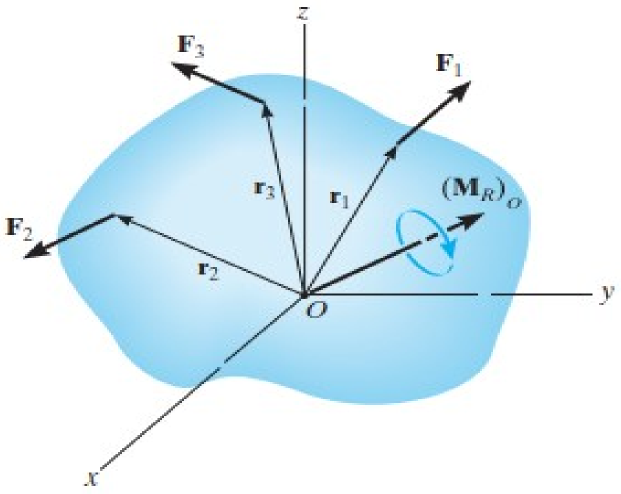
- **Resultant Moment of a System of Forces.**

- If a body is acted upon by a system of forces, the resultant moment of the forces about point O can be determined by vector addition of the moment of each force.

- This resultant can be written as

$$(M_R)_O = \sum(\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}_1 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}_2 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}_3$$



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

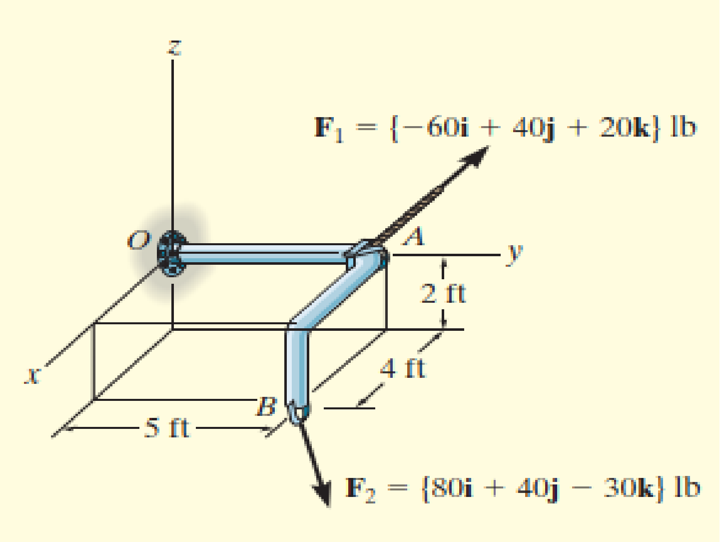
MOMENT OF A FORCE

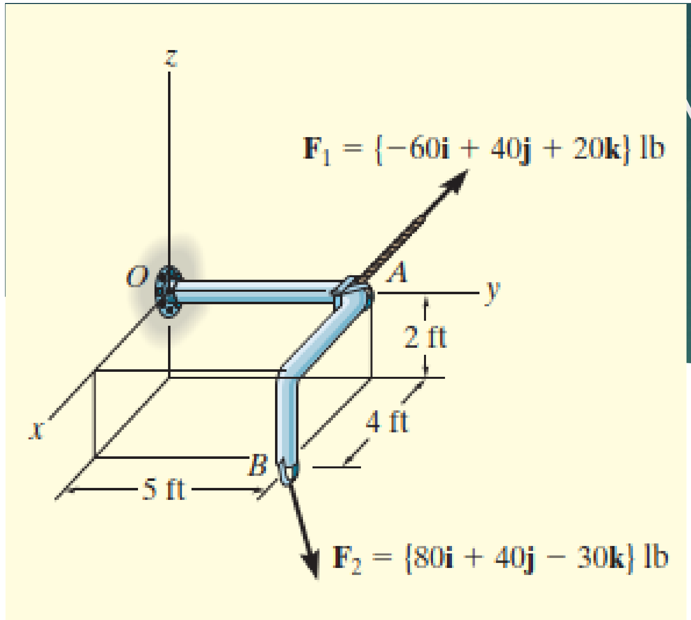
Example 4.4

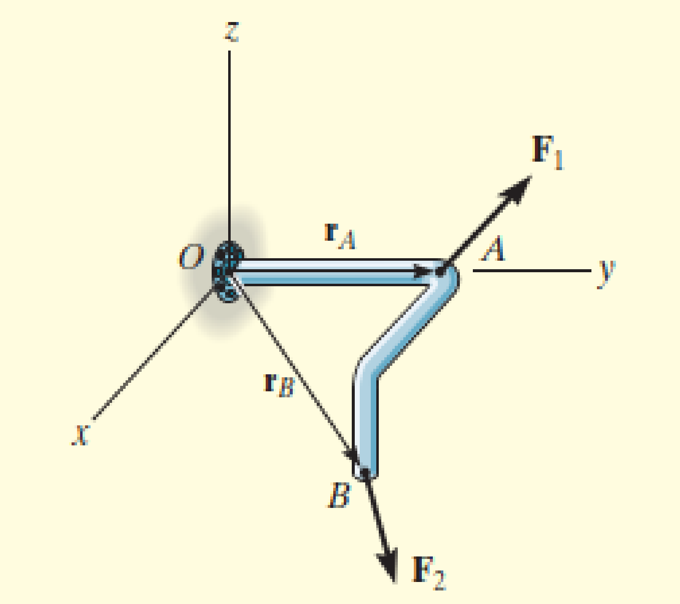
$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Question

- Two forces act on the rod shown in Fig.
- Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.







Position vectors are directed from point *O* to each force as shown in Fig. 4–15*b*. These vectors are

$$\mathbf{r}_A = \{5\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

The resultant moment about *O* is therefore

$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$

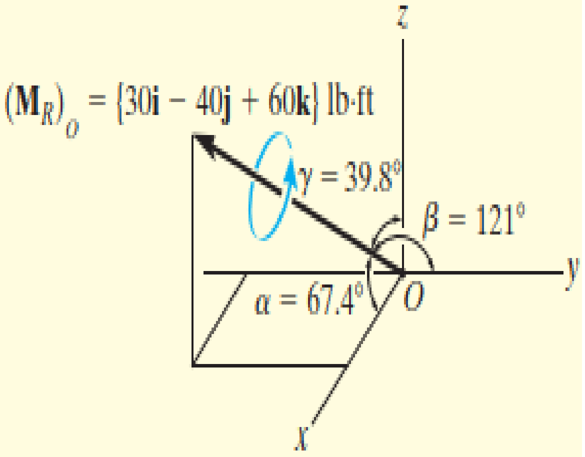
$$= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}$$

$$+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}$$

$$= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\text{ lb}\cdot\text{ft}$$ *Ans.*



(c)

Fig. 4–15

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

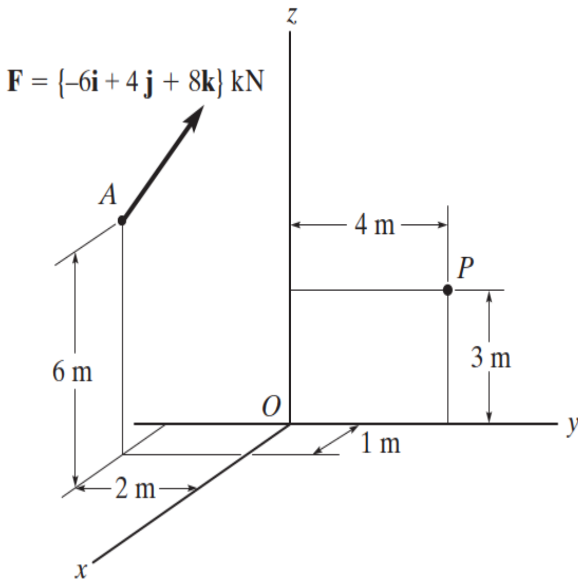
MOMENT OF A FORCE

Example 4.5

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Question

- Determine the moment of the force F about point O and P. Express the result as a Cartesian vector.
- Answer $\mathbf{r}_{OA} = \{i - 2j + 6k\}$ m & $\mathbf{r}_{PA} = (1 - 0)i + (-2 - 4)j + (6 - 3)k = \{i - 6j + 3k\}$ m



$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 6 \\ -6 & 4 & 8 \end{vmatrix}$$

$$M_P = \mathbf{r}_{PA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -6 & 3 \\ -6 & 4 & 8 \end{vmatrix}$$

$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \{-40i - 44j - 8k\}$kN.m

$\mathbf{M}_P = \mathbf{r}_{PA} \times \mathbf{F} = \{-60i - 26j - 32k\}$kN.m

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

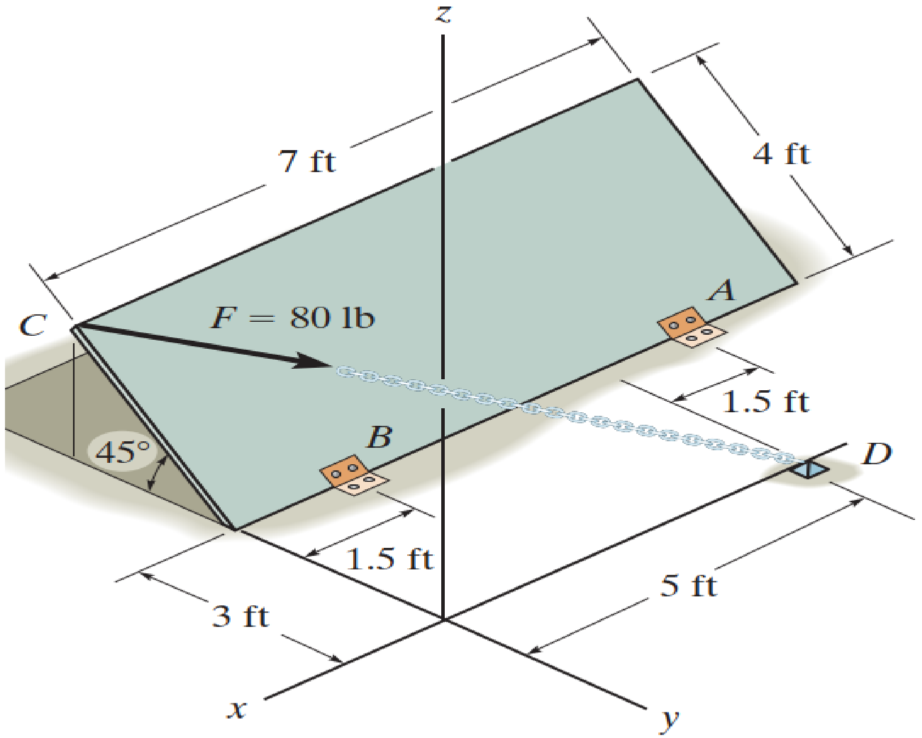
MOMENT OF A FORCE

Example 4.6

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Question

- Determine the moment of the force F about the door hinges at A and B. Express the result as a Cartesian vector.



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE

Example 4.6

$$A \times B = (A_yB_z - A_zB_y)\mathrm{i} - (A_xB_z - A_zB_x)\mathrm{j} + (A_xB_y - A_yB_x)\mathrm{k}$$

Solution

Position Vectors And Force Vector. The coordinates of points A, C and D are $A\,(-6.5, -3, 0)$ ft, $C\,[0, -(3 + 4\cos 45°), 4\sin 45°]$ ft and $D\,(-5, 0, 0)$ ft, respectively. Thus,

$$\mathbf{r}_{AC} = [0 - (-6.5)]\mathbf{i} + [-(3 + 4\cos 45°) - (-3)]\mathbf{j} + (4\sin 45° - 0)\mathbf{k}$$

$$= \{6.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AD} = [-5 - (-6.5)]\mathbf{i} + [0 - (-3)]\mathbf{j} + (0 - 0)\mathbf{k} = \{1.5\mathbf{i} + 3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{CD} = (-5 - 0)\mathbf{i} + \{0 - [-(3 + 4\cos 45°)]\}\mathbf{j} + (0 - 4\sin 45°)\mathbf{k}$$

$$= \{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 80\left(\frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}}\right)$$

$$= \{-48.88\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\}\text{ lb}$$

Moment of F About Point A.

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} + 41.47\mathbf{j} + 232.10\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} + 41.5\mathbf{j} + 232\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

OR

$$\mathbf{M}_A = \mathbf{r}_{AD} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 3 & 0 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} + 41.47\mathbf{j} + 232.10\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} + 41.5\mathbf{j} + 232\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE

Example 4.6

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Solution

Position Vectors And Force Vector. The coordinates of points B, C and D are $B\,(-1.5, -3, 0)$ ft, $C\,[0, -(3 + 4\cos 45°), 4\sin 45°]$ ft and $D\,(-5, 0, 0)$ ft, respectively. Thus,

$$\mathbf{r}_{BC} = [0 - (-1.5)]\mathbf{i} + [-(3 + 4\cos 45°) - (-3)]\mathbf{j} + (4\sin 45° - 0)\mathbf{k}$$

$$= \{1.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{BD} = [-5 - (-1.5)]\mathbf{i} + [0 - (-3)]\mathbf{j} + (0 - 0)\mathbf{k} = \{-3.5\mathbf{i} + 3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{CD} = (-5-0)\mathbf{i} + \{0 - [-(3 + 4\cos 45°)]\}\mathbf{j} + (0 - 4\sin 45°)\mathbf{k}$$

$$= \{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 80\left(\frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}}\right)$$

$$= \{-48.88\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\}\text{ lb}$$

Moment of *F* About Point *B*.

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

or

$$\mathbf{M}_B = \mathbf{r}_{BD} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3.5 & 3 & 0 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

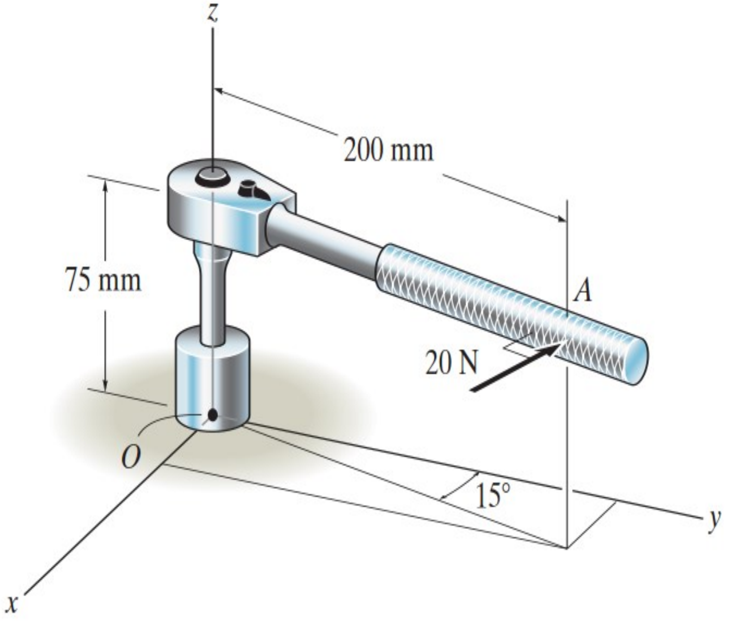
MOMENT OF A FORCE

Example 4.7

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Question

- A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point O.



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE

Example 4.7

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Solution

$$\mathbf{r}_A = 0.2 \sin 15°\mathbf{i} + 0.2 \cos 15°\mathbf{j} + 0.075\mathbf{k}$$

$$= 0.05176\mathbf{i} + 0.1932\mathbf{j} + 0.075\mathbf{k}$$

$$\mathbf{F} = -20 \cos 15°\mathbf{i} + 20 \sin 15°\mathbf{j}$$

$$= -19.32\mathbf{i} + 5.176\mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.05176 & 0.1932 & 0.075 \\ -19.32 & 5.176 & 0 \end{vmatrix}$$

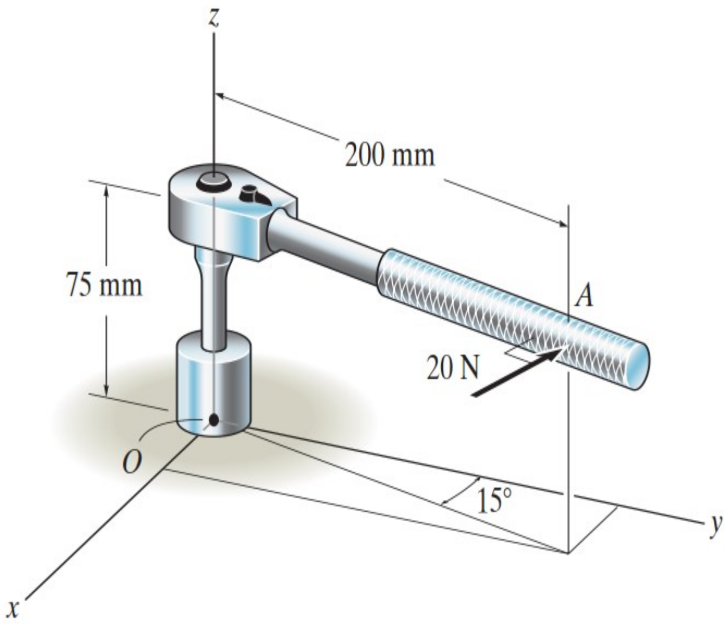
$$= \{-0.3882\mathbf{i} - 1.449\mathbf{j} + 4.00\mathbf{k}\}\ \text{N}\cdot\text{m}$$

$$M_O = 4.272 = 4.27\ \text{N}\cdot\text{m}$$

$$\alpha = \cos^{-1}\left(\frac{-0.3882}{4.272}\right) = 95.2°$$

$$\beta = \cos^{-1}\left(\frac{-1.449}{4.272}\right) = 110°$$

$$\gamma = \cos^{-1}\left(\frac{4}{4.272}\right) = 20.6°$$



MOMENT OF A COUPLE

HOME WORK EXERCISE

4-4, 4-12, 4-29, 4-34, 4-36, 4-38, 4-43, 4-46, 4-48, 4-50, 4-51, 4-54, 4-56, 4-58, 4-66, 4-69, 4-72, 4-80, 4-83, 4-96, 4-97, 4-102, 4-108, 4-115 & 4-134.