CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

Lecture A1

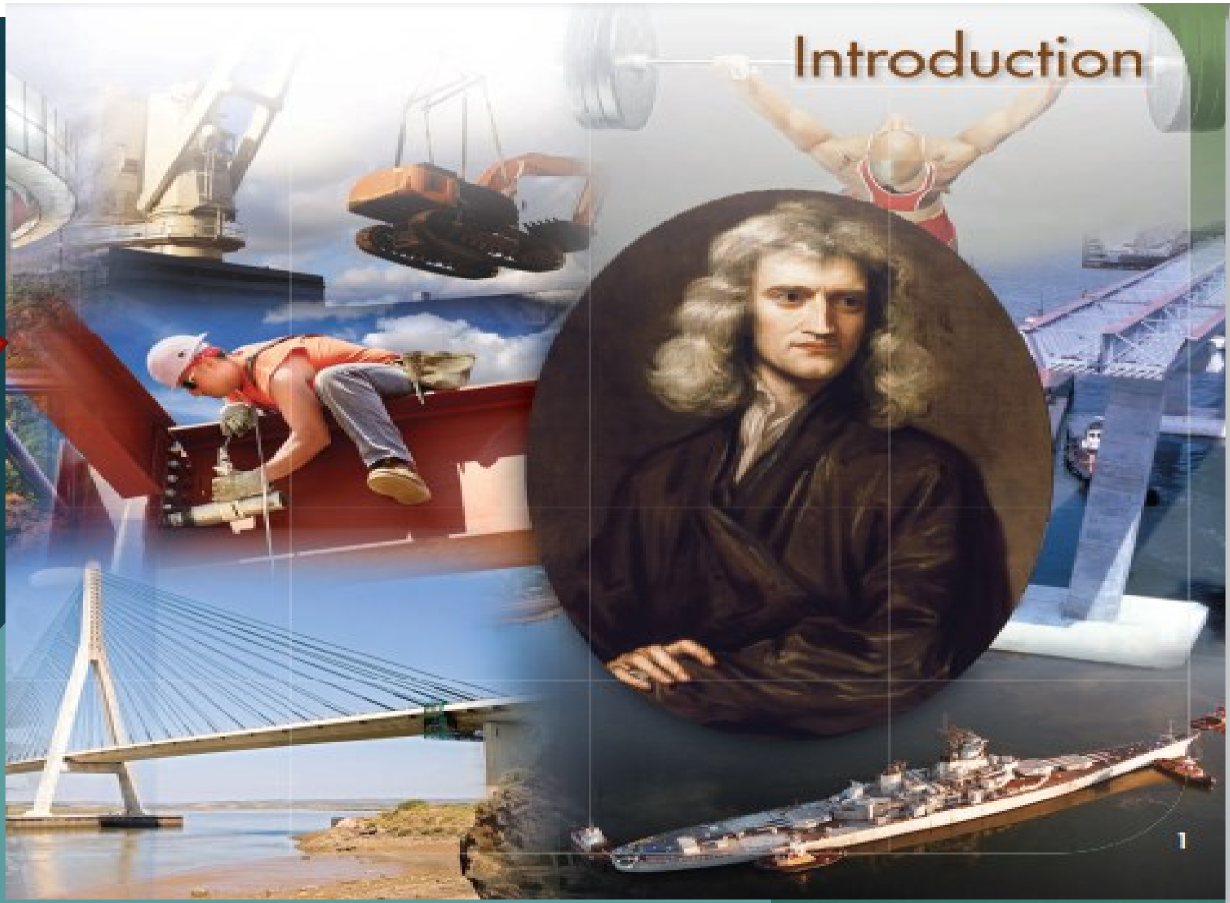
- TOPIC INTRODUCTION - GENERAL PRINCIPLES/BASIC CONCEPTS
- COMPOSITION & RESOLUTION OF FORCE

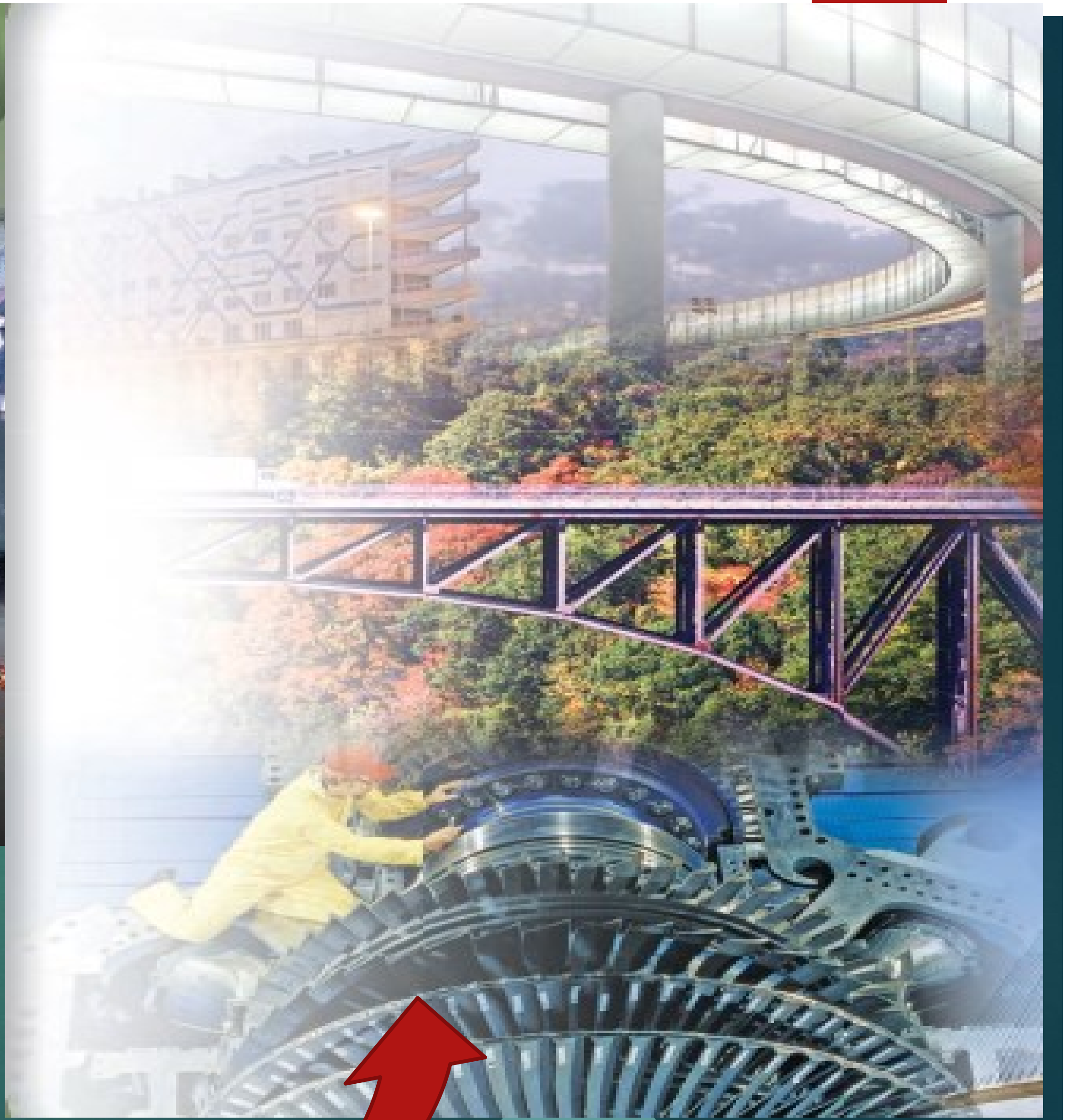
COURSE MAIN OBJECTIVES

- The main objective of statics and introduction to mechanics of materials course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This course is designed for a course that combines statics and mechanics of material (strength of materials) offered to engineering/mines students in second-year

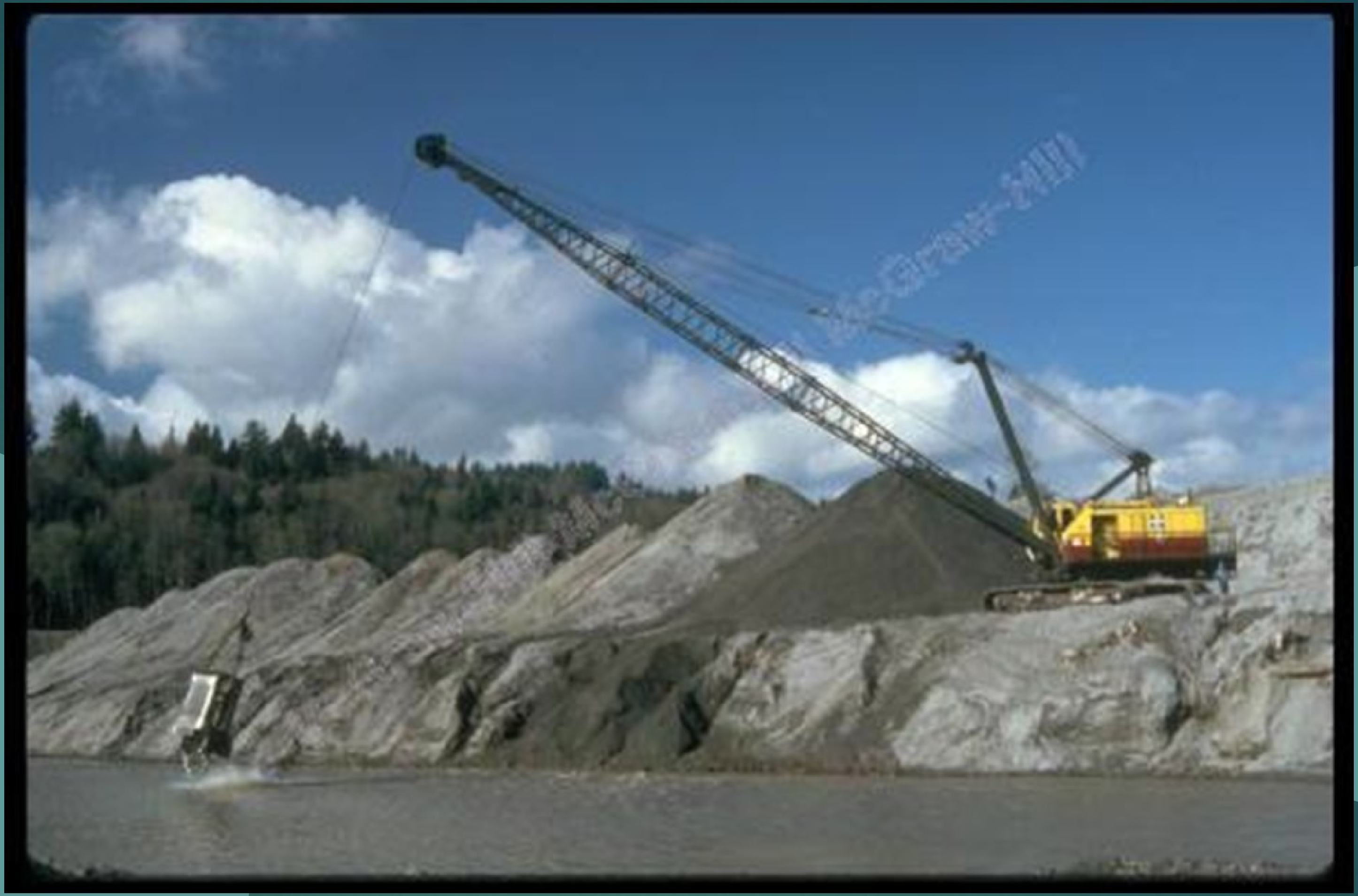
LECTURE OBJECTIVES

- To provide an introduction to the basic quantities and idealizations of mechanics
  To state Newton's Laws of Motion
  To present a general guide for solving problems
  To show how to add forces and resolve them into components





Sir Isaac Newton stated the fundamental principles of mechanics, which are the foundation of much of today's engineering.



❖ Engineers designing this crane will need to determine the forces that act on this body under various conditions.

OVERVIEW TO MECHANICS

MECHANICS

- *Mechanics is the science that describes and predicts the conditions of rest or of bodies in motion under the action of forces.*
- Mechanics can also be defined as the branch of physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces.
- *Mechanics is divided into three branches:*
  - *Mechanics of rigid bodies*
  - *Mechanics of deformable bodies*
  - *Mechanics of fluids*

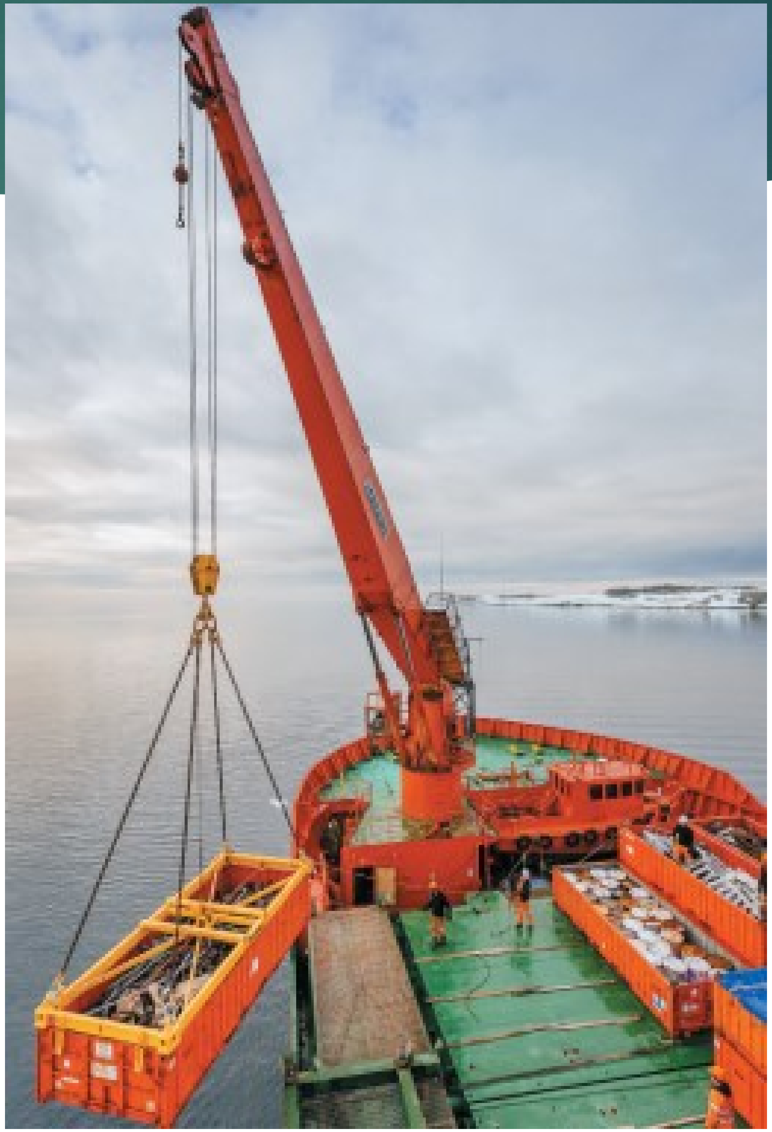
OVERVIEW TO MECHANICS

- Two parts of Mechanics of rigid bodies
  - Statics: Deals with bodies at rest or move with constant velocity
  - Dynamics: Deals with bodies in accelerated motion.
- In this part of the study of mechanics, bodies are assumed to be perfectly rigid
- However, bodies are never absolutely rigid and deform under the loads to which they are subjected
- But these deformations are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure

- Mechanics of deformable bodies
  - Deformations are important, though, as far as the resistance of the structure to failure is concerned and are studied in mechanics of materials, which is a part of the mechanics of deformable bodies.
- Mechanics of fluids (Fluid Mechanics)
  - This is the study of incompressible and compressible fluids - An important subdivision of the study of incompressible fluids is hydraulics. Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible.
- The main purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications

OVERVIEW TO MECHANICS

- In this course we will study two important branches of mechanics,
  - Statics and
  - Strength of materials
- These subjects form a suitable basis for the design and analysis of many types of structural, mechanical, or electrical devices encountered in engineering



Large cranes such as shown are required to lift extremely large loads. Their design is based on the basic principles of statics and dynamics, which form the subject matter of ***engineering mechanics***.

OVERVIEW TO MECHANICS

Principles of statics

Since for the design and analysis of any structural or mechanical element it is first necessary to determine the forces acting both on and within its various members. (Internal & External forces)

Mechanics of Materials

Once principles of statics are understood, ie determination of internal forces, the size of the members, their deflection and their stability can then be determined using the fundamentals of mechanics of materials

OVERVIEW TO MECHANICS

- Note that statics deals with the equilibrium of bodies, that is, it is used to determine the forces acting either external to the body or within it that are necessary to keep the body in equilibrium.
- Mechanics of materials studies the relationships between the external loads and the distribution of internal forces acting within the body. This subject is also concerned with finding the deformations of the body, and it provides a study of the body's stability.

OVERVIEW TO MECHANICS

- The course as it states STATICS will only discuss rigid body mechanics with ZERO acceleration.
- Statics is very imported as it’s understanding forms the basis for design and analysis of Dynamic bodies, Mechanics of deformable bodies and Fluid bodies which are used in structural, mechanical, electrical engineering design
- Statics needs spacial education in engineering since most of the objects/infrastructure are designed with the intention that they remain in equilibrium wiyhout failure

BASIC CONCEPTS AND PRINCIPLES

- The basic quantities used in mechanics are:
  - *space*
  - *Time*
  - *mass* and
  - *force*
- However, you also need to understand other fundermental concepts and principles ie Idealization, Newton‘s Law of Motion, Weight, etc

BASIC CONCEPTS AND PRINCIPLES

- *Space*: The concept of space is associated with the notion of the point P. The position P may be defined by three lengths measured from a reference point (point of origin). Space is the region occupied by the bodies. We set up a coordinate system to specify where the object is by the position and its posture by the orientation
- *Time*: To define an event, the time of the event should be stated in addition to its position. Time is the measure of the succession of events. Often, we are more interested in the change of physical quantities with respect to time, e.g. $v = dr/dt$, instead of time variable itself.

BASIC CONCEPTS AND PRINCIPLES

- *Mass*: The concept of mass is used to characterise and compare bodies on the basis of some fundamental experiements, e.g two bodies of the same mass will offer the same resistance to a change in translational motion.
- Mass is a measure of the quantity of matter. It is related to the *inertia* of the body and is usually considered a constant.

BASIC CONCEPTS AND PRINCIPLES

- *Force*: Represents the action of one body on another. This term is applied to any action on a body which tends to make it *move*, change its *motion*, or change its *size* and *shape*
- A force usually produces acceleration

Idealizations.

- *These are models or idealizations used in mechanics in order to simplify application of the theory.*
- *Here we will consider three important idealizations.*
  - *Particle*
  - *Rigid body and*
  - *Concentrated force*

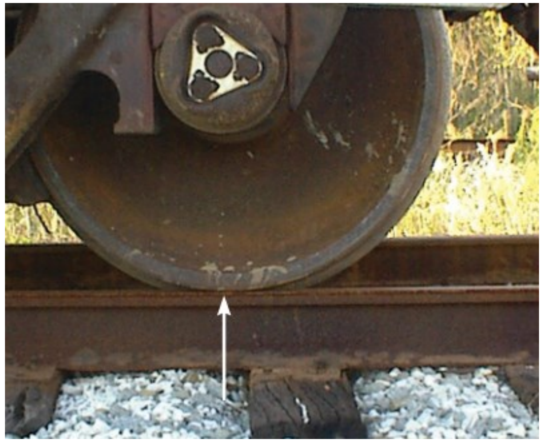
BASIC CONCEPTS AND PRINCIPLES

- *Particle*: by particle we mean a very small amount of matter which may be assumed to occupy a single point in space.
- *Rigid body*: This is a combination of a large number of particles occupying fixed positions with respect to each other.
- The results obtained for a particle can be used directly in a large number of problems dealing with the conditions of rest or motion of actual bodies.

The three force analysis can be assumed to be represented as a particle

BASIC CONCEPTS AND PRINCIPLES

- *Concentrated Force*: A concentrated force represents the effect of a loading which is assumed to act at a point on a body.
- We can represent a load by a concentrated force, *provided the area over which the load is applied is very small compared to the overall size of the body.*
- An example would be the contact force between a wheel and the ground.



We consider the railroad wheel to be a rigid body acted upon by the concentrated force of the rail

Other important nomenclature/terminology used in mechanics

- *Weight:* Weight is the force with which a body is attracted toward the center of the earth by gravitational pull
- *Load*: This term is used to indicate that a body of some weight is applying a force against some supporting structure or part of a structure.
  - For example, a load weighing 100 lb is applied on a beam supported at two ends. Or, a beam itself can be considered a certain load on part of a structure

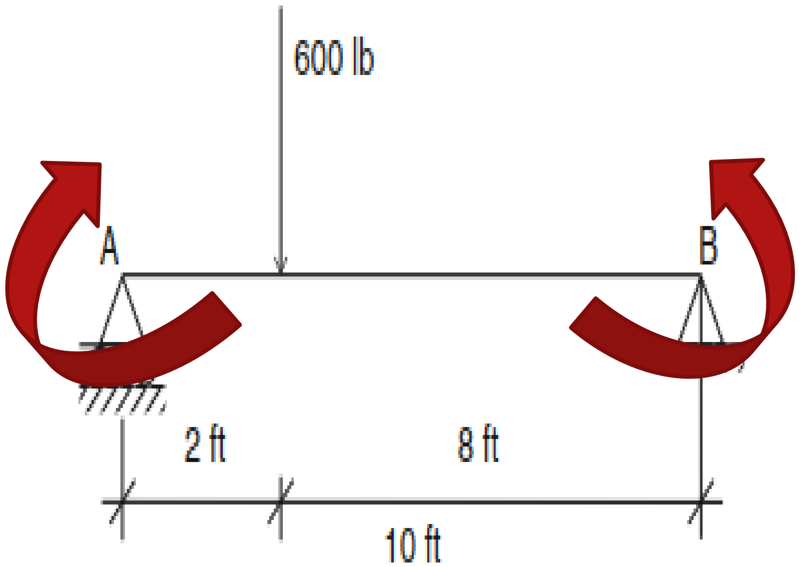
BASIC CONCEPTS AND PRINCIPLES

- *Moment:* The tendency of a force to cause rotation about an axis through some point is known as moment. Moment (M) of a force (F) about a given point (O), is the product of the force and its perpendicular distance r from the line of action between the force and point O.
- The point or axis of rotation is called the center of moments. The perpendicular distance between the line of action and the center of rotation is called the moment arm.
- *A moment* can be formulated as: $M = F \times r$
- Moment of Force = Magnitude of Force x Moment Arm

BASIC CONCEPTS AND PRINCIPLES

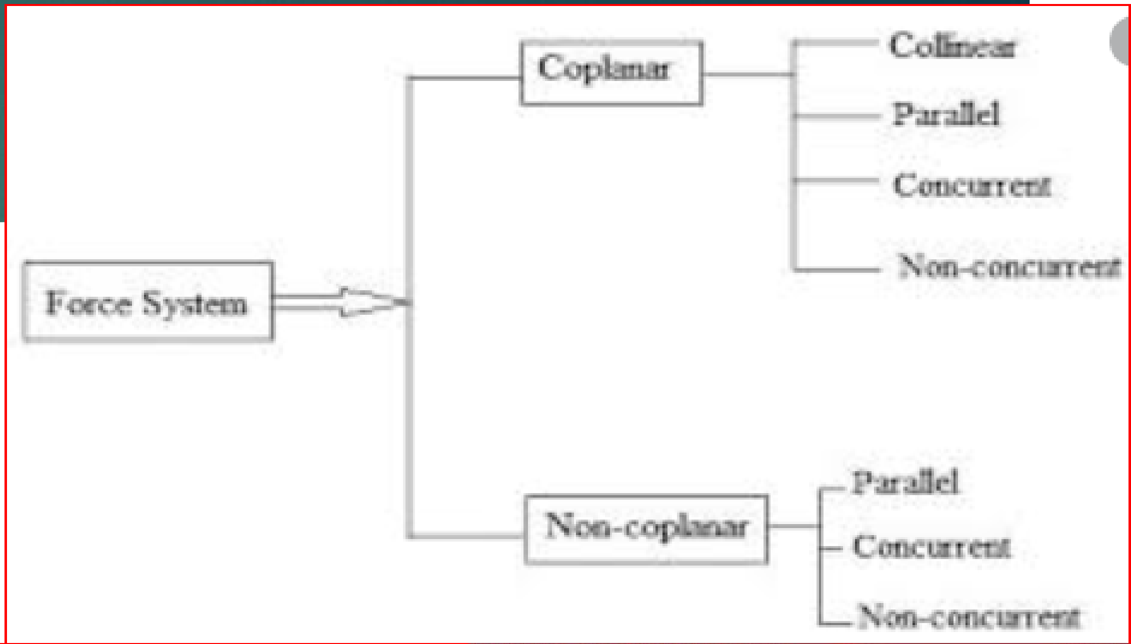
- *Moment*: Example - A 10 ft beam has a load of 600 lb at a distance of 2 ft from the left end of the beam. Calculate the moment of load about each end point

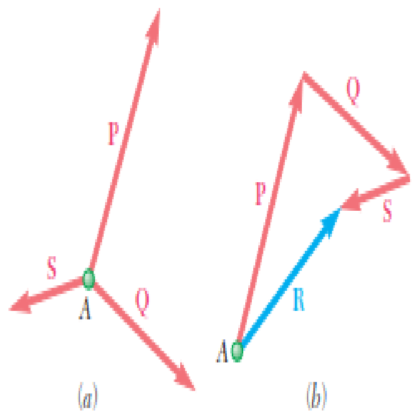
- Solution Moment of force @ A = 600 lb x 2 ft = + 1,200 ft-lb
- Moment of force @ B = 600 lb x 8 ft = - 4,800 ft-lb
- Notice the sign of moment clockwise (+) and counterclockwise (-).

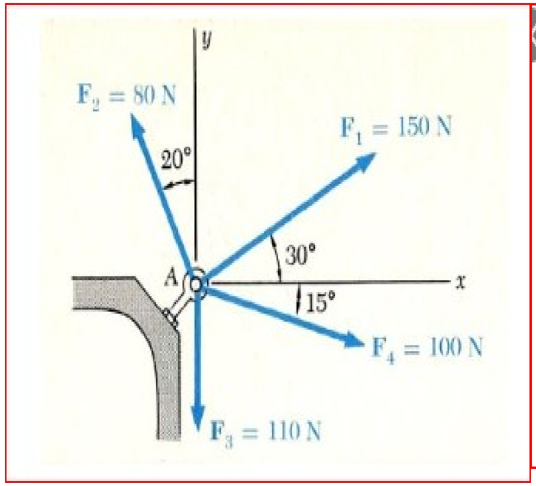


BASIC CONCEPTS AND PRINCIPLES

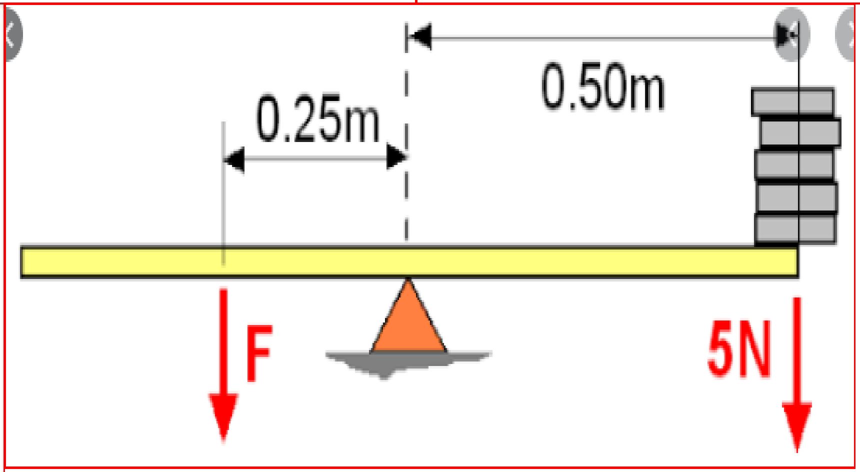
- *Concurrent forces* – These are forces which act through one point
- *Coplanar forces* –These are forces which act in one plane



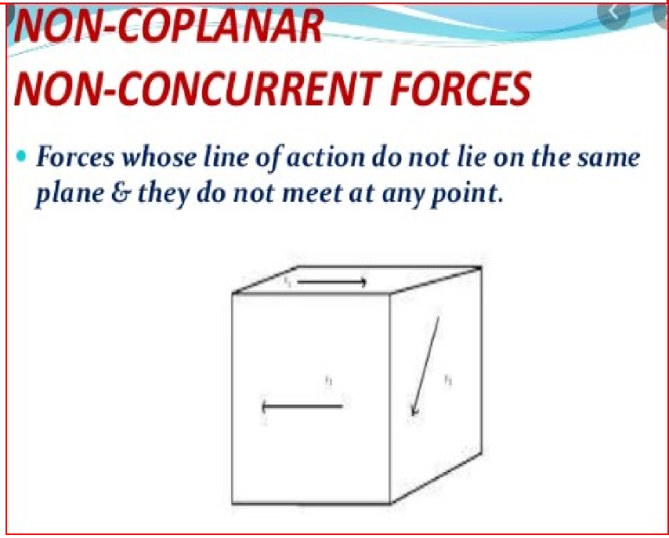




Concurrent and coplanar

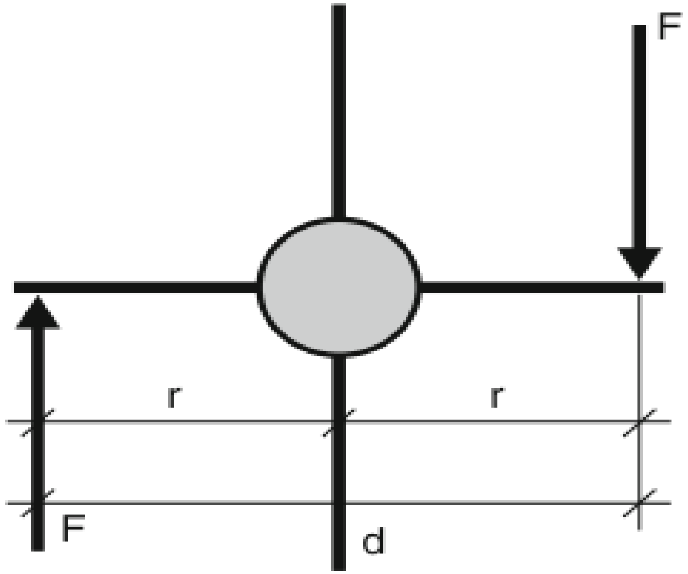


Non-Concurrent and coplanar



BASIC CONCEPTS AND PRINCIPLES

- *Couple*: A pair of parallel forces equal in magnitude and opposite in direction is called a couple. Their only effect is to produce a moment. The only motion a couple can cause is rotation
- Note that the moment of a couple is equal to the product of one of its forces F and the perpendicular distance d between the forces $M = F \times d$



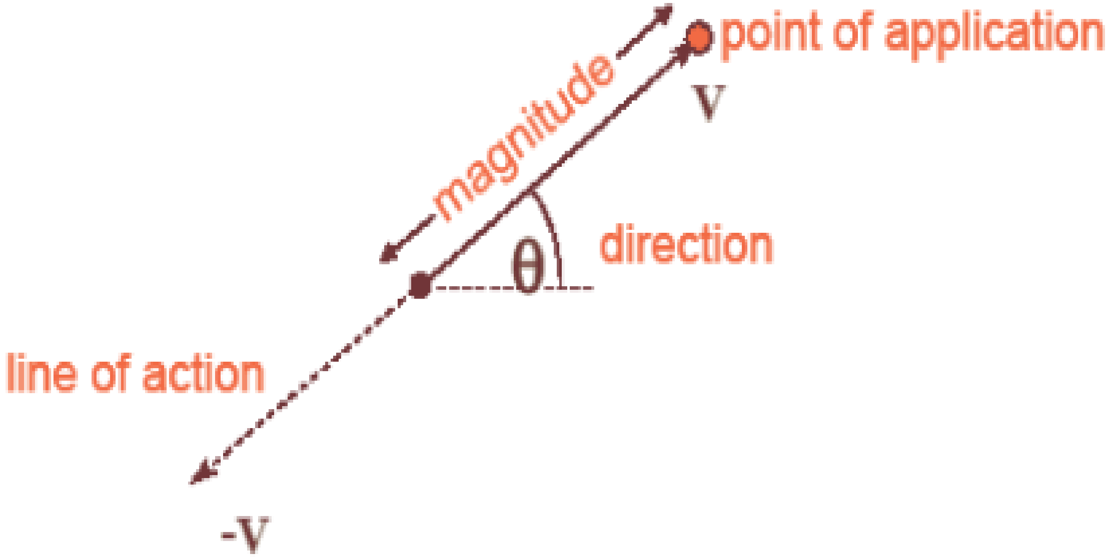
BASIC CONCEPTS AND PRINCIPLES

- *Vector*: Vectors are quantities for which both the magnitude and the direction are needed to describe them completely
- Examples include force, weight, displacement, velocity, moment, acceleration, momentum, etc.
- *Scalar:* Quantities that have magnitude only is called scalar . They are complete without a direction or Scalars are quantities for which only the magnitude can describe them completely
- Examples include mass, density, area, volume, distance, speed, time, temperature, power, energy, etc.

BASIC CONCEPTS AND PRINCIPLES

- Representation of Vectors
  - There are many notations to represent a vector quantity
  - If we would like to tell only the magnitude, |v| or v may be used.
  - Keep in mind that the complete representation of a vector must be able to determine its magnitude, direction, line of action, and point of application.

$\mathbf{v}, \bar{v}, \vec{v}, \text{or } \underline{v}.$



BASIC CONCEPTS AND PRINCIPLES

- *The study of elementary mechanics rests on six fundamental principles based on experimental evidence***.

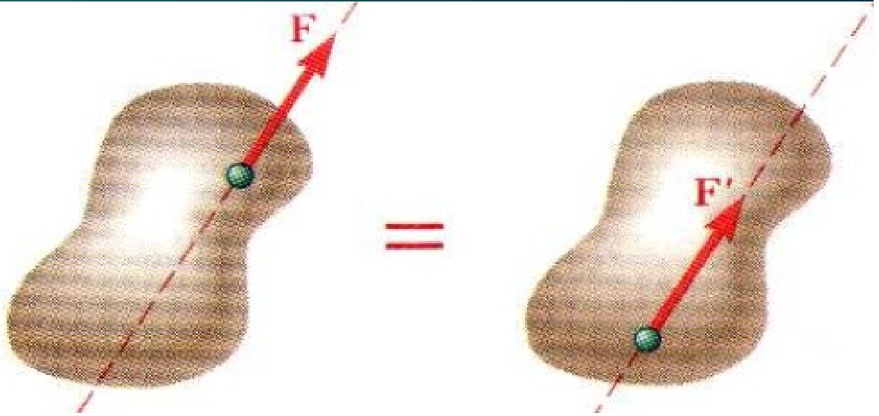
- The Parallelogram Law for the Addition of Forces.

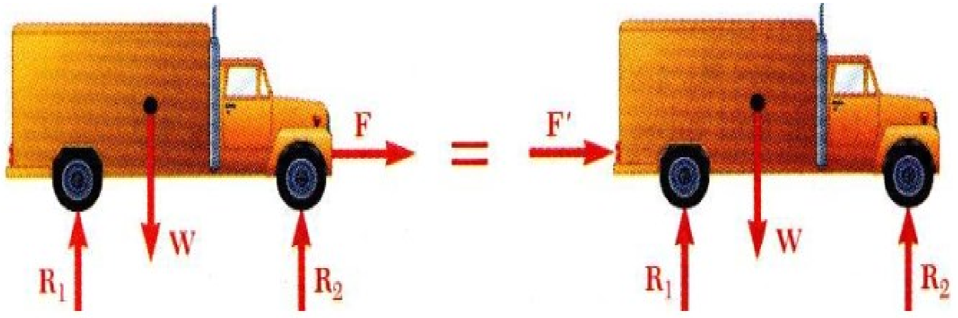
- This states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces

BASIC CONCEPTS AND PRINCIPLES

The Principle of Transmissibility.

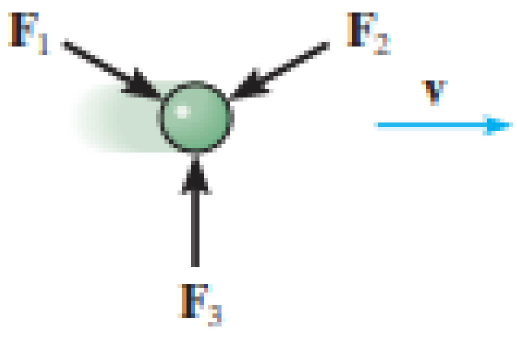
- This states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action





BASIC CONCEPTS AND PRINCIPLES

- The Newton's Three Fundamental Laws.
- Formulated by Sir Isaac Newton in the latter part of the 17th century, these laws can be stated as follows:



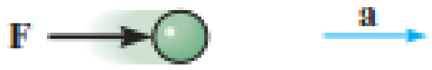
- FIRST LAW.

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion) *(*

BASIC CONCEPTS AND PRINCIPLES

- SECOND LAW.

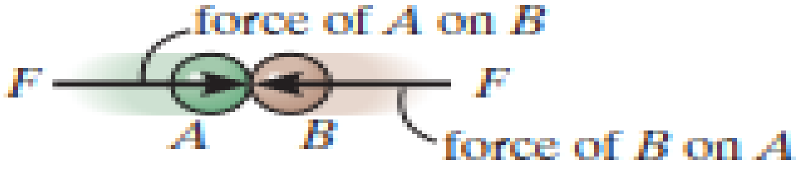
✓ If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force



Accelerated motion

✓ This law can be stated as $F = ma$, where F, m, and a represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle, expressed in a consistent system of units

BASIC CONCEPTS AND PRINCIPLES



Action – reaction

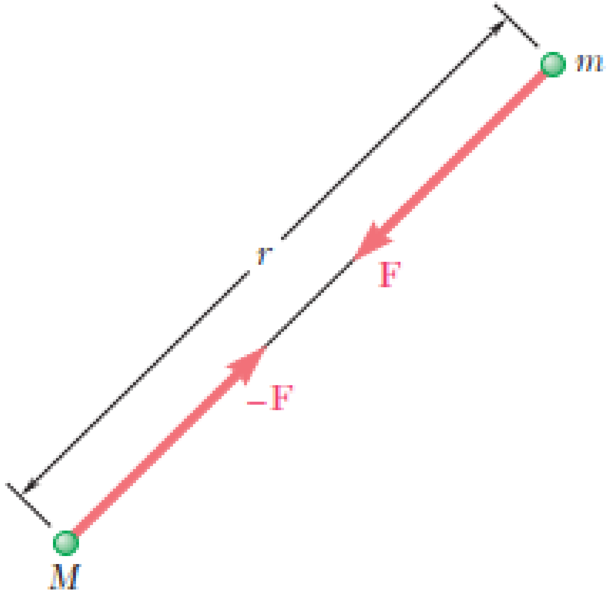
- THIRD LAW.

The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense

BASIC CONCEPTS AND PRINCIPLES

- **Newton's Law of Gravitation**

- This states that two particles of mass M and m are mutually attracted with equal and opposite forces F and –F



- **Note that the six fundamental principles are based on experimental evidence. Except for Newton's first law and the principle of transmissibility, they are independent principles which can-not be derived mathematically from each other or from any other elementary physical principle

Units of Measurement

- The four basic quantities (length, time, mass, and force) are not all independent from one another; in fact, they are related by Newton's second law of motion, $F = ma$
- Hence the units used to measure these quantities cannot all be selected arbitrarily
- The equality $F = ma$ is maintained only if three of the four units, called base units, are defined and the fourth unit is then derived from the equation

Units of Measurement

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.

Prefixes

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Equivalents

TABLE 1.3 U.S. Customary Units and Their SI Equivalents

Quantity	U.S. Customary Unit	SI Equivalent
Acceleration	ft/s^2	$0.3048\ m/s^2$
	$in./s^2$	$0.0254\ m/s^2$
Area	ft^2	$0.0929\ m^2$
	in^2	$645.2\ mm^2$
Energy	ft · lb	1.356 J
Force	kip	4.448 kN
	lb	4.448 N
	oz	0.2780 N
Impulse	lb · s	4.448 N · s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	oz mass	28.35 g
	lb mass	0.4536 kg
	slug	14.59 kg
	ton	907.2 kg
Moment of a force	lb · ft	1.356 N · m
	lb · in.	0.1130 N · m
Moment of inertia		
Of an area	in^4	$0.4162 \times 10^6\ mm^4$
Of a mass	$lb \cdot ft \cdot s^2$	$1.356\ kg \cdot m^2$
Momentum	lb · s	4.448 kg · m/s
Power	ft · lb/s	1.356 W
	hp	745.7 W
Pressure or stress	lb/ft^2	47.88 Pa
	lb/in^2 (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Volume	ft^3	$0.02832\ m^3$
	in^3	$16.39\ cm^3$
Liquids	gal	3.785 L
	qt	0.9464 L
Work	ft · lb	1.356 J

General Procedure for Analysis

- Attending lectures, reading books, and studying the example problems helps, but the most effective way of learning the principles of engineering mechanics is to solve problems!
- You should approach a problem in mechanics as you would approach an actual engineering situation
- To be successful in solving these problems, it is important to always present the work in a logical and orderly manner, as suggested by the following sequence of steps:
  - *Read the problem carefully and try to correlate the actual physical situation with the theory studied.*

General Procedure for Analysis

- *Make sure the statement of a problem is clear and precise. It should contain the given data and indicate what information is required*
- *Tabulate the problem data and draw a neat drawing showing all quantities involved. These diagrams should be drawn for all bodies involved, indicating clearly the forces acting on each body. These diagrams are known as free-body diagrams and are described in detail in the other sections*
- *Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous*
- *Each equation should be clearly related to one of the free-body diagrams*

BASIC CONCEPTS AND PRINCIPLES

General Procedure for Analysis

- *Solve the necessary equations, and report the answer with no more than three significant figures and recording neatly the various steps taken*
- *Study the answer with technical judgment and common sense to determine whether or not it seems reasonable. Mistakes in reasoning can often be detected by checking the units carefully*
- *Errors in computation will usually be found by substituting the numerical values obtained into an equation which has not yet been used and verifying that the equation is satisfied*

BASIC CONCEPTS AND PRINCIPLES

Important Points to Remember

- Statics is the study of bodies that are at rest or move with constant velocity
- A particle has a mass but a size that can be neglected, and a rigid body does not deform under load
- A force is considered as a "push" or "pull" of one body on another
- Concentrated forces are assumed to act at a point on a body
- Newton's three laws of motion should be understood & memorized.
- Mass is measure of a quantity of matter that does not change from one location to another.

BASIC CONCEPTS AND PRINCIPLES

Important Points to Note

- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- In the SI system the unit of force, the newton, is a derived unit.
- The meter, second, and kilogram are base units.
- Perform numerical calculations with several significant figures, and then report the final answer to three significant figures.
- Algebraic manipulations of an equation can be checked in part by verifying that the equation remains dimensionally homogeneous

BASIC CONCEPTS AND PRINCIPLES

Example

EXAMPLE 1.1

Convert 2 km/h to m/s How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\ \text{km/h} = \frac{2\ \cancel{\text{km}}}{\cancel{\text{h}}}\left(\frac{1000\ \text{m}}{\cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$$= \frac{2000\ \text{m}}{3600\ \text{s}} = 0.556\ \text{m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\ \text{m/s} = \left(\frac{0.556\ \cancel{\text{m}}}{\text{s}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \cancel{\text{m}}}\right)$$

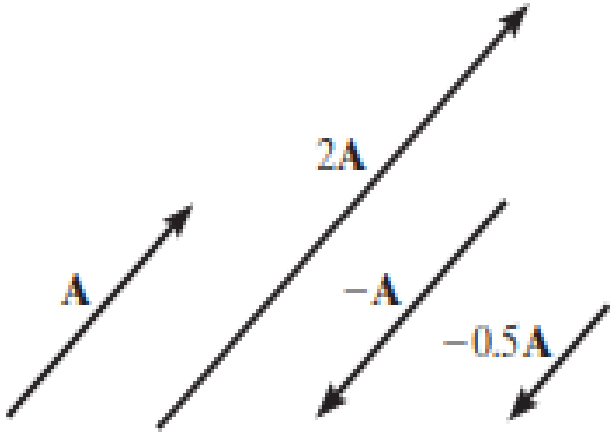
$$= 1.82\ \text{ft/s} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

COMPOSITION & RESOLUTION OF FORCE

Vector Operation

- *Multiplication and Division of Vectors by a Scalar*
  - *When a vector is multiplied by a scalar quantity, its magnitude will be changed.*
  - *Depending on the positive or negative values of the scalar, the magnitude of the vector will be increased or decreased.*
  - *In the same manner, we use this operation if we divide a vector by any positive or negative scalar quantity.*



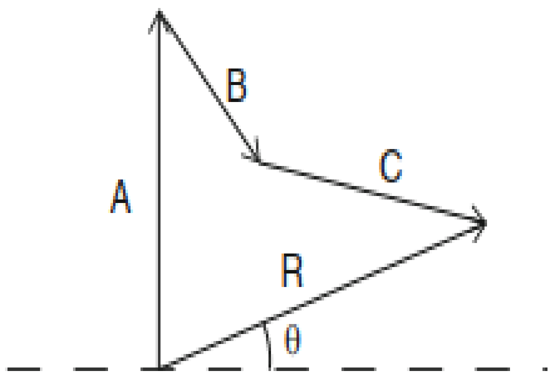
Scalar multiplication and division

Vector Operation

- *Addition of Vectors*
- *There are two common graphical methods for finding the geometric sum of vectors*
- *Polygon method*
- *Parallelogram method*

Vector Operation

- *Polygon Method*
  - *This method is mostly used in applications dealing with the addition of more than two vectors*
  - *Use a ruler and protractor to measure the size (magnitude) and direction of the vector*
  - *Measurements must be done to proper scale*
  - *Continue this process for each vector until you find the magnitude and direction of the resultant vector*

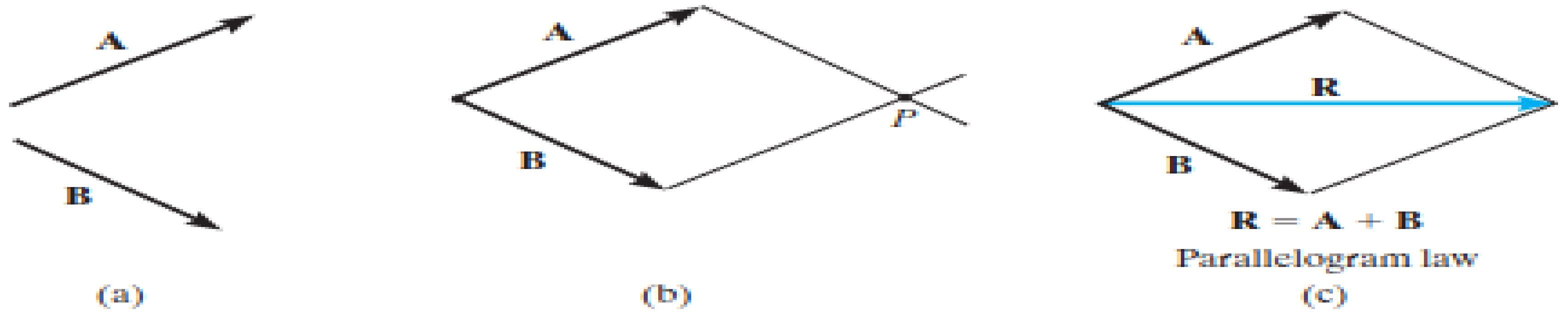


COMPOSITION & RESOLUTION OF FORCE

Vector Operation

- *Parallelogram Method*

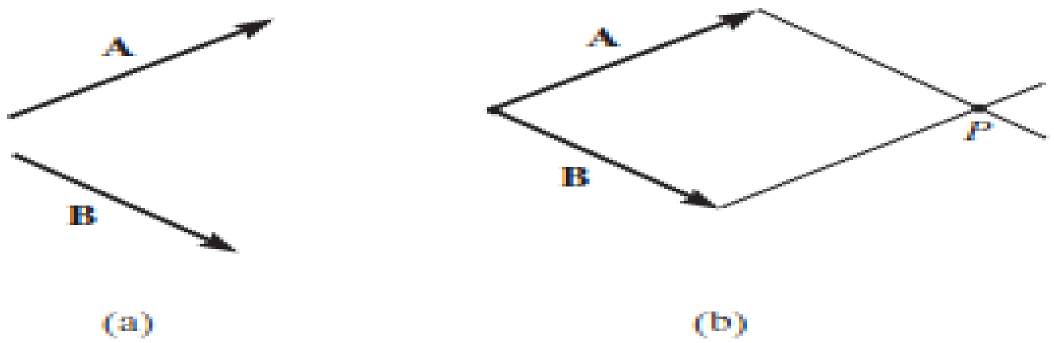
- *When adding two vectors together it is important to account for both their magnitudes and their directions*
- *To illustrate, the two component vectors A and B are added to form a resultant vector R = A + B using the following procedure:.*



Vector Operation

- *Parallelogram Method*

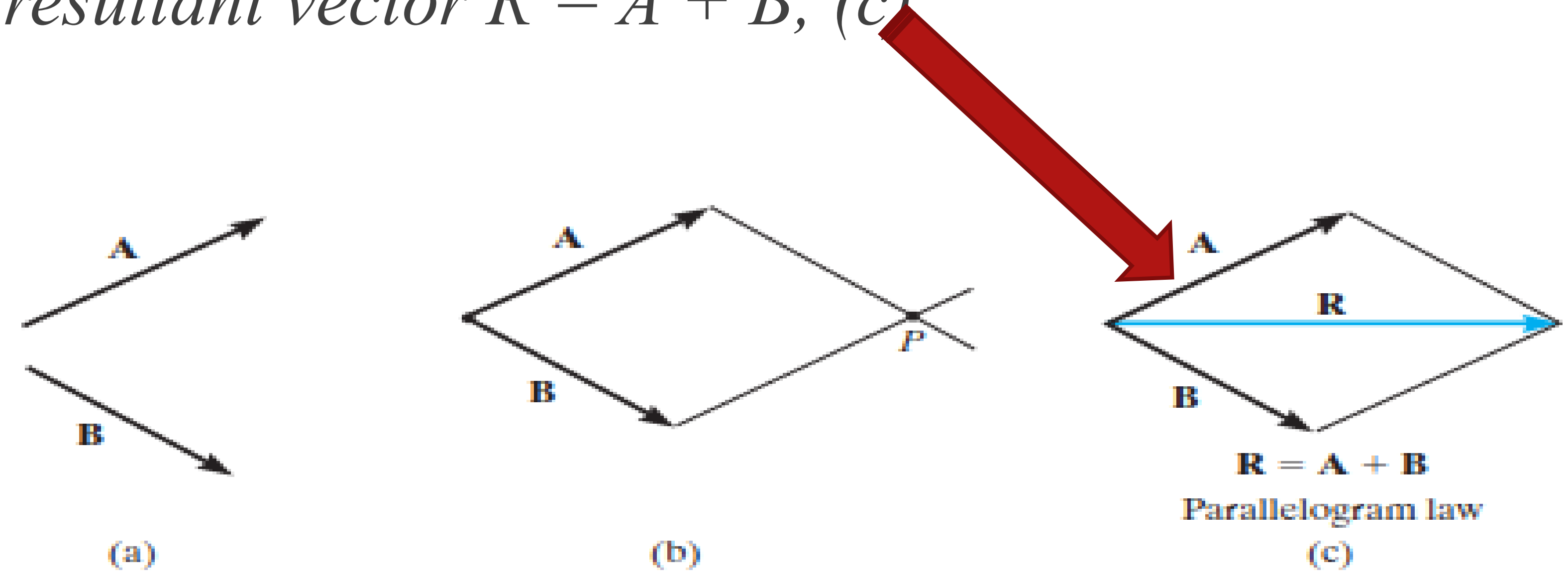
- *First join the tails of the components at a point to make them concurrent (b).*
- *From the head of B, draw a line parallel to A. Draw another line from the head of A that is parallel to B. These two lines intersect at point P to form the adjacent sides of a parallelogram.*



Vector Operation

- *Parallelogram Method*

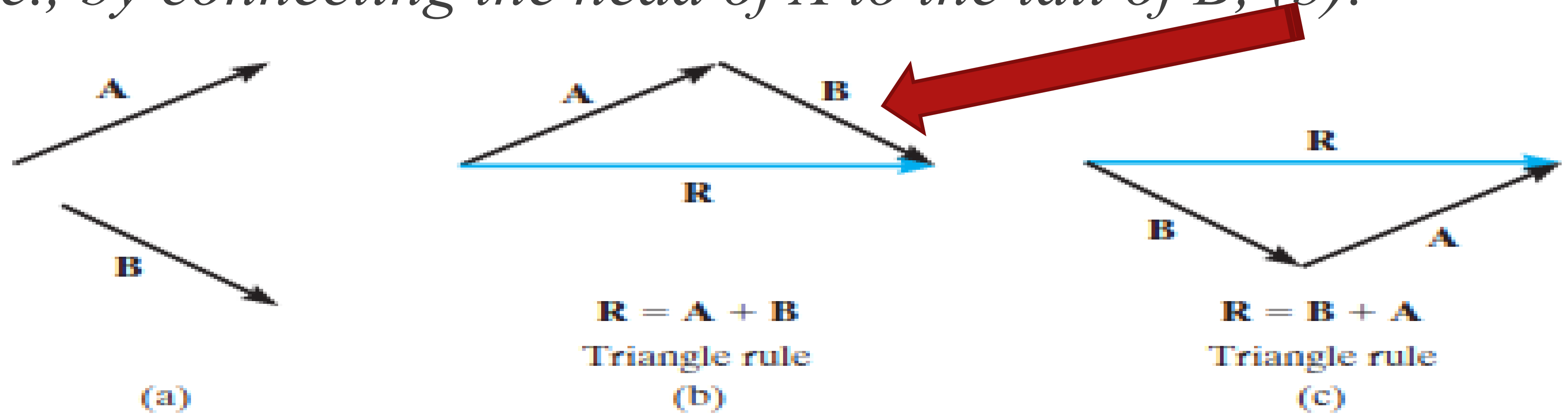
- *The diagonal of this parallelogram that extends to P forms R, which then represents the resultant vector R = A + B, (c)*



Vector Operation

- *Parallelogram Method – Triangle rule*

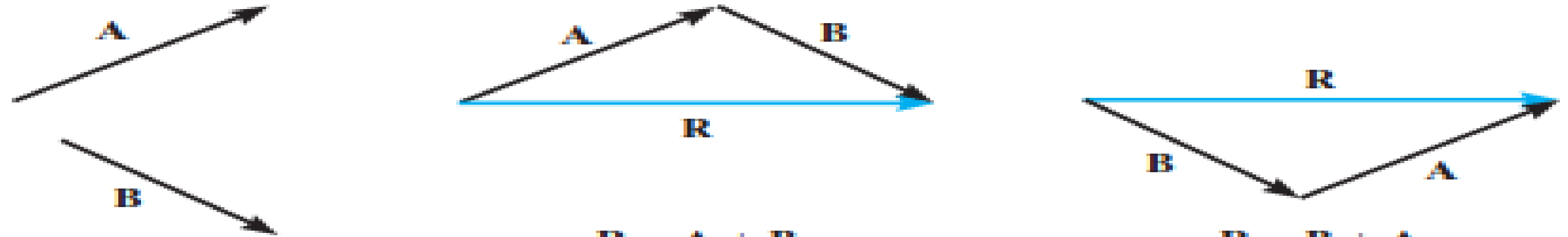
- *We can also add B to A, using the triangle rule, which is a special case of the parallelogram law, whereby vector B is added to vector A in a "head-to-tail" fashion, i.e., by connecting the head of A to the tail of B, (b).*



Vector Operation

- *Parallelogram Method – Triangle rule*

- *The resultant R extends from the tail of A to the head of B. In a similar manner, R can also be obtained by adding A to B, (c)*
- By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e., R = A + B = B + A.



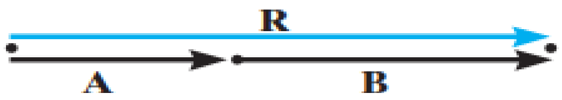
R = A + B
Triangle rule
(b)

R = B + A
Triangle rule
(c)

Vector Operation

Parallelogram Method

- *As a special case, if the two vectors A and B are collinear, i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B*



$R = A + B$

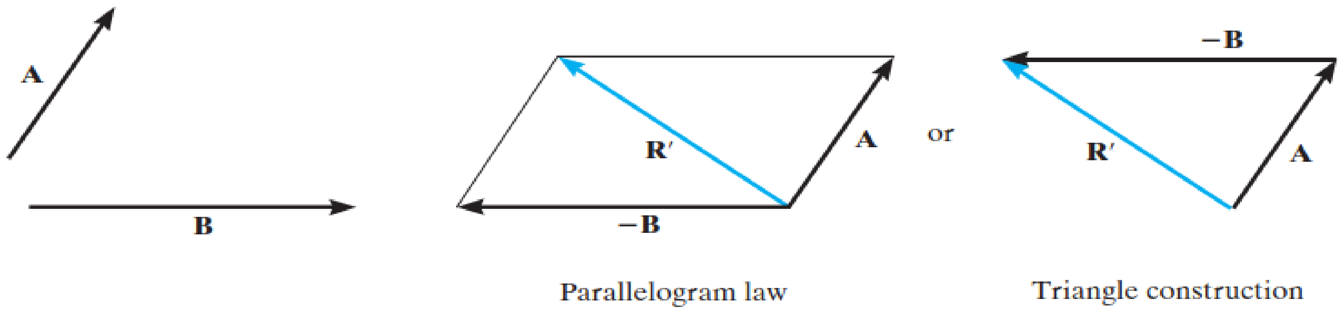
Addition of collinear vectors

Vector Operation

- *Subtraction of Vectors*

- *The resultant of the difference between two vectors A and B of the same type may be expressed as R' = A - B = A + (-B)*

- *Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.*

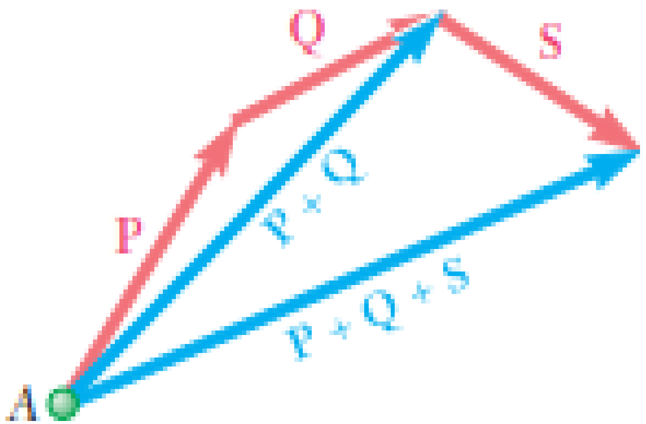


Vector subtraction

COMPOSITION & RESOLUTION OF FORCE

Vector Operation

- *Parallelogram Method for more than two Vectors*

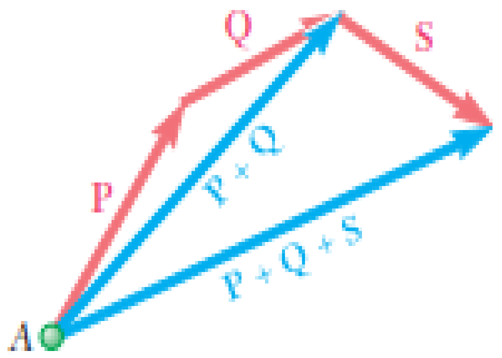


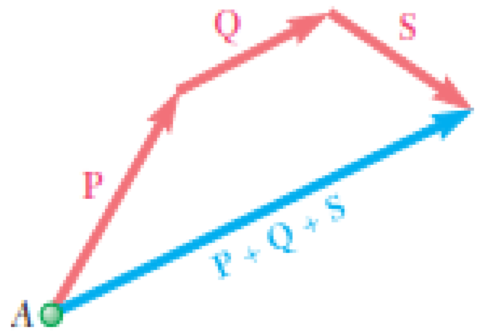
- *The sum of three vectors P, Q, and S will, by definition, be obtained by first adding the vectors P and Q and then adding the vector S to the vector P + Q*
- *We thus write P + Q + S = (P + Q) + S*
- *Similarly, the sum of four vectors will be obtained by adding the fourth vector to the sum of the first three*
- *It follows that the sum of any number of vectors can be obtained by applying repeatedly the parallelogram law to successive pairs of vectors until all the given vectors are replaced by a single vector.*

COMPOSITION & RESOLUTION OF FORCE

Vector Operation

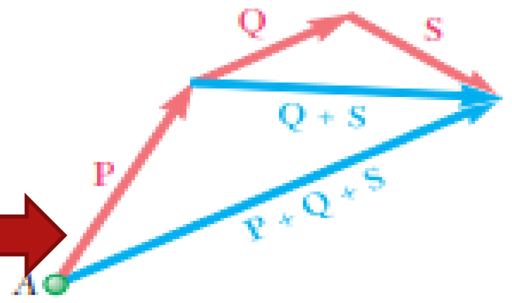
- *Parallelogram Method for more than two Vectors*
  - *If the given vectors are coplanar, their sum can be easily obtained graphically.*
  - *For this case, the repeated application of the triangle rule is preferred to the application of the parallelogram law.*
  - *The determination of the vector $P + Q$, however, could have been omitted and the sum of the three vectors could have been obtained directly by using polygon rule*





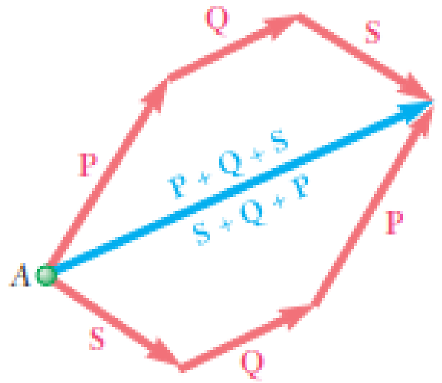
Vector Operation

- *Parallelogram Method for more than two Vectors*



- *We observe that the result obtained would have been unchanged if the vectors Q and S had been replaced by their sum $Q + S$. We may thus write $P + Q + S = (P + Q) + S = P + (Q + S)$ which expresses the fact that vector addition is associative.*

- *You Recall that vector addition has also been shown, in the case of two vectors, to be commutative*

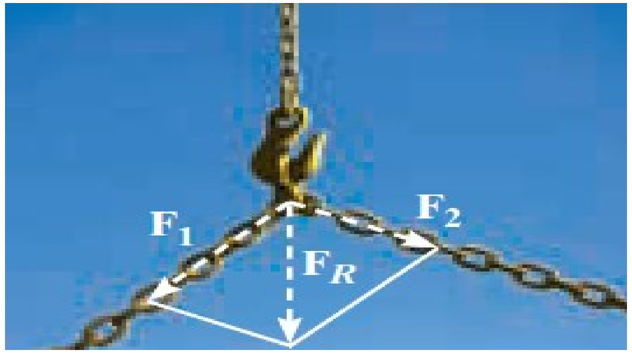


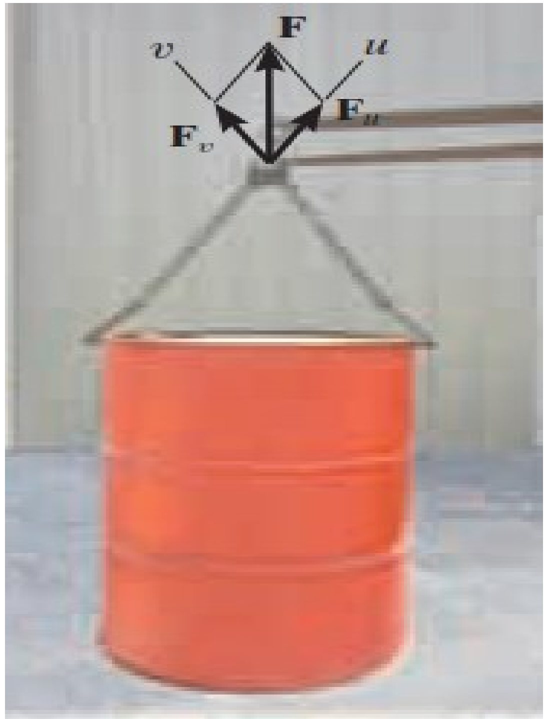
- *This shows that the order in which several vectors are added together is immaterial*

COMPOSITION & RESOLUTION OF FORCE

Vector Addition of Forces

- *Two common problems in statics involve either*
  - *finding the resultant force, knowing its components, or*
  - *resolving a known force into two components.*



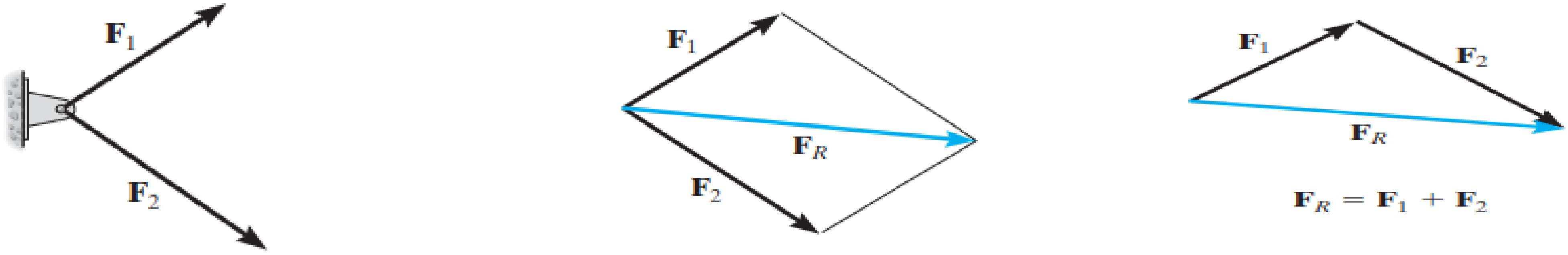


- *We will now describe how each of these problems is solved using the parallelogram law.*

COMPOSITION & RESOLUTION OF FORCE

Vector Addition of Forces

- *Finding a Resultant Force. The two component forces F1 and F2*
- *acting on the pin can be added together to form the resultant force* $F_R = F_1 + F_2$.
- *From this construction, or using the triangle rule, we can apply the law of* ***cosines*** *or the law of* ***sines*** *to the triangle in order to obtain the magnitude of the resultant force and its direction.*



COMPOSITION & RESOLUTION OF FORCE

Vector Addition of Forces

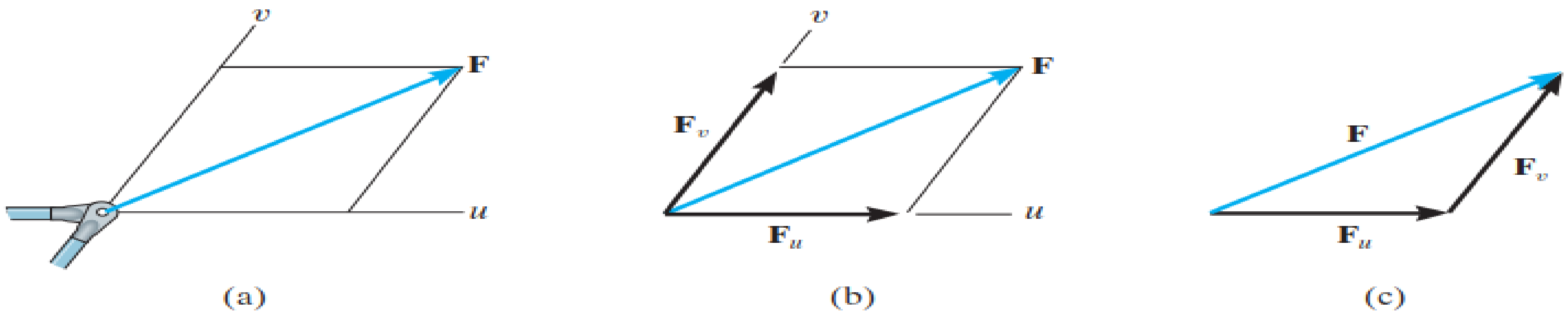
- *Finding the Components of a Force. Sometimes it is necessary to resolve a force into two components in order to study its pulling or pushing effect in two specific directions.*

- *For example, F is to be resolved into two components along u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of F, one line parallel to u, and the other line parallel to v. These lines then intersect with the v and u axes, forming a parallelogram*

Vector Addition of Forces

- *The force components Fu and Fv are then established by simply joining the tail of F to the intersection points on the u and v axes.*

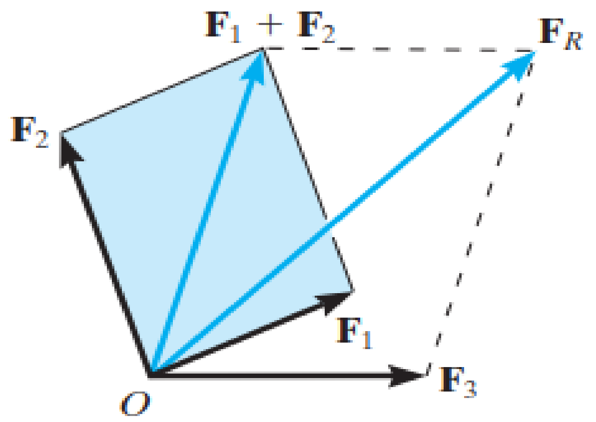
- *This parallelogram can then be reduced to a triangle, which represents the triangle rule. From this, the law of sines can then be applied to determine the unknown magnitudes of the components*



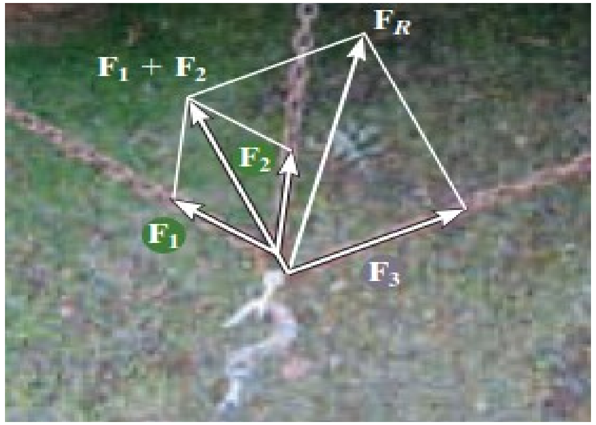
COMPOSITION & RESOLUTION OF FORCE

Vector Addition of Forces

- *Addition of Several Forces*. *If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force.*

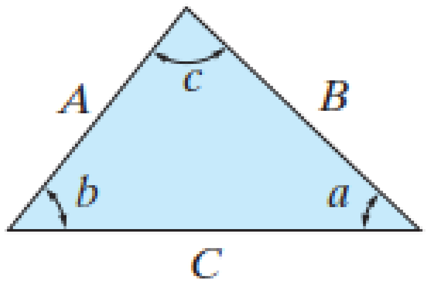


- *For example, if three forces F_1, F_2, F_3 act at a point O, the resultant of any two of the forces is found, say, $F_1 + F_2$ - and then this resultant is added to the third force, yielding the resultant of all three forces; i.e., $F_R = (F_1 + F_2) + F_3$.*



COMPOSITION & RESOLUTION OF FORCE

- Vectors can be added by Algebraic solution:
- Cosine or Sine Rule
  - Cosine Rule $a^2 = b^2 + c^2 - 2bcCosA$
  - Sine Rule $\dfrac{a}{SinA} = \dfrac{b}{SinB} = \dfrac{c}{SinC}$
- You can also use rectangular components: The components are oriented in the X and Y direction.



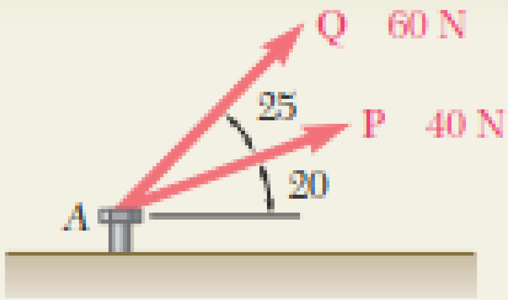
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

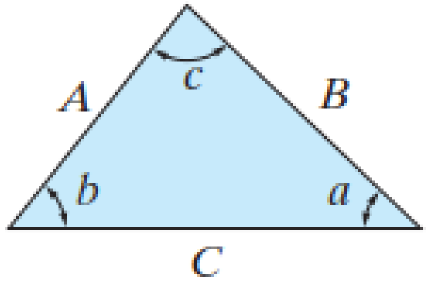
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

COMPOSITION & RESOLUTION OF FORCE



SAMPLE PROBLEM 2.1

The two forces **P** and **Q** act on a bolt *A*. Determine their resultant.



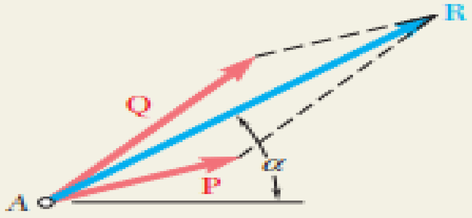
- Cosine Rule $a^2 = b^2 + c^2 - 2bcCosA$
- Sine Rule $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

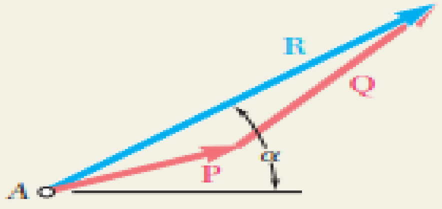
Cosine law:

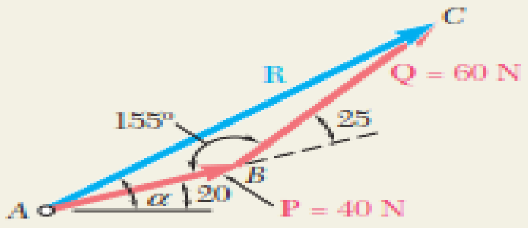
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

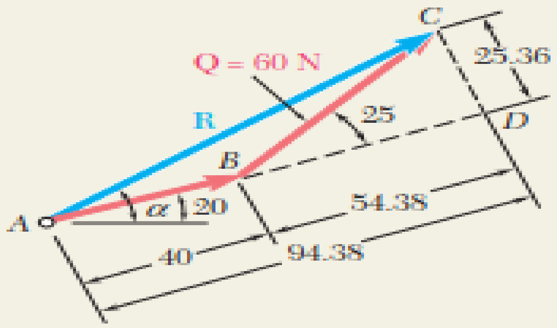
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$









SOLUTION

Graphical Solution. A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35°$$

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98\text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98\text{ N} \measuredangle 35°$$

Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$R^2 = (40\text{ N})^2 + (60\text{ N})^2 - 2(40\text{ N})(60\text{ N})\cos 155°$$
$$R = 97.73\text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\text{ N}} = \frac{\sin 155°}{97.73\text{ N}} \qquad (1)$$

Solving Eq. (1) for sin *A*, we have

$$\sin A = \frac{(60\text{ N})\sin 155°}{97.73\text{ N}}$$

Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

We use 3 significant figures to record the answer (cf. Sec. 1.7):

$$\mathbf{R} = 97.7\text{ N} \measuredangle 35.0°$$

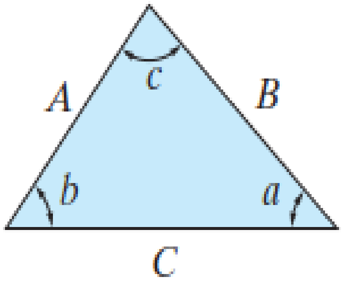
Alternative Trigonometric Solution. We construct the right triangle *BCD* and compute

$$CD = (60\text{ N})\sin 25° = 25.36\text{ N}$$
$$BD = (60\text{ N})\cos 25° = 54.38\text{ N}$$

Then, using triangle *ACD*, we obtain

$$\tan A = \frac{25.36\text{ N}}{94.38\text{ N}} \qquad A = 15.04°$$
$$R = \frac{25.36}{\sin A} \qquad R = 97.73\text{ N}$$

Again, $\alpha = 20° + A = 35.04°$ $\quad \mathbf{R} = 97.7\text{ N} \measuredangle 35.0°$



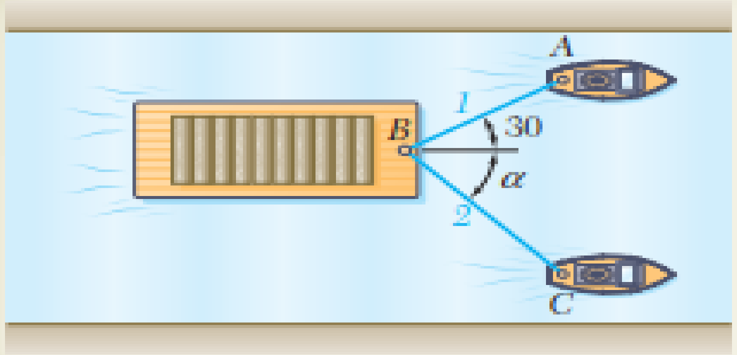
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

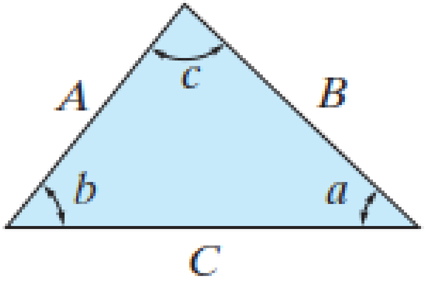
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

COMPOSITION & RESOLUTION OF FORCE



SAMPLE PROBLEM 2.2

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$, (*b*) the value of α for which the tension in rope 2 is minimum.



- Cosine Rule $a^2 = b^2 + c^2 - 2bcCosA$
- Sine Rule $\dfrac{a}{SinA} = \dfrac{b}{SinB} = \dfrac{c}{SinC}$

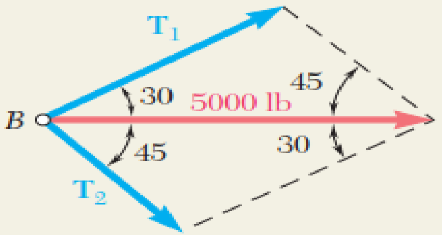
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

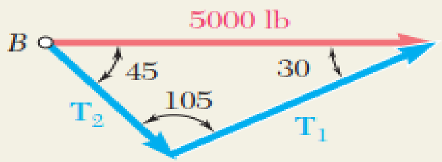
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

SOLUTION



a. Tension for $\alpha = 45°$. ***Graphical Solution.*** The parallelogram law is used; the diagonal (resultant) is known to be equal to 5000 lb and to be directed to the right. The sides are drawn parallel to the ropes. If the drawing is done to scale, we measure

$$T_1 = 3700 \text{ lb} \qquad T_2 = 2600 \text{ lb}$$

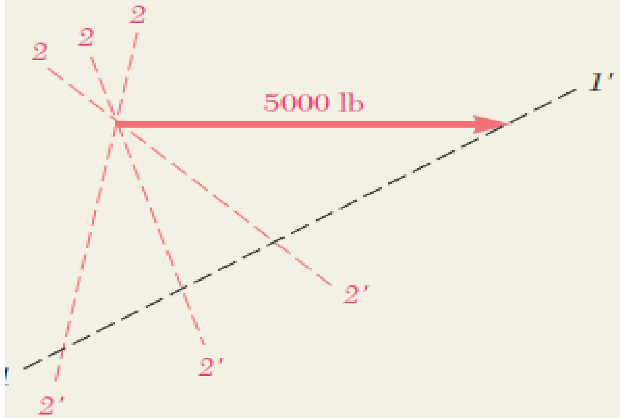


Trigonometric Solution. The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines, we write

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lb}}{\sin 105°}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 45° and sin 30°, we obtain

$$T_1 = 3660 \text{ lb} \qquad T_2 = 2590 \text{ lb}$$



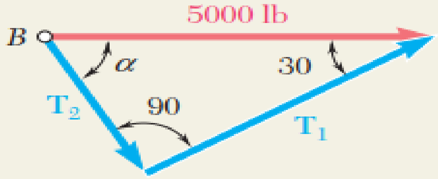
b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line *1-1′* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2′*. We note that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular. The minimum value of T_2 is

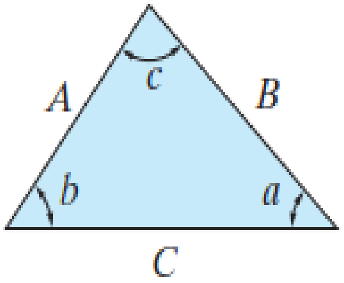
$$T_2 = (5000 \text{ lb}) \sin 30° = 2500 \text{ lb}$$

Corresponding values of T_1 and α are

$$T_1 = (5000 \text{ lb}) \cos 30° = 4330 \text{ lb}$$

$$\alpha = 90° - 30° \qquad \alpha = 60°$$





Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the u and v axes and determine the magnitudes of these components.

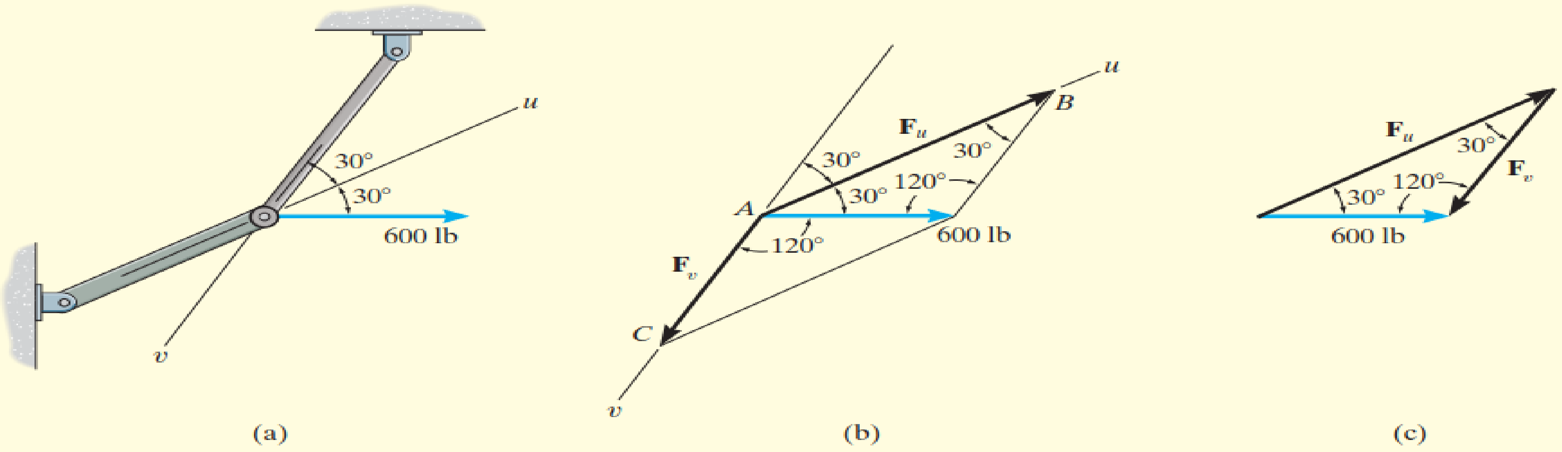


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the v axis until it intersects the u axis at point B, Fig. 2–12*b*. The arrow from A to B represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the u axis intersects the v axis at point C, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

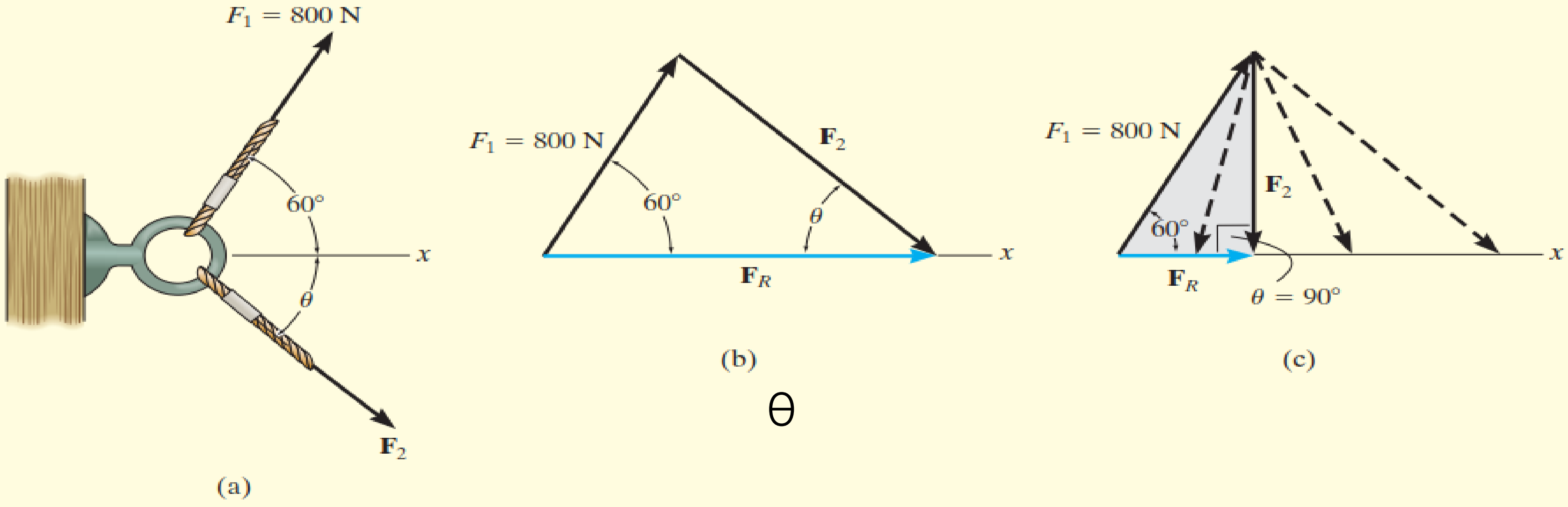
$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

NOTE: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive x axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.



θ

Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90^\circ \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800\text{ N})\cos 60^\circ = 400\text{ N} \qquad \textit{Ans.}$$

$$F_2 = (800\text{ N})\sin 60^\circ = 693\text{ N} \qquad \textit{Ans.}$$