# CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

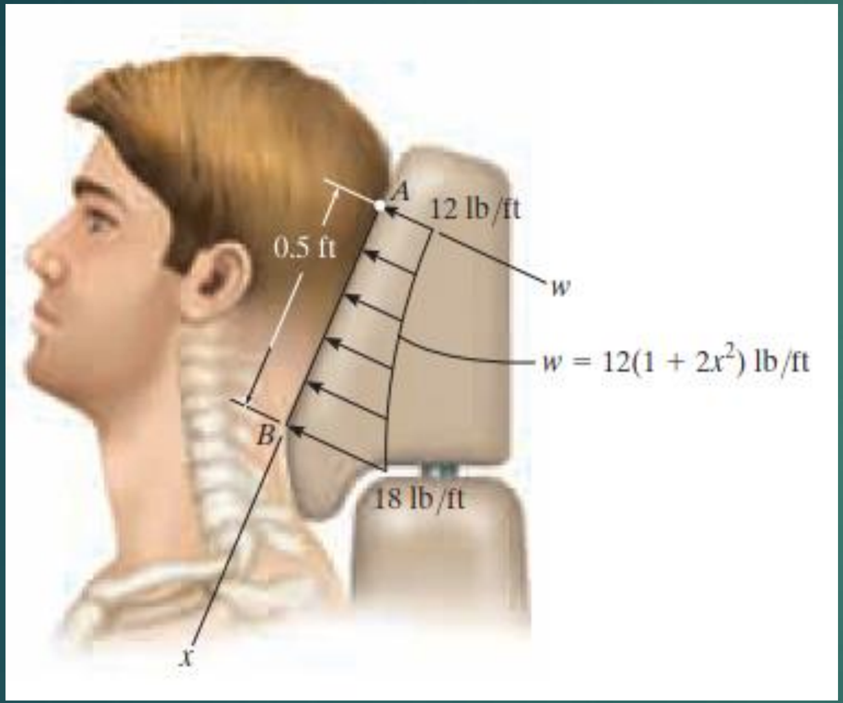


# Lecture A5 -Sup

- REDUCTION OF A SIMPLE DISTRIBUTED LOADING

# LECTURE OBJECTIVES

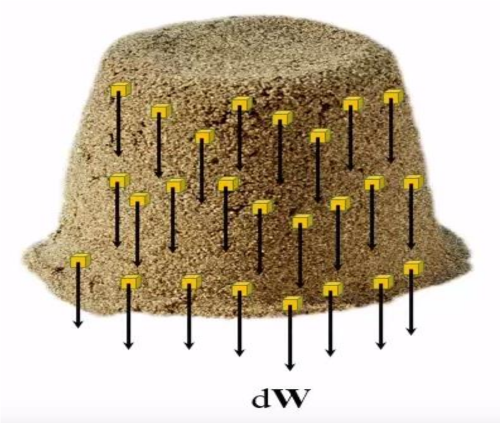


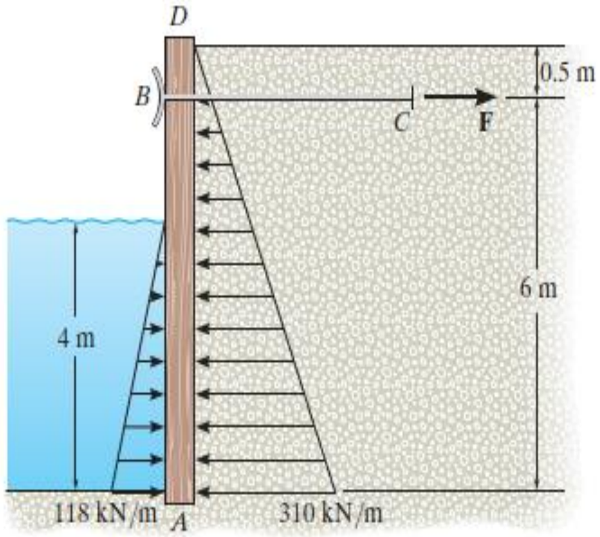



- ❖ To indicate how to reduce a simple distributed loading to a resultant force acting at a specified location.
- ❖ To calculate and solve Moments/Couples, Resultants and using Equilibrium Equations for rigid bodies with distributed loading.

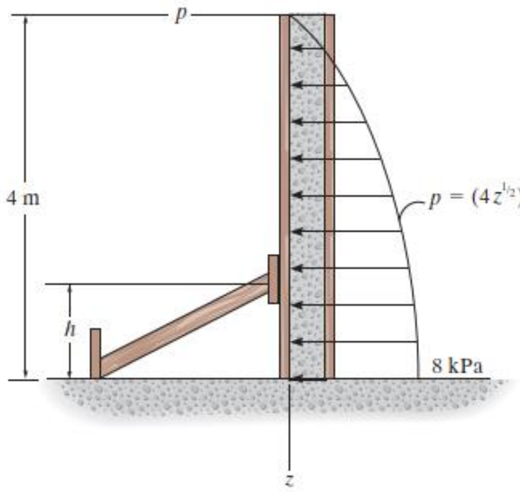
# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

- Sometimes, a body may be subjected to a loading that is distributed over its surface.
- For example, the pressure of the wind on the face of a solid wall, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all distributed loadings.
- The pressure exerted at each point on the surface indicates the intensity of the loading.
- It is measured using pascals Pa (or N/m2) in SI units or lb/ft2.



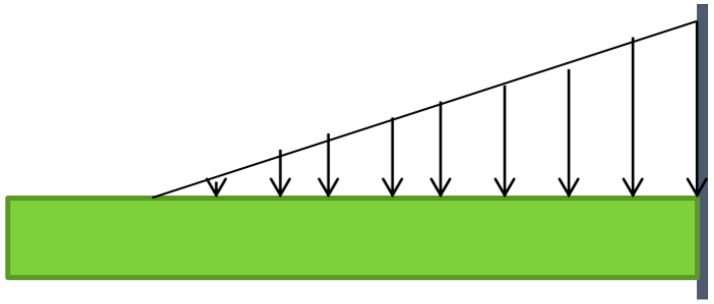



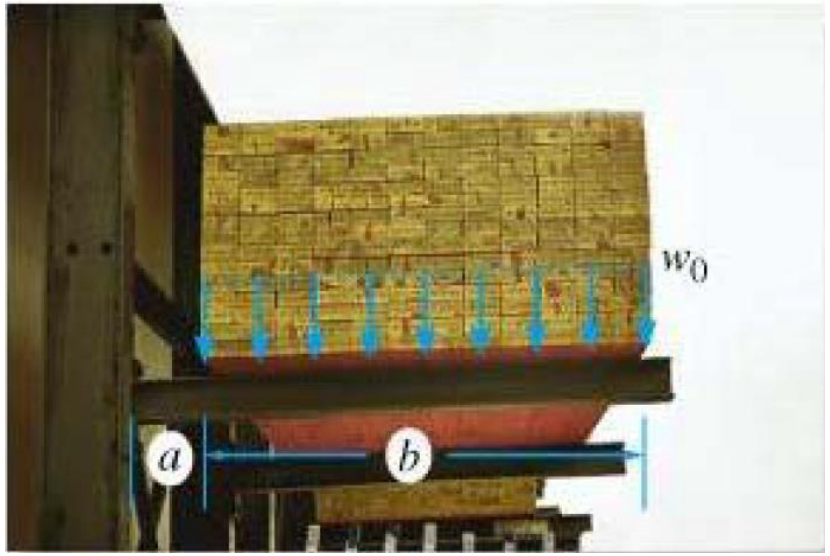

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD




- WHAT IS A DISTRIBUTED LOAD?

A distributed load is a weight (load) applied across a length or area instead of at one point

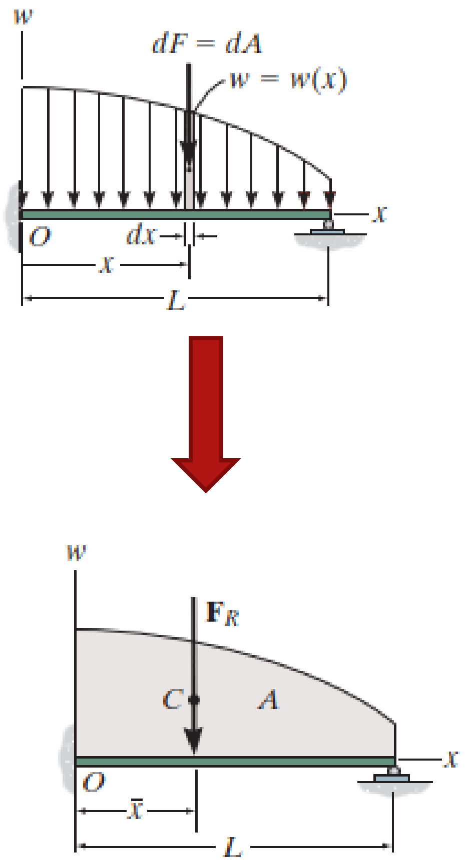


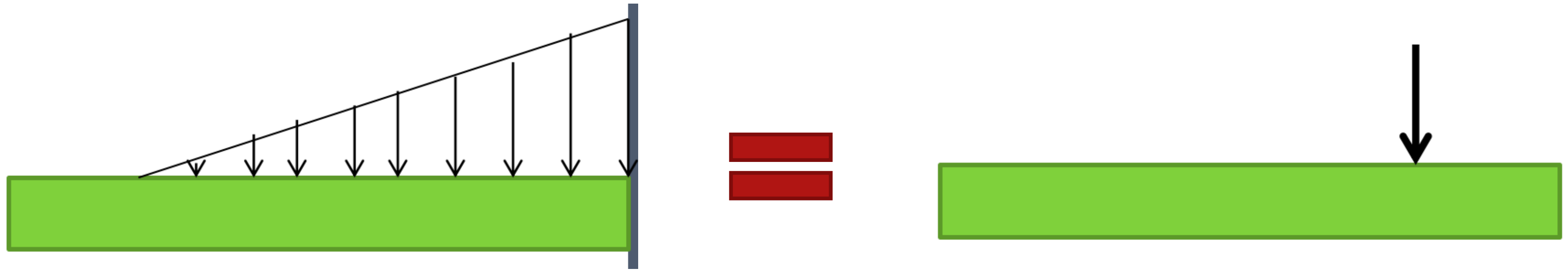


# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Analysing Distributed Loads

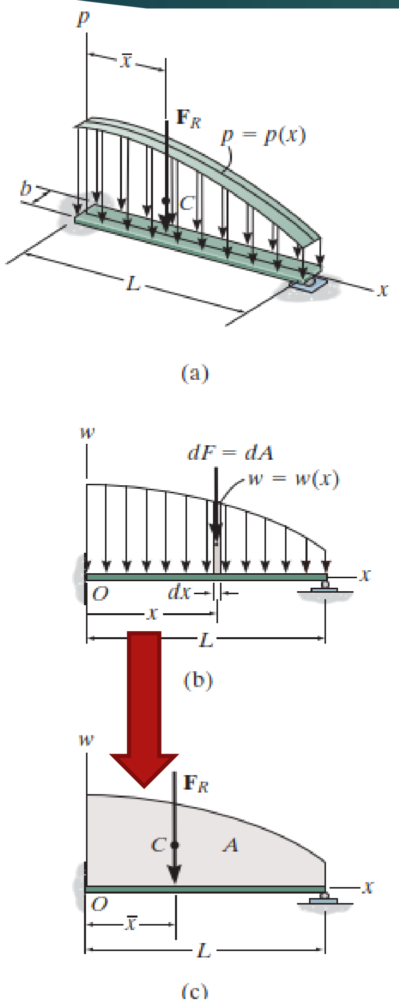
- A distributed load can be equated with a concentrated load applied at a specific point along the bar



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

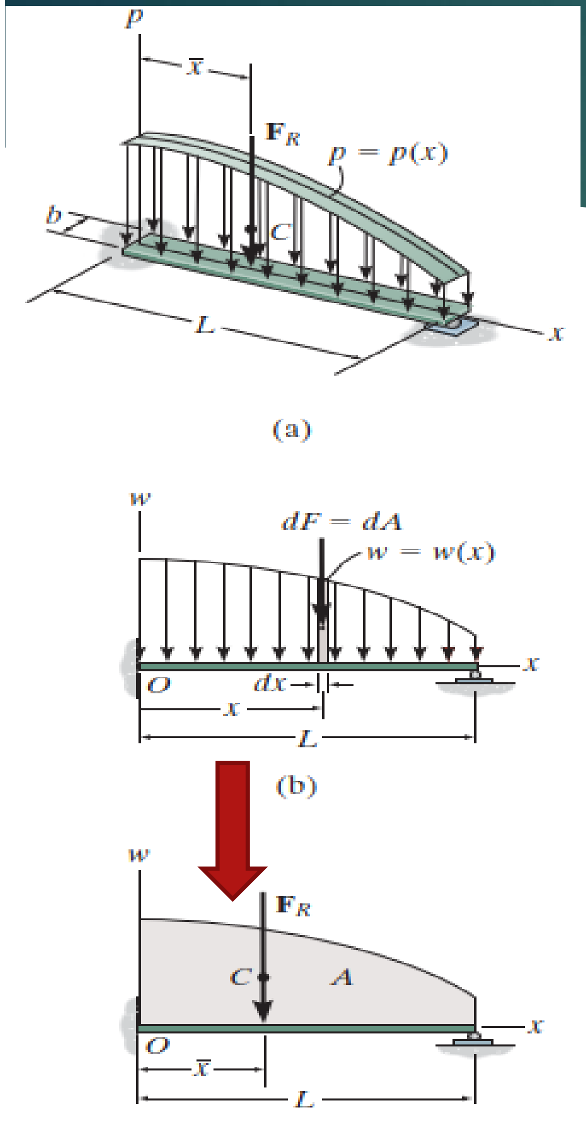
## Analysing Distributed Loads




- The most common type of distributed loading encountered in engineering practice can be represented along a single axis.
- For example, consider the beam (or plate) in Fig. 'a' that has a constant width and is subjected to a pressure loading that varies only along the x axis.
- This loading can be described by the function p = p(x) N/m2.
- It contains only one variable x, and for this reason, we can also represent it as a coplanar distributed load.

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Analysing Distributed Loads

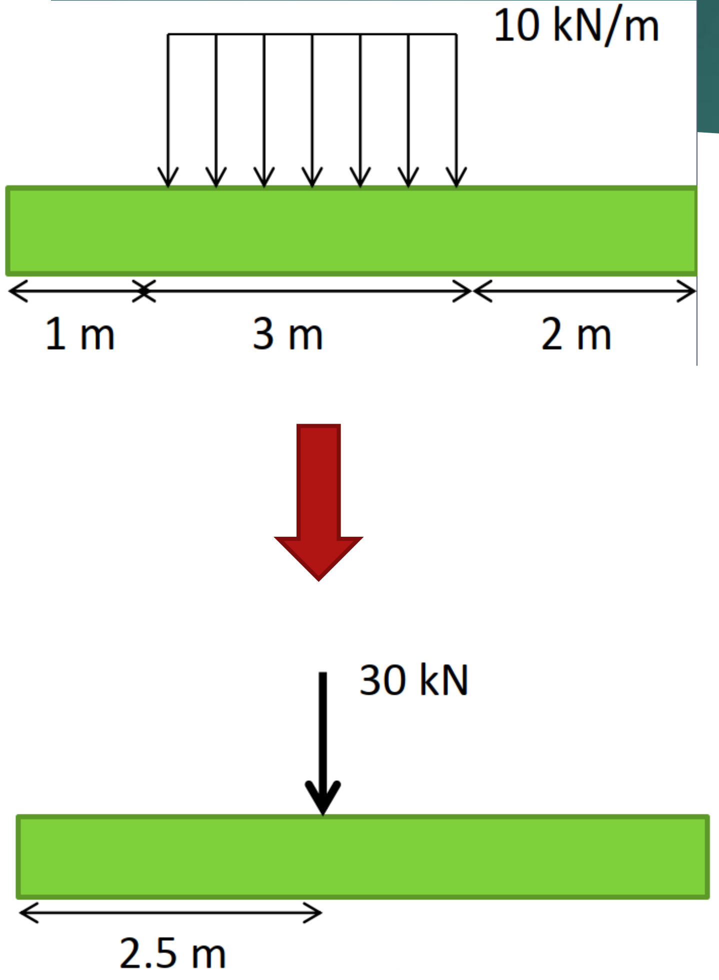



(a) (b) (c)

- To do so, we multiply the loading function by the width ‘b’ m of the beam, so that w(x) = p(x)b N/m, Fig. ‘b’
- Using this methods, we can replace this coplanar parallel force system with a single equivalent resultant force $F_R$ acting at a specific location on the beam as shown in Fig. ‘c’

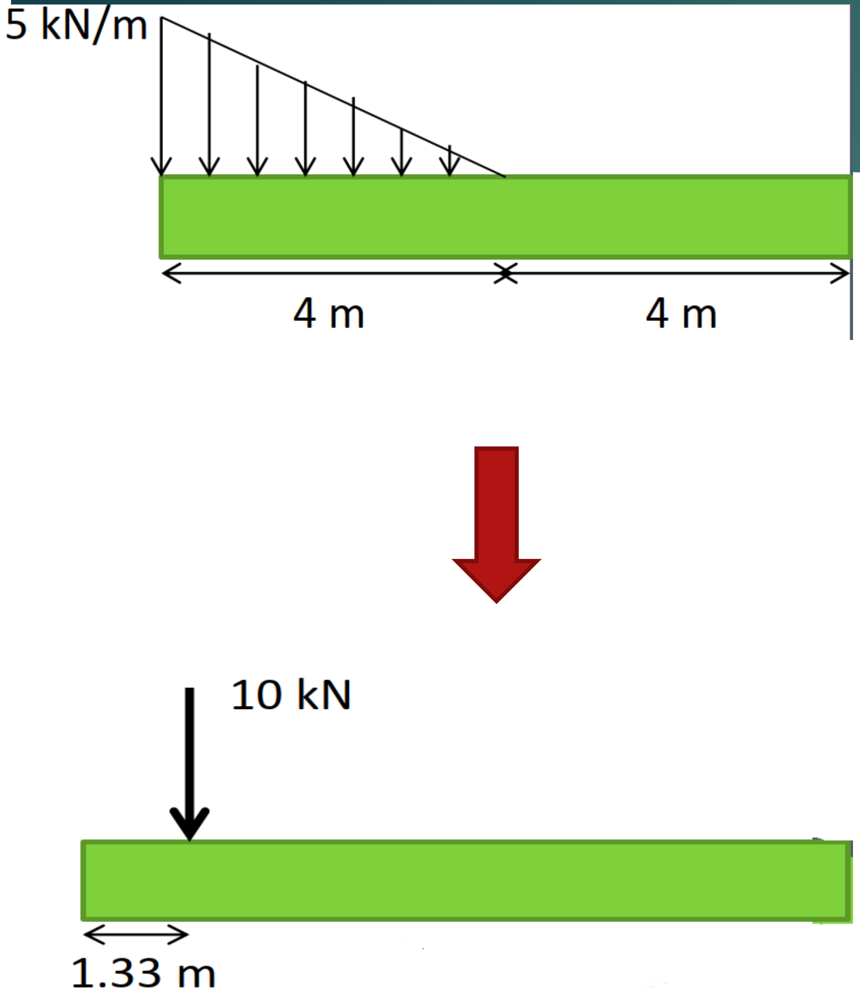
# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Geometry Method



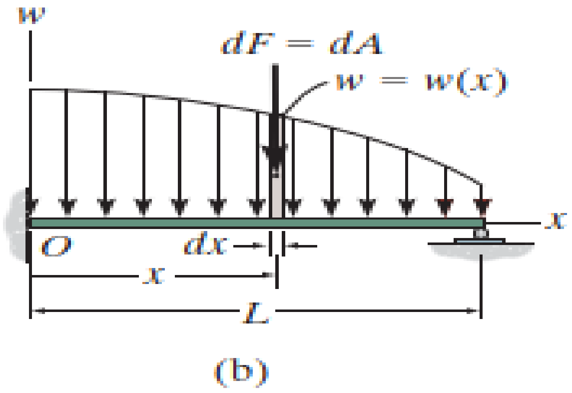

- The magnitude of the resultant force is equivalent to the area under the curve of the distributed load
- $F_R$= b*h = 10 kN/m*3m = 30 kN
- The location of the resultant force is at the centre of mass of the distributed load
- $\bar{x} = x_0$ + ½ b = 1m + ½ * 3m = 2.5m

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## Geometry Method

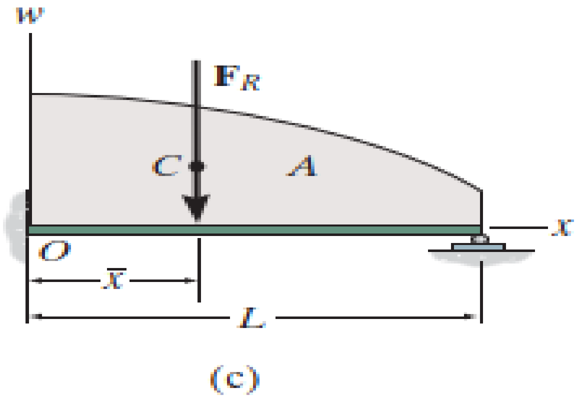



- For a triangular distributed load, the magnitude of the resultant force is the area of the triangle, ½*b*h
- $F_R$= 1/2b*h = ½*5 kN/m*4m = 10 kN
- The location of the resultant force for a triangle is 1/3 of the length of the load from the larger end
- $\bar{x} = x_0$+1/3 b = 0m+1/3 * 4m = 4/3m

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## Integral Method



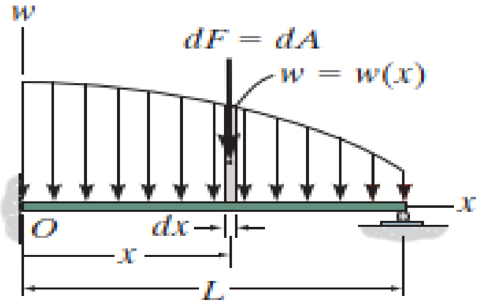
(b)



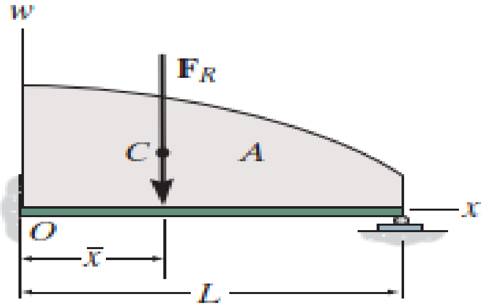
(c)

- The magnitude $of\ F_R$ is equivalent to the sum of all the forces in the system.
- In this case integration must be used since there is an infinite number of parallel forces dF of different magnitude acting on the beam, Fig. 4–48b.
- Since dF is acting on an element of length dx, and w(x) is a force per unit length,
- then dF = w(x) dx = dA.

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## Integral Method




(b)




(c)

- dF = w(x) dx = dA.
- In other words, the magnitude of dF is determined from the colored differential area dA under the loading curve.
- For the entire length L,

$$+\downarrow F_R = \Sigma F;$$

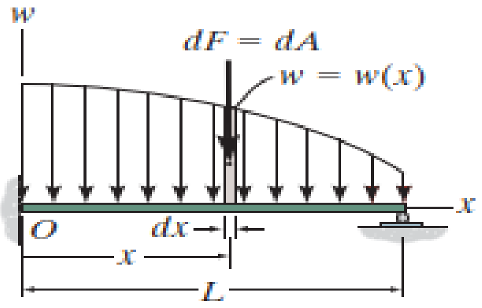
$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

- Therefore, the magnitude of the resultant force is equal to the area A under the loading diagram, Fig. 4–48c.

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

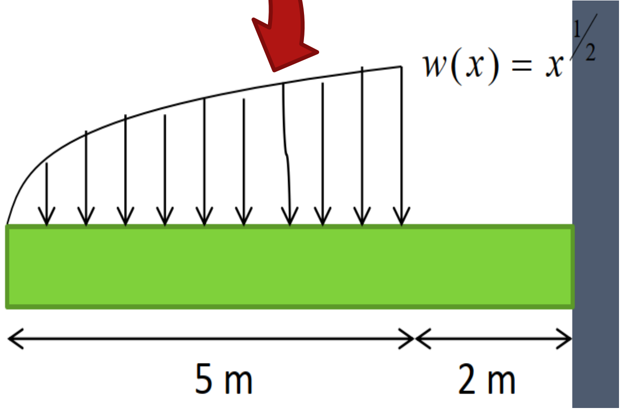
## Integral Method

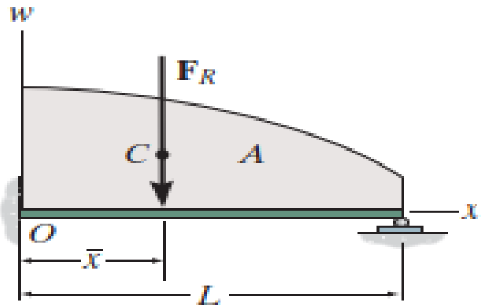
$$+\downarrow F_R = \Sigma F; \qquad F_R = \int_L w(x)\,dx = \int_A dA = A$$




(b)

- The magnitude of the resultant force is given by the integral of the curve defining the force, w(x)



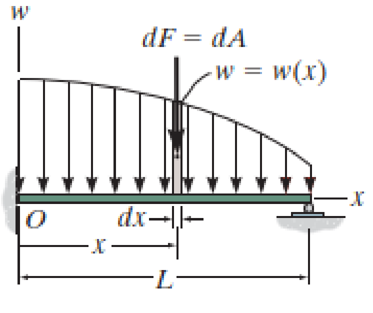

(c)

$$F_r = \int_{x=0}^{5\,m} w(x)\,dx = \int_{x=0}^{5\,m} x^{1/2}\,dx \qquad F_r = \frac{2}{3}[x^{3/2}]_0^{5\,m} = 7.45\ kN$$

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

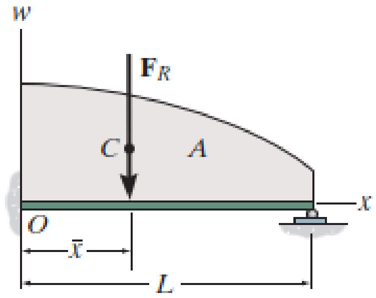
## Integral Method

$+\downarrow F_R = \Sigma F;$ $\quad F_R = \int_L w(x)\,dx = \int_A dA = A$




(b)

- To determine the location of the resultant Force, the location x of the line of action of $F_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O (the y axis).




(c)

- Since dF produces a moment of x dF = xw(x) dx about O, Fig. 4–48b, then for the entire length, Fig. 4–48c,

$$\zeta + (M_R)_O = \Sigma M_O; \qquad -\bar{x}F_R = -\int_L xw(x)\,dx$$

- Solving for $\bar{x}$, we have

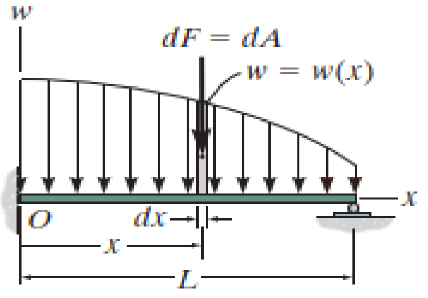
$$\bar{x} = \frac{\int_L xw(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

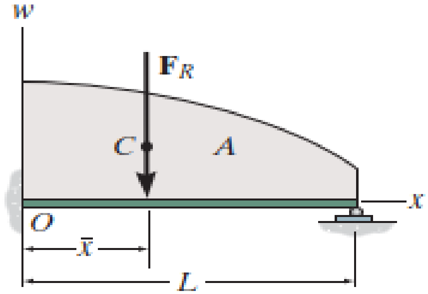
## Integral Method

$+\downarrow F_R = \Sigma F;\quad F_R = \int_L w(x)\,dx = \int_A dA = A$

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$



(b)



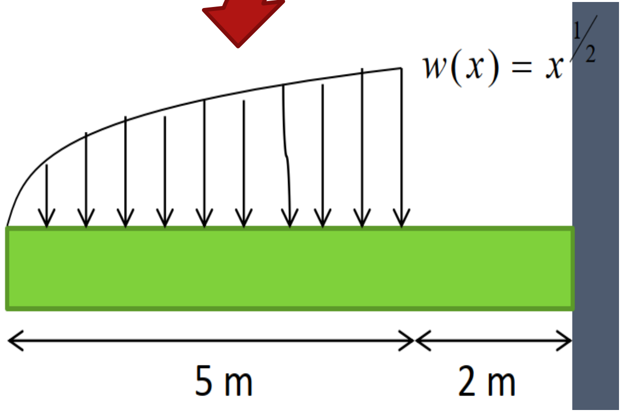
(c)

- This coordinate x, locates the geometric centre or centroid of the area under the distributed loading.
- In other words, the resultant force has a line of action which passes through the centroid C (geometric centre) of the area under the loading diagram, Fig. 4–48c.
- Detailed treatment of the integration techniques for finding the location of the centroid for areas will be discussed in the future lectures.

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Integral Method

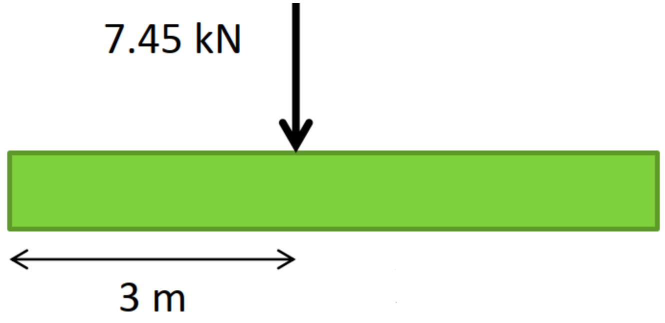
$$+\downarrow F_R = \Sigma F; \qquad F_R = \int_L w(x)\,dx = \int_A dA = A$$




- The location of the resultant force is given by the centroid of the area under the curve w($x$) = $x^{1/2}$

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$

$$\bar{x} = \frac{\int_0^L w(x)\,x dx}{\int_0^L w(x)\,dx} = \frac{\int_0^{5\,m} x^{3/2}\,dx}{7.45}$$




$$\bar{x} = \frac{2}{5 * 7.45}[x^{5/2}]_0^{5\,m} = 3.0\,m$$

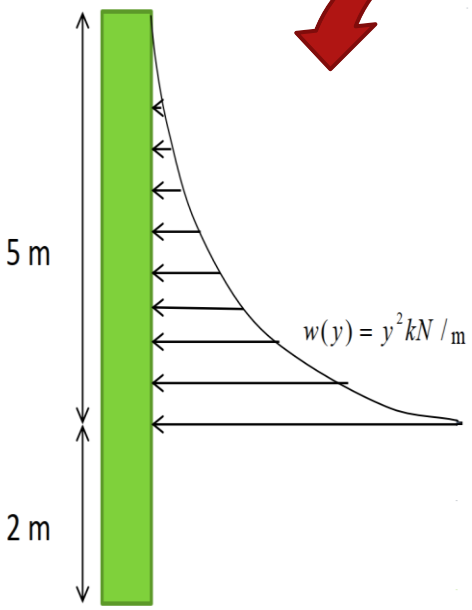
# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.1-Sup

$+\downarrow F_R = \Sigma F;$

$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$




- Question

For a vertical bar, determine the resultant force and its location

- Solution

For the resultant force, simply integrate with respect to y instead of x

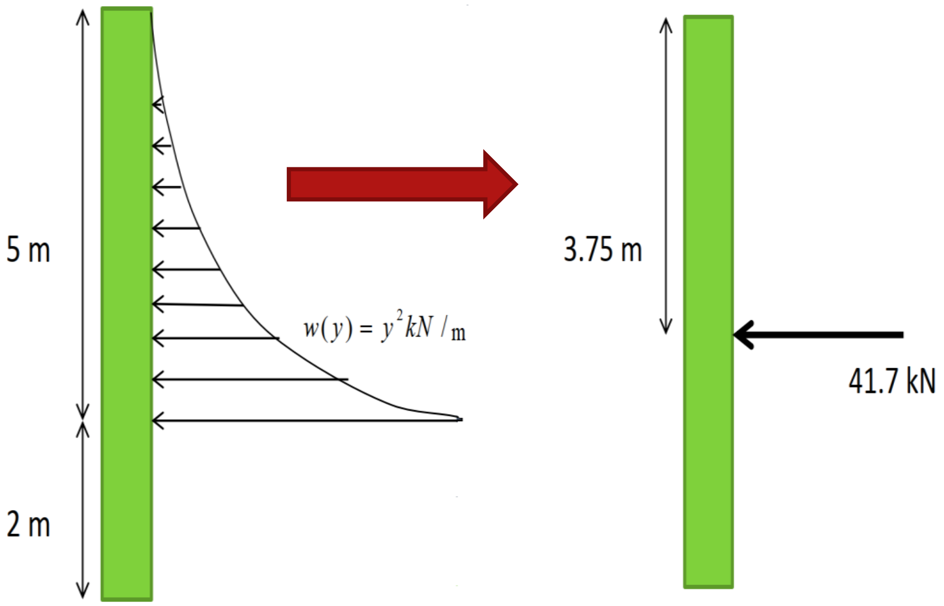
$$F_r = \int_{y=0}^{5\,m} w(y)dy = \int_{y=0}^{5\,m} y^2 dy$$

$$F_r = \frac{1}{3}[y^3]_0^{5\,m} \qquad F_r = 41.7\,kN$$

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.1-Sup

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$




- Solution

For the location of the $F_r$, simply integrate with respect to y instead of x

$$\bar{y} = \frac{\int_0^L w(y)y\,dy}{\int_0^L w(y)\,dy} = \frac{\int_0^{5m} y^3\,dy}{41.7}$$

$$\bar{y} = \frac{1}{4 * 41.7}[y^4]_0^{5m}$$

$$\bar{y} = 3.75\,m$$

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

$+\downarrow F_R = \Sigma F;$

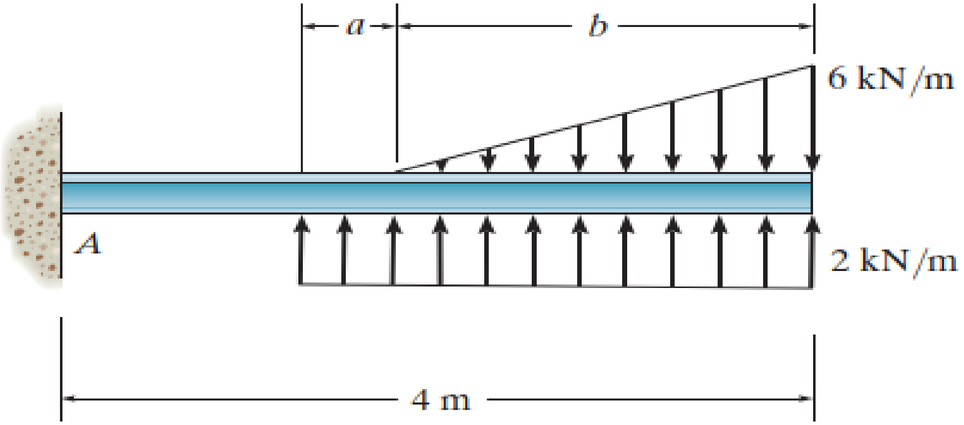
$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

## Example 5.2-Sup

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$

- Question

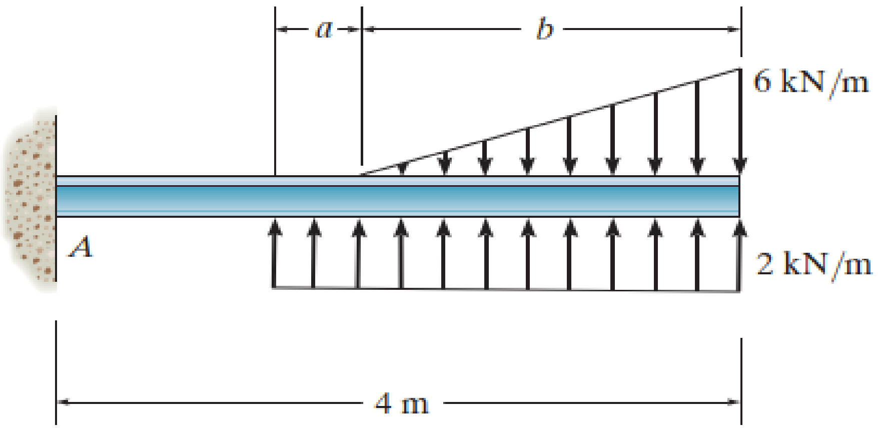
Determine the length b of the triangular load and its position ‘a’ on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN. m clockwise.



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

$$+\downarrow F_R = \Sigma F; \qquad F_R = \int_L w(x)\,dx = \int_A dA = A$$

## Example 5.2-Sup




## Solution

**Equivalent Resultant Force And Couple Moment At Point A.** Summing the forces along the $y$ axis by referring to Fig. $a$, with the requirement that $\mathbf{F}_R = 0$,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 0 = 2(a + b) - \frac{1}{2}(6)(b)$$

$$2a - b = 0 \qquad (1)$$

Summing the moments about point $A$, with the requirement that $(M_R)_A = 8\ \text{kN}\cdot\text{m}$,

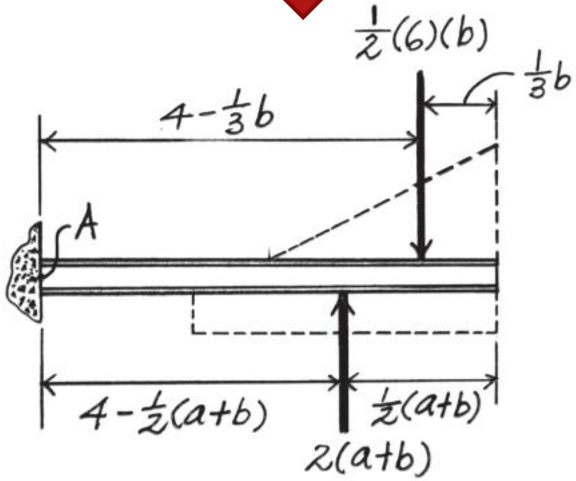
$$\circlearrowleft +(M_R)_A = \Sigma M_A; \ -8 = 2(a + b)\left[4 - \frac{1}{2}(a + b)\right] - \frac{1}{2}(6)(b)\left(4 - \frac{1}{3}b\right)$$

$$-8 = 8a - 4b - 2ab - a^2 \qquad (2)$$

Solving Eqs (1) and (2),

$$a = 1.264\ \text{m} = 1.26\ \text{m} \qquad \textbf{Ans.}$$

$$b = 2.530\ \text{m} = 2.53\ \text{m} \qquad \textbf{Ans.}$$



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

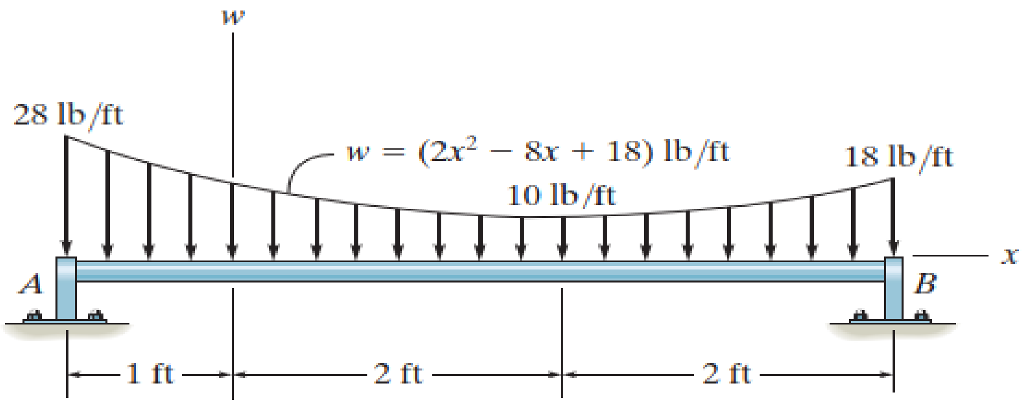
## Example 5.3-Sup

$+\downarrow F_R = \Sigma F;$ $\quad F_R = \int_L w(x)\,dx = \int_A dA = A$

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$

- Question

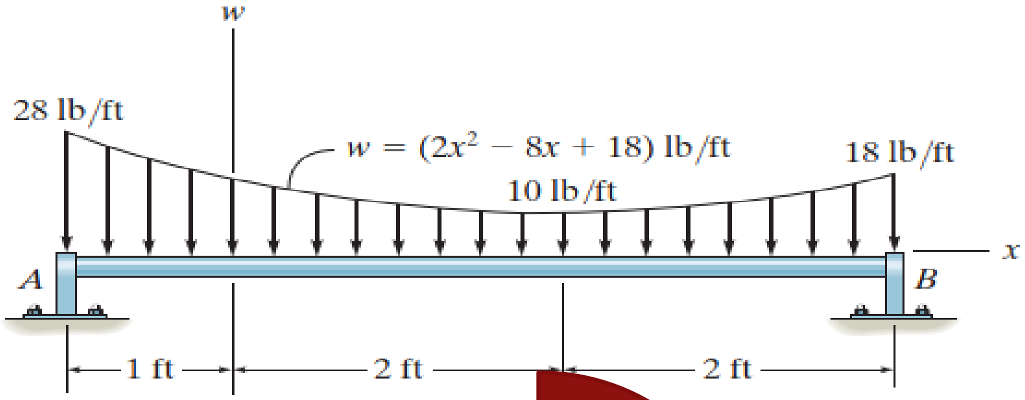
The distributed load acts on the shaft as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from the support, A.



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.3-Sup

$+\downarrow F_R = \Sigma F;$ $\quad F_R = \int_L w(x)\,dx = \int_A dA = A$



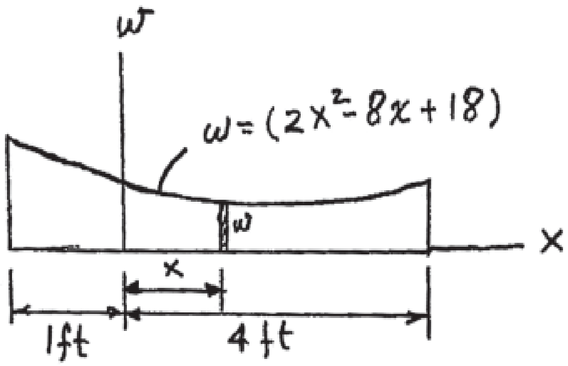

- Solution

$$F_R = \int_{-1}^{4} (2x^2 - 8x + 18)\,dx = \frac{2}{3}x^3 - \frac{8x^2}{2} + 18x\Big|_{-1}^{4} = 73.33 = 73.3\text{ lb} \quad \textbf{Ans.}$$

$$\int x\,dF = \int_{-1}^{4} (2x^3 - 8x^2 + 18x)\,dx = \frac{2}{4}x^4 - \frac{8}{3}x^3 + \frac{18}{2}x^2\Big|_{-1}^{4} = 89.166\text{ lb}\cdot\text{ft}$$

$$x = \frac{89.166}{73.3} = 1.22\text{ ft}$$

$$d = 1 + 1.22 = 2.22\text{ ft} \quad \textbf{Ans.}$$



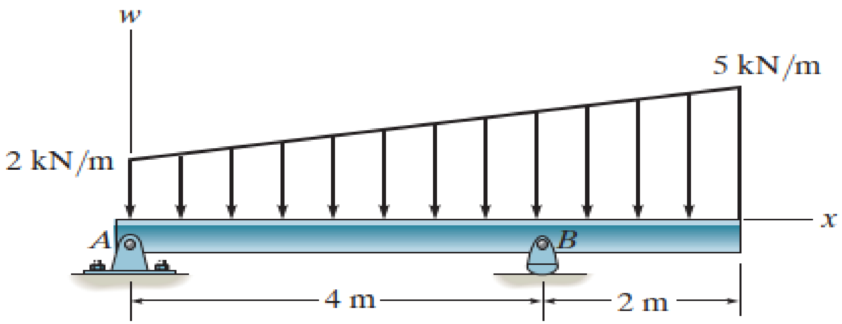
# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.4-Sup



- Question

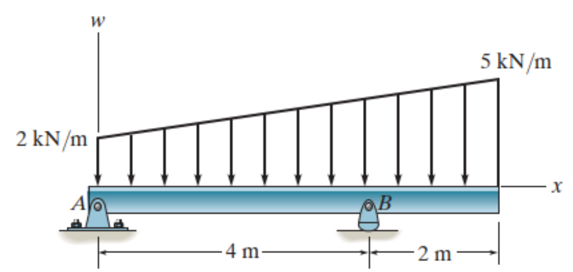
Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point A

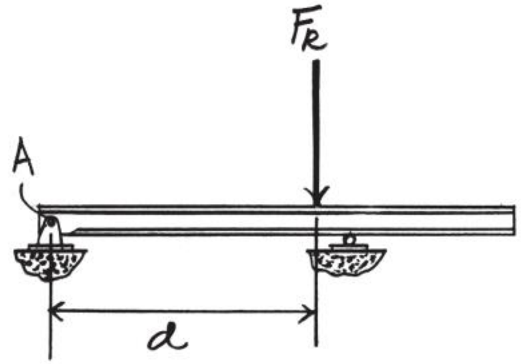


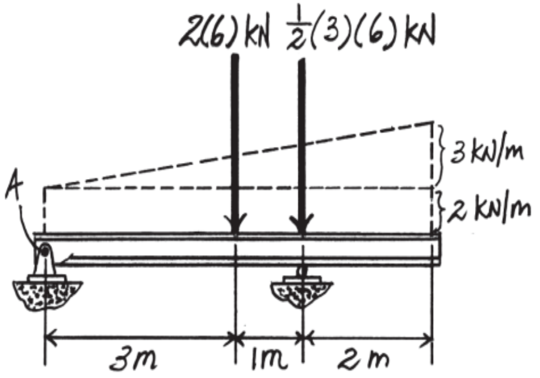
# REDUCTION OF A SIMPLE DISTRIBUTED LOAD



## Example 5.4-Sup




## ▶ Solution

**Equivalent Resultant Force.** Summing the forces along the *y* axis by referring to Fig. *a*

$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(3)(6) \qquad \textbf{Ans.}$$

$$F_R = 21.0\ \text{kN}\downarrow \qquad \textbf{Ans.}$$

**Location of the Resultant Force.** Summing the moments about point *A*,

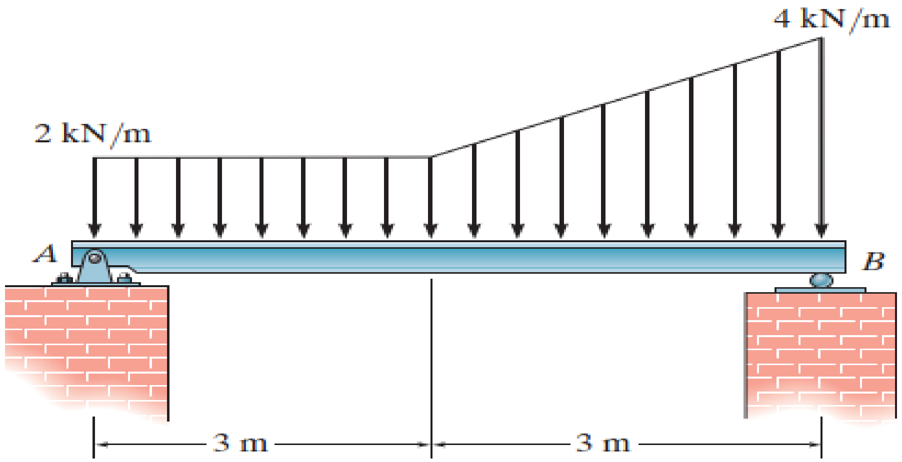
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -21.0(d) = -2(6)(3) - \frac{1}{2}(3)(6)(4)$$

$$d = 3.429\ \text{m} = 3.43\ \text{m} \qquad \textbf{Ans.}$$

# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## Example 5.5-Sup

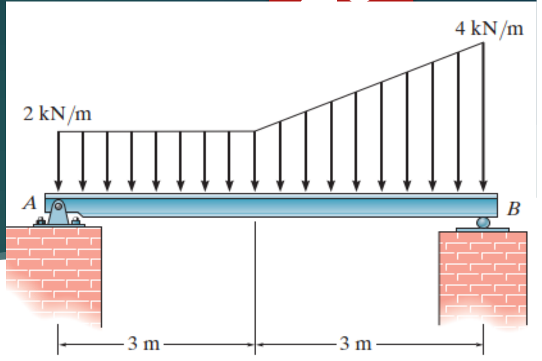
- Question

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.

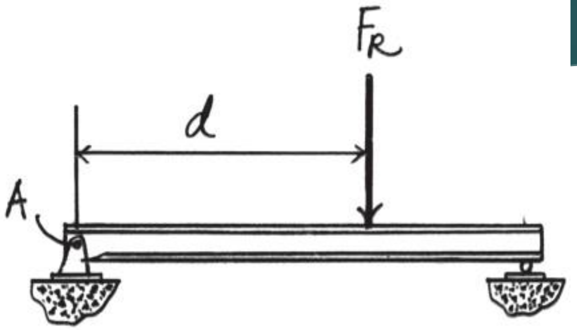


# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.5-Sup




## Solution




**Equivalent Resultant Force.** Summing the forces along the $y$ axis by referring to Fig. $a$,

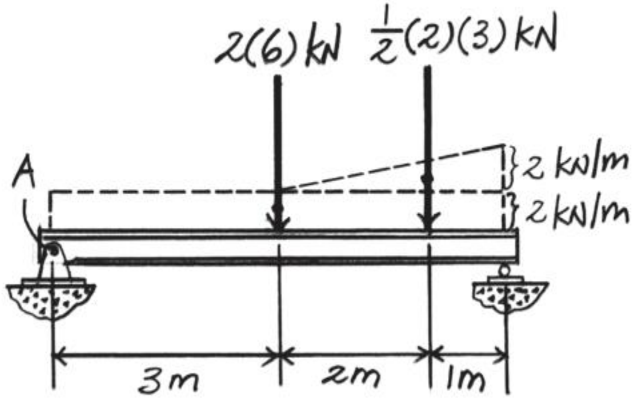
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(2)(3)$$

$$F_R = 15.0 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

**Location of the Resultant Force.** Summing the Moments about point $A$,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -15.0(d) = -2(6)(3) - \frac{1}{2}(2)(3)(5)$$

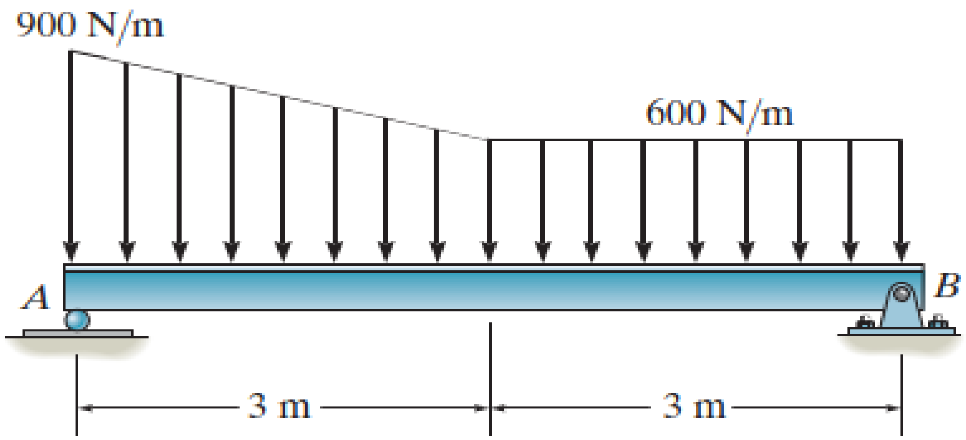
$$d = 3.40 \text{ m} \qquad \textbf{Ans.}$$



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## Example 5.6-Sup

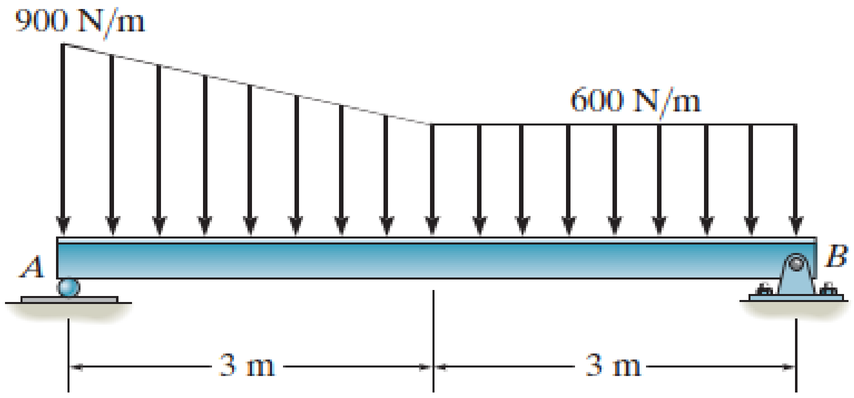
- Question

Determine the reactions at the supports



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.6-Sup




## ▶ Solution

**Equations of Equilibrium.** $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points *B* and *A*, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \quad 600(6)(3) + \frac{1}{2}(300)(3)(5) - N_A(6) = 0$$

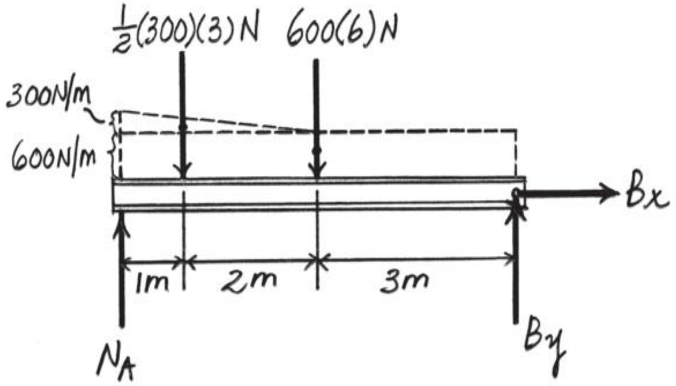
$$N_A = 2175 \text{ N} = 2.175 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(6) - \frac{1}{2}(300)(3)(1) - 600(6)(3) = 0$$

$$B_y = 1875 \text{ N} = 1.875 \text{ kN} \qquad \textbf{Ans.}$$

Also, $\mathbf{B}_x$ can be determined directly by writing the force equation of equilibrium along the *x* axis.

$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0 \qquad \textbf{Ans.}$$

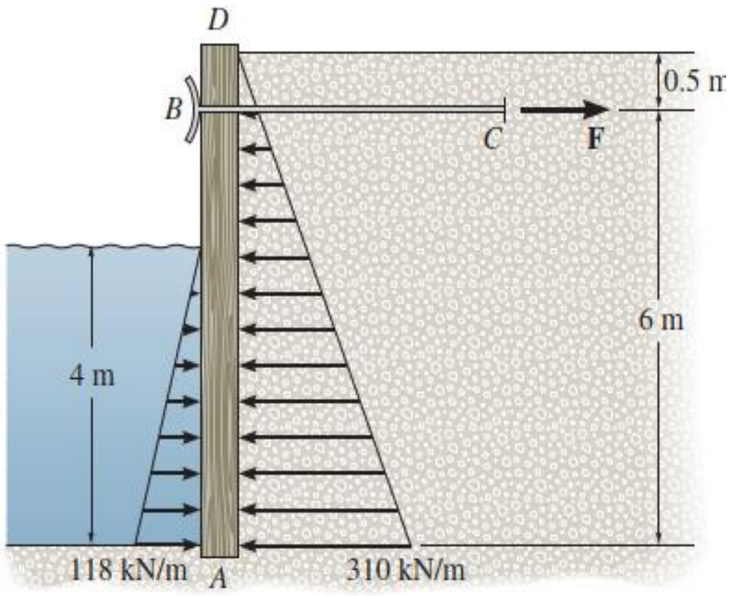


# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.7-Sup

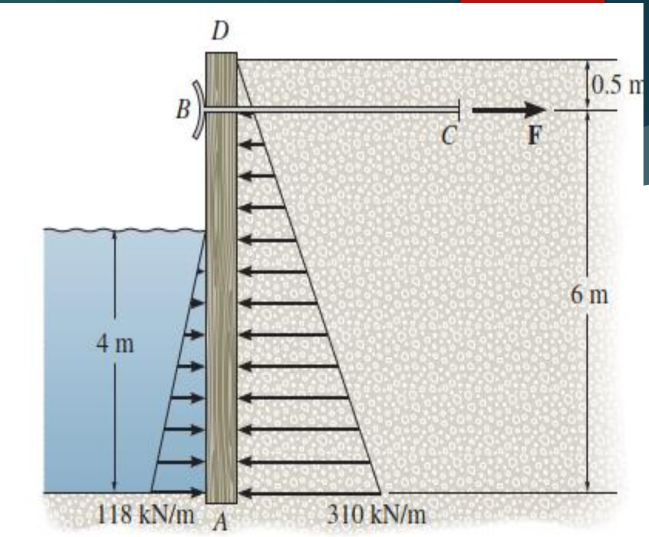
- Question

The bulk head AD is subjected to both water and soil-backfill pressures. Assuming AD is "pinned" to the ground at A, determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulk head has a mass of 800 kg.



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD

## Example 5.7-Sup




- Solution

***Equations of Equilibrium:*** The force in ground anchor $BC$ can be obtained directly by summing moments about point $A$.

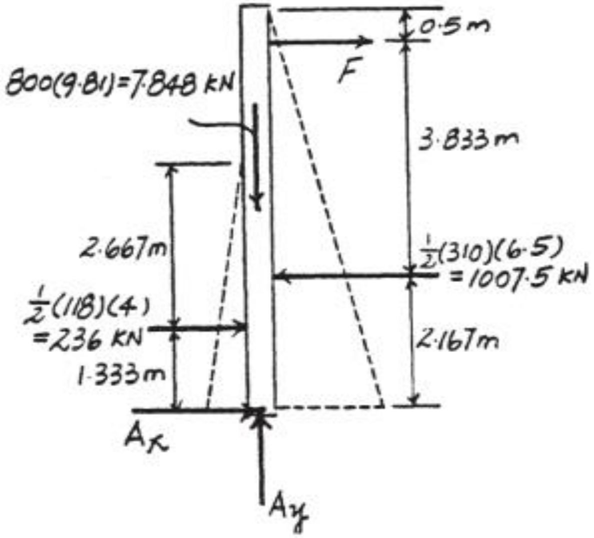
$$\zeta+\Sigma M_A = 0; \qquad 1007.5(2.167) - 236(1.333) - F(6) = 0$$

$$F = 311.375 \text{ kN} = 311 \text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x + 311.375 + 236 - 1007.5 = 0$$

$$A_x = 460 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 7.848 = 0 \qquad A_y = 7.85 \text{ kN} \qquad \textbf{Ans.}$$



# REDUCTION OF A SIMPLE DISTRIBUTED LOAD
## HOME WORK EXERCISE

4-138, 4-141, 4-143, 4-48, 4-155, 4-160, 5-14, 5-51, 5-58, & 5-72

***THE END – THANK YOU***