Lecture A6

CEE 2219: STATICS AND INTRODUCTION TO MECHANICS OF MATERIALS

STRUCTURAL ANALYSIS

Dept. of Civil & Environ Engineering, SOE.

MR. DENIS MWABA MSc, B.Eng., R.Eng., PEIZ

CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

Lecture A6

- ❖ STRUCTURAL ANALYSIS – <u>METHODS OF SECTIONS</u>
- ✓ PLANE TRUSSES (2D)
- ✓ SPACE TRUSSES (3D)

LECTURE OBJECTIVES

- To show how to determine the forces in the members of a truss using the method of sections.

CHAPTER INTRODUCTION

- Structure refers a system of connected parts used to support loads, such as buildings, bridges, dams, towers etc
- Structural analysis is the prediction (design) of the performance of a given structure under prescribed loads and/or other external effects such as support movements and temperature changes
- *Structural Engineers*
  - Analyze and design new structures
  - Investigate the capacity and serviceability of existing structures
  - Develop retrofit methods for existing structures with inadequate capacity
  - Forensic investigations
  - Research and development

CHAPTER INTRODUCTION

CHAPTER INTRODUCTION

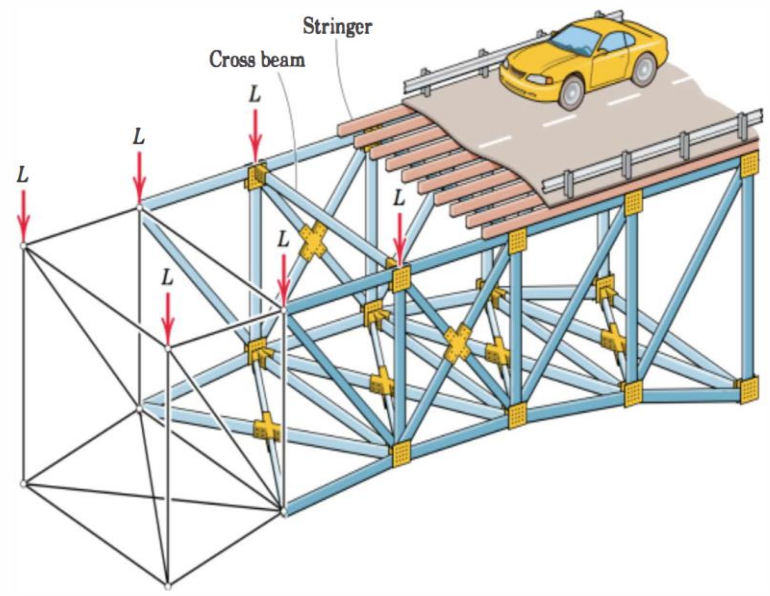
- In the previous lecture we discussed the equilibrium of a rigid body i.e a system of connected members treated as a single rigid body.
- We first drew a FBD of the body showing all forces external to the isolated body, and then we applied the force and moment equations of equilibrium.
- In lecture A6 & A7 we will focus on the determination of the forces internal to a structure - that is, forces of action and reaction between the connected members.
- An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

CHAPTER INTRODUCTION

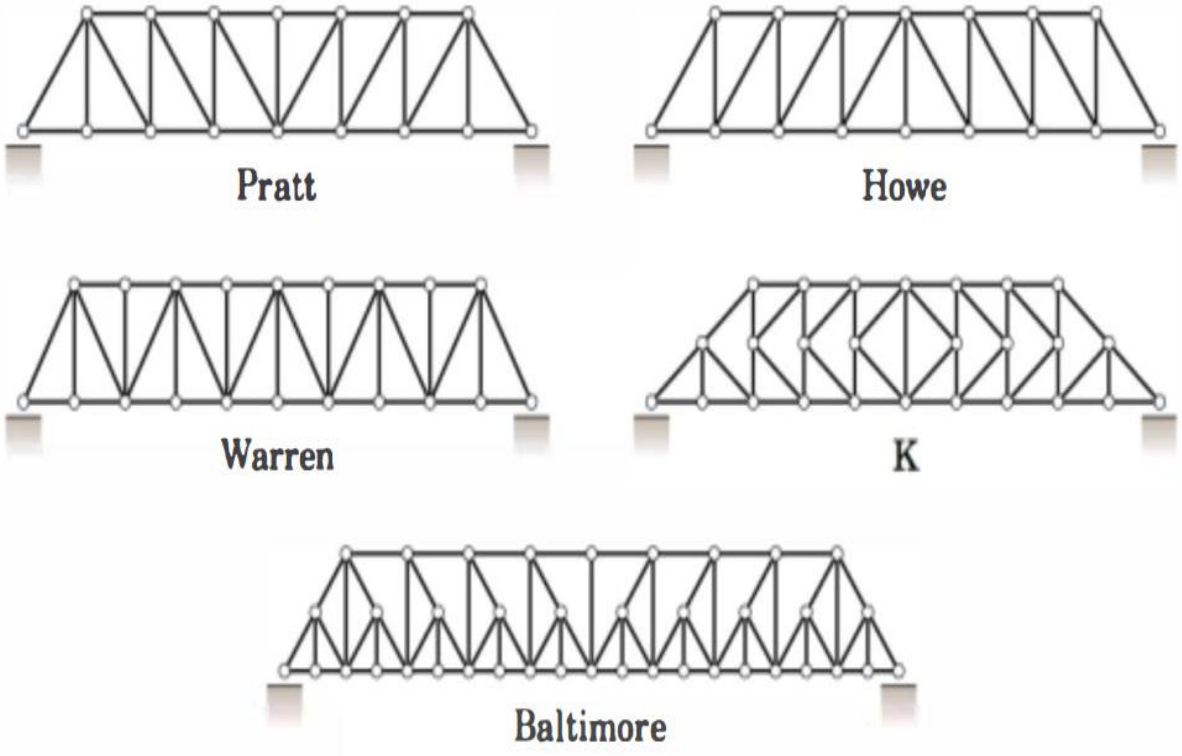
- To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate FBD of individual members or combinations of members
- This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.
- In this chapter we will analyze the internal forces acting in several types of structures (trusses, frames, and machines)
- In this treatment we consider only statically determinate structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration

CHAPTER INTRODUCTION

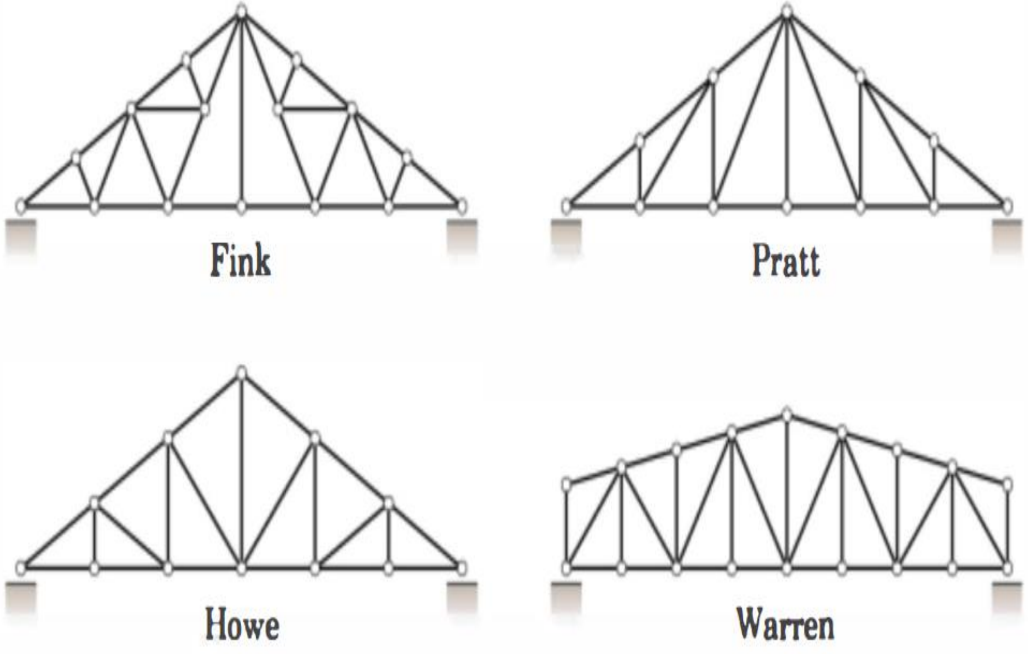
- The analysis of trusses, frames and machines, and beams under concentrated loads constitutes a straightforward application of the material developed in the previous lectures. However, only trusses will be discussed in this lecture
- The basic procedure developed in Chapter 3 and Chapter 5 for isolating a body by constructing a correct free-body diagram is essential for the analysis of statically determinate structures.



CHAPTER INTRODUCTION



Commonly Used Bridge Trusses



Commonly Used Roof Trusses

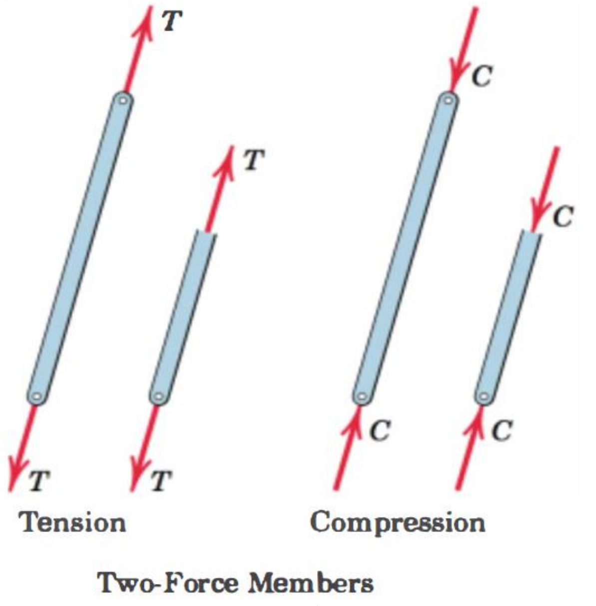
PLANE TRUSSES
Assumptions for Design

- To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading.
- To do this we will make two important assumptions;

✓The members are assumed to be connected only by frictionless pins.

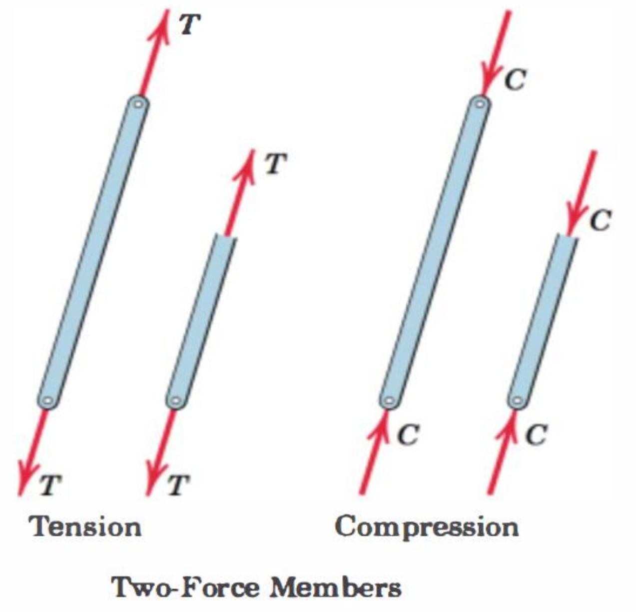
✓The loads must be applied at the joints

- Because of these two assumptions, each truss member will act as a two-force member, and therefore the force acting at each end of the member will be directed along the axis of the member.



PLANE TRUSSES
Assumptions for Design

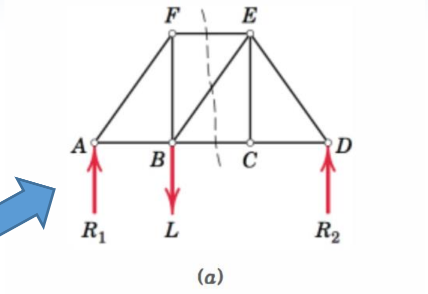
- If the force tends to elongate the member, it is a tensile force (T), whereas if it tends to shorten the member, it is a compressive force (C)
- In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive.
- Often, compression members must be made thicker than tension members because of the buckling or column effect that occurs when a member is in compression.



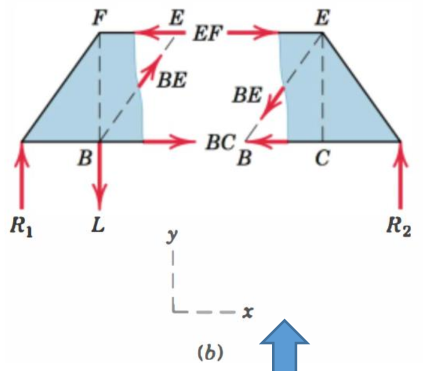
SIMPLE TRUSSES
Method of Section

- When we need to find the force in only a few members of a truss, we can analyze the truss using the *method of sections.*
- It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium
- This method of sections has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has cut that member
- Thus, it is not necessary to proceed with the calculation from joint to joint until the mem-ber in question has been reached.
- In choosing a section of the truss, we note that, in general, *not more than three members whose forces are unknown should be cut*, since there are only three available independent equilibrium relations.

SIMPLE TRUSSES
Method of Section

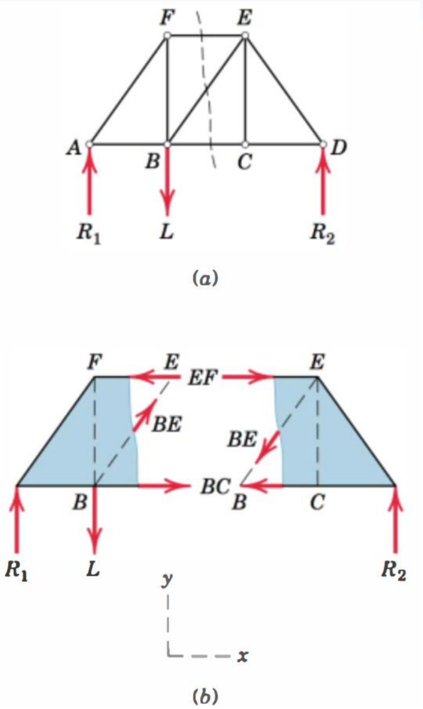


- The Fig. shown will be used to illustrate how the section method works.
- The external reactions are first computed as with the method of joints, by considering the truss as a whole.
- Let us determine the force in the member BE, for example.
- An imaginary section, indicated by the dashed line, is passed through the truss, cutting it into two parts



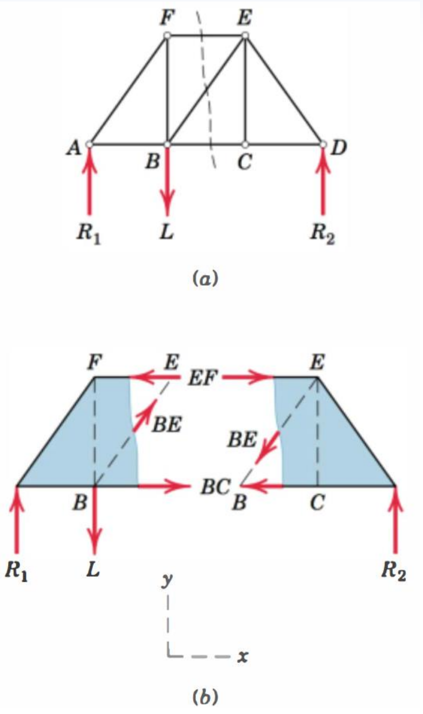
- This section has cut three members whose forces are initially unknown.
- In order for the portion of the truss on each side of the section to remain in equilibrium, it is necessary to apply to each cut member the force which was exerted on it by the member cut away.

SIMPLE TRUSSES
Method of Section



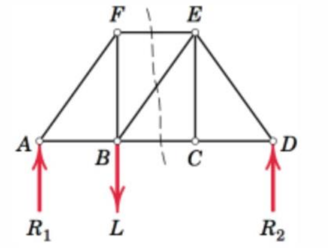
- For simple trusses composed of straight two-force members, these forces, either tensile (T) or compressive (C), will always be in the directions of the respective members.
- The left-hand section is in equilibrium under the action of the applied load L, the end reaction R1 and the three forces exerted on the cut members by the right-hand section which has been removed.
- We can usually draw the forces with their proper senses by a visual approximation of the equilibrium requirements. Thus, in balancing the
- moments about point B for the left-hand section, the force EF is clearly to the left, which makes it compressive, because it acts toward the cut section of member EF.

Method of Section

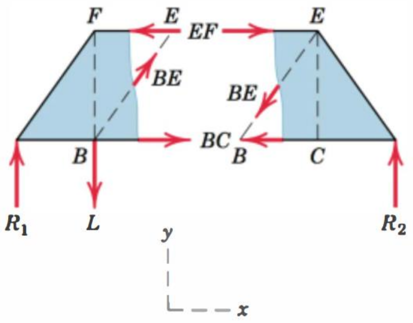


- The load L is greater than the reaction Rl so that the force BE must be up and to the right to supply the needed upward component for vertical equilibrium.
- Force BE is therefore tensile, since it acts away from the cut section.
- The equation of moments about joint B eliminates three forces from the relation, and EF can be determined directly.
- The force BE is calculated from the equilibrium equation for the y-direction. Finally, we determine BC by balancing moments about point E.
- In this way each of the three unknowns has been determined independently of the other two.

SIMPLE TRUSSES
Method of Section



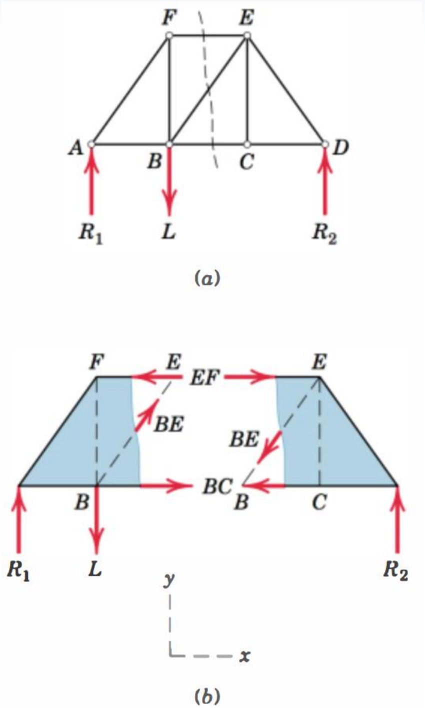
(a)



(b)

- The right-hand section of the truss is also in equilibrium under the action of R2 and the same three forces in the cut members applied in the directions opposite to those for the left section.
- The proper sense for the horizontal forces can easily be seen from the balance of moments about points B and E.
- Note that cutting a section is preferably passing through the *members and not the joints*.
- We may use either portion of a truss for the calculations, but the one involving the smaller number of forces will usually yield the simpler solution.

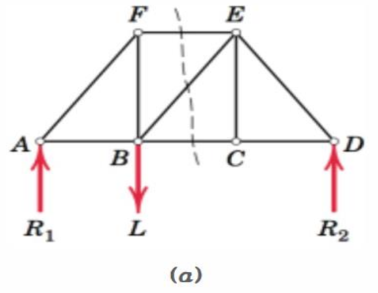
SIMPLE TRUSSES
Method of Section

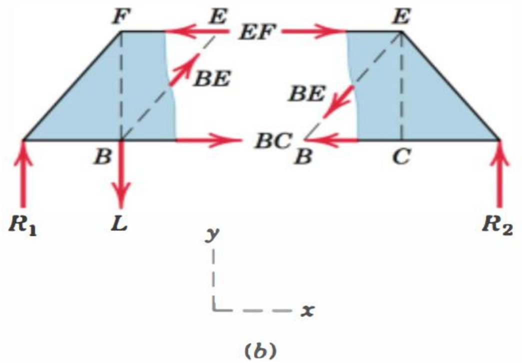


- *In some cases the methods of sections and joints can be combined for an efficient solution.*
- For example, suppose we wish to find the force in a central member of a large truss. Furthermore, suppose that it is not possible to pass a section through this member without passing through at least four unknown members.
- *It may be possible to determine the forces in nearby members by the method of sections and then progress to the unknown member by the method of joints.*

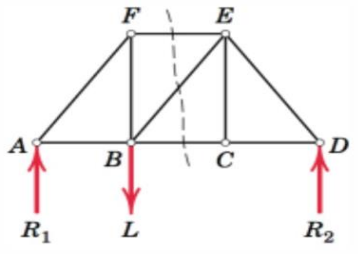
SIMPLE TRUSSES
Method of Section

- Such a combination of the two methods may be more expedient than exclusive use of either method.
- The moment equations are used to great advantage in the method of sections.
- One should choose a moment centre, either on or off the section, through which as many unknown forces as possible pass.
- Recall that it is not always possible to assign the proper sense of an unknown force when the FBD of a section is initially drawn.

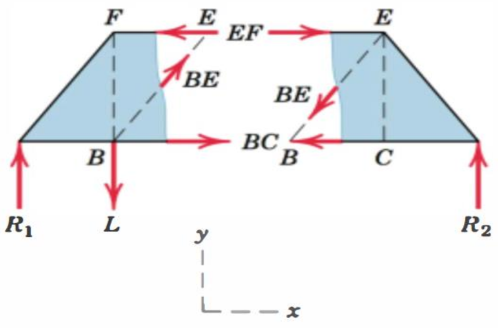




SIMPLE TRUSSES
Method of Section



(a)



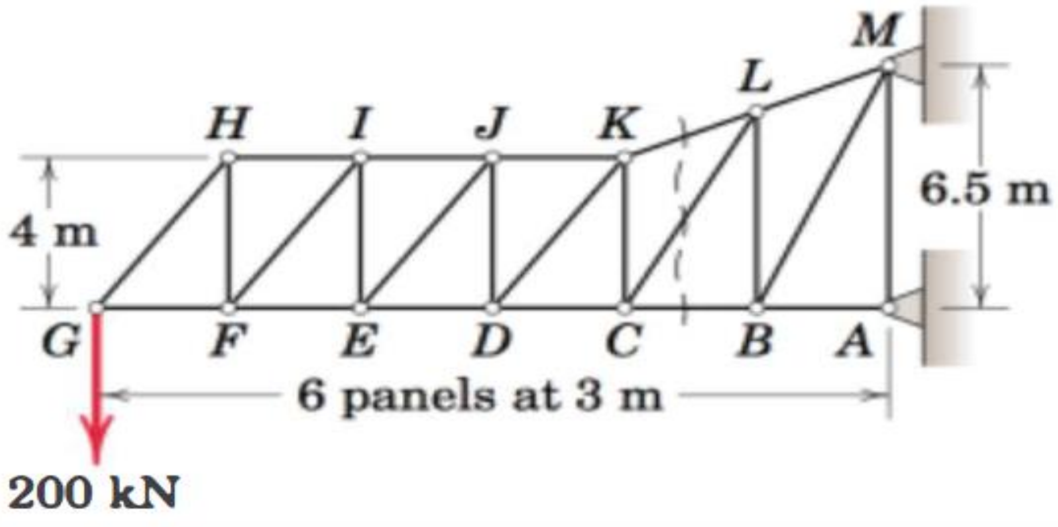
(b)

- Once an arbitrary assignment is made, a positive answer will verify the assumed sense, and a negative result will indicate that the force is in the sense opposite to that assumed.
- An alternative notation preferred by some is to assign all unknown forces arbitrarily as positive in the tension direction (away from the section) and let the algebraic sign of the answer distinguish between tension and compression.
- Thus, a plus sign would signify tension and a minus sign compression.

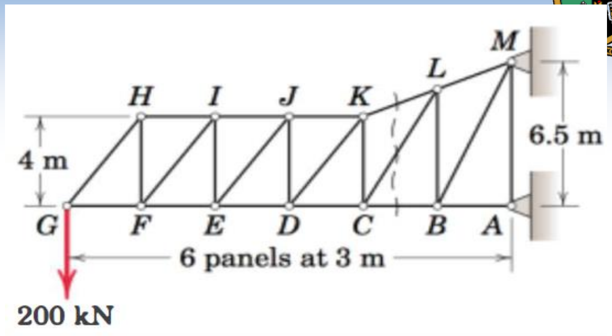
SIMPLE TRUSSES
Example 6.4

Question

- Calculate the forces induced in members KL, CL, and CB by the 200-kN load on the cantilever truss.

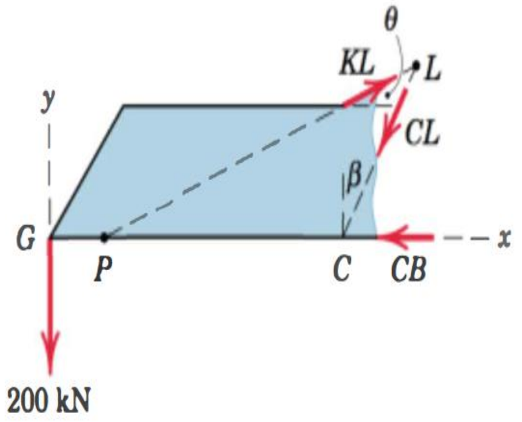


SIMPLE TRUSSES
Example 6.4

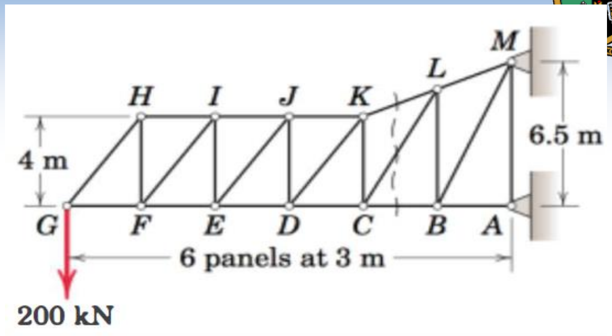


Solution

- Although the vertical components of the reactions at A and M are statically indeterminate with the two fixed supports, all members other than AM are statically determinate.
- We may pass a section directly through members KL, CL, and CB and analyze the portion of the truss to the left of this section as a statically determinate rigid body.
- The FBD of the portion of the truss to the left of the section is shown.
- A moment sum about L quickly verifies the assignment of CB as compression, and a moment sum about C quickly discloses that KL is in tension.

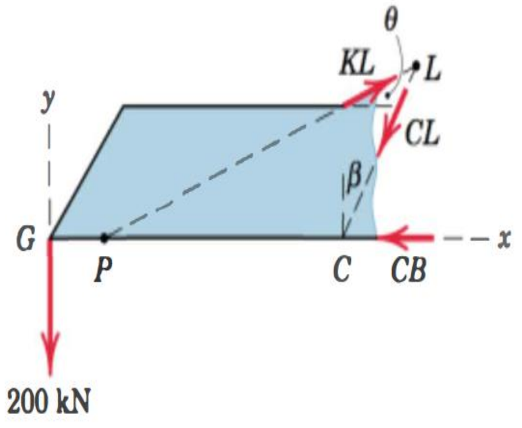


SIMPLE TRUSSES
Example 6.4



Solution

- The direction of CL is not quite so obvious until we observe that KL and CB intersect at a point P to the right of G.
- A moment sum about P eliminates reference to KL and CB and shows that CL must be compressive to balance the moment of the 200-k.N force about P.
- With these considerations in mind the solution becomes straightforward, as we now see how to solve for each of the three unknowns independently of the other two.



SIMPLE TRUSSES
Example 6.4

Solution

Summing moments about L requires finding the moment arm $\overline{BL} = 4 + (6.5 - 4)/2 = 5.25$ m. Thus,

$[\Sigma M_L = 0]$ $\quad 200(5)(3) - CB(5.25) = 0 \qquad CB = 571 \text{ kN } C$

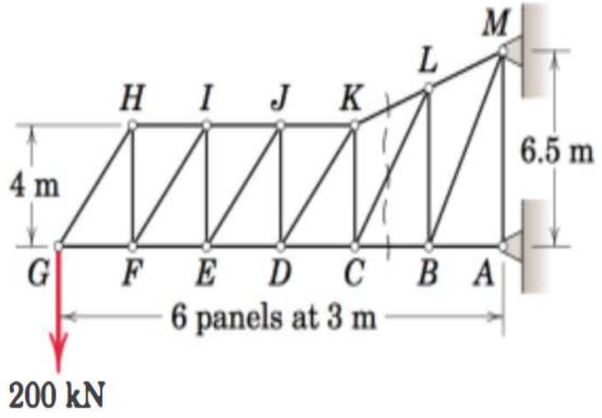
Next we take moments about C, which requires a calculation of $\cos\theta$. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$. Therefore,

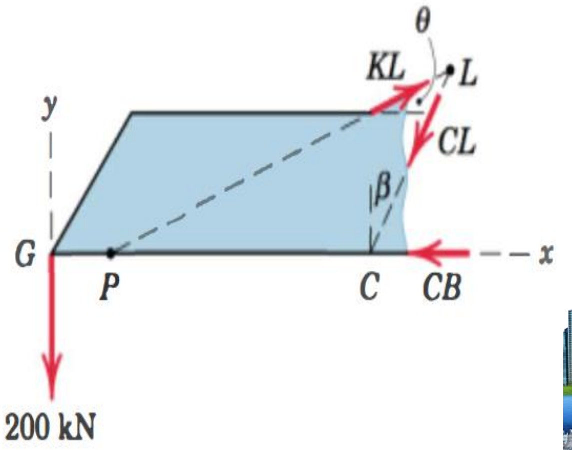
$[\Sigma M_C = 0]$ $\quad 200(4)(3) - \frac{12}{13}KL(4) = 0 \qquad KL = 650 \text{ kN } T$

Finally, we may find CL by a moment sum about P, whose distance from C is given by $\overline{PC}/4 = 6/(6.5 - 4)$ or $\overline{PC} = 9.6$ m. We also need which is given by $\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(3/5.25) = 29.7°$ and $\cos\beta = 0.868$. We now have

$[\Sigma M_P = 0]$ $\quad 200(12 - 9.6) - CL(0.868)(9.6) = 0$

$CL = 57.6 \text{ kN } C$

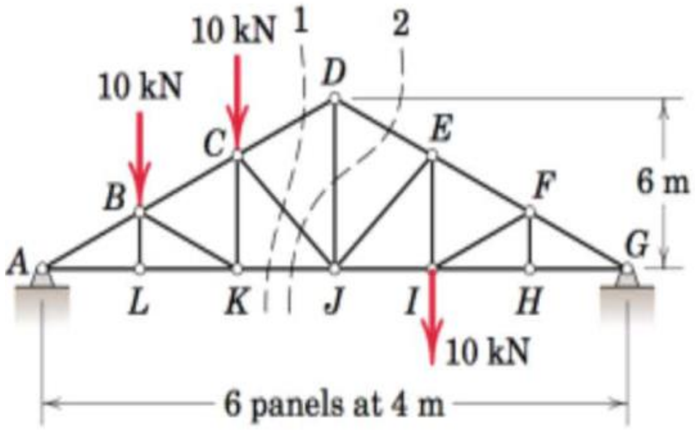




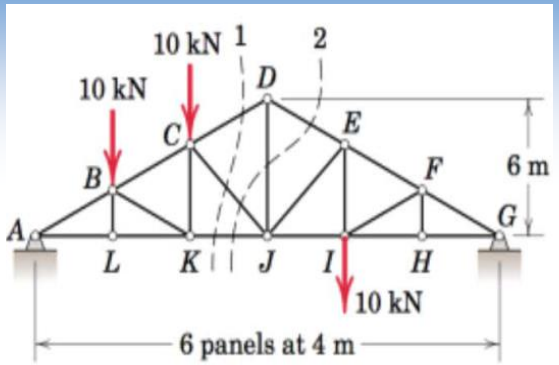
SIMPLE TRUSSES
Example 6.5

Question

- Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.

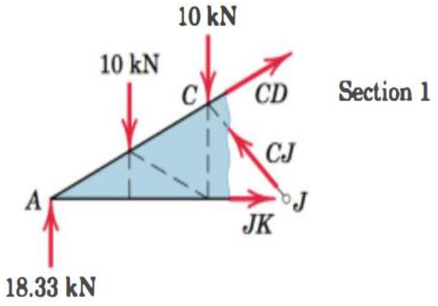


SIMPLE TRUSSES
Example 6.5

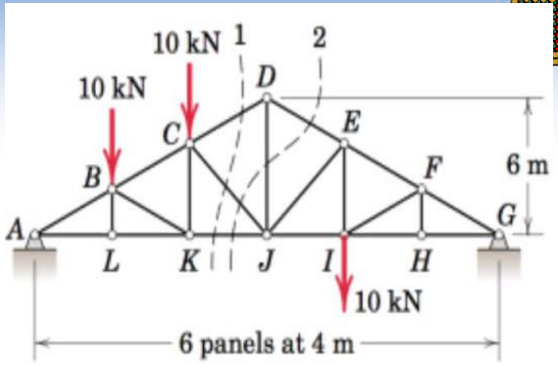


Solution

- It is not possible to pass a section through DJ without cutting four members whose forces are unknown
- Although three of these cut by section 2 are concurrent at J and therefore the moment equation about J could be used to obtain DE, the force in DJ cannot be obtained from the remaining two equilibrium principles.
- It is necessary to consider first the adjacent section 1 before analyzing section 2.

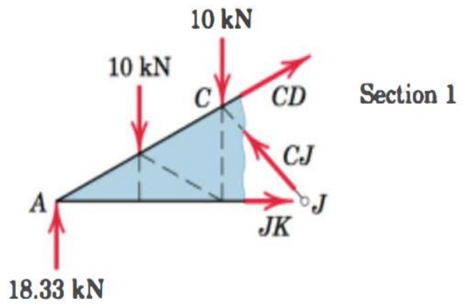


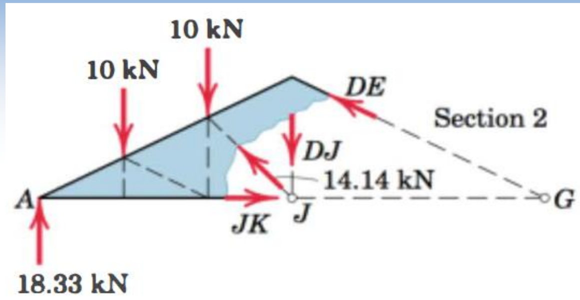
SIMPLE TRUSSES
Example 6.5



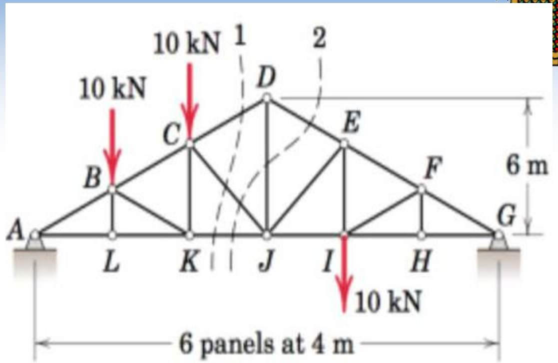
Solution

- The FBD for section 1 is drawn and includes the reaction of 18.33 kN at A, which is previously calculated from the equilibrium of the truss as a whole.
- In assigning the proper directions for the forces acting on the three cut members, we see that a balance of moments about A eliminates the effects of CD and JK and clearly requires that CJ be up and to the left.
- A balance of moments about C eliminates the effect of the three forces concurrent at C and indicates that JK must be to the right to supply sufficient counterclockwise moment.
- Again it should be fairly obvious that the lower chord is under tension because of the bending tendency of the truss.



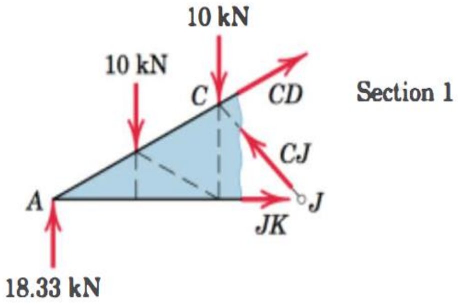


SIMPLE TRUSSES
Example 6.5



Solution

- Although it should also be apparent that the top chord is under compression, for purposes of illustration the force in CD will be arbitrarily assigned as tension.



By the analysis of section 1, CJ is obtained from

$[\Sigma M_A = 0]$ $\quad 0.707CJ(12) - 10(4) - 10(8) = 0 \qquad CJ = 14.14 \text{ kN } C$

In this equation the moment of CJ is calculated by considering its horizontal and vertical components acting at point J. Equilibrium of moments about J requires

$[\Sigma M_J = 0]$ $\quad 0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0$

$CD = -18.63 \text{ kN}$

The moment of CD about J is calculated here by considering its two components as acting through D. The minus sign indicates that CD was assigned in the wrong direction.

Hence, $\quad CD = 18.63 \text{ kN } C$

From the free-body diagram of section 2, which now includes the known value of CJ, a balance of moments about G is seen to eliminate DE and JK. Thus,

$[\Sigma M_G = 0]$ $\quad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$

$DJ = 16.67 \text{ kN } T$ *Ans.*

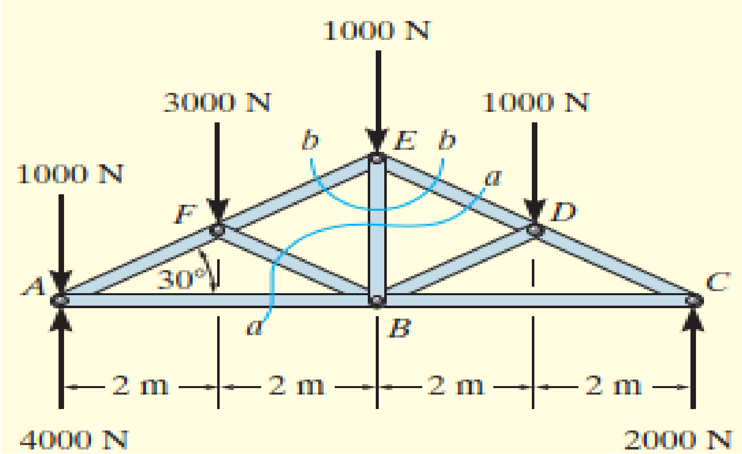
Again the moment of CJ is determined from its components considered to be acting at J. The answer for DJ is positive, so that the assumed tensile direction is correct.

An alternative approach to the entire problem is to utilize section 1 to determine CD and then use the method of joints applied at D to determine DJ.

SIMPLE TRUSSES
Example 6.6

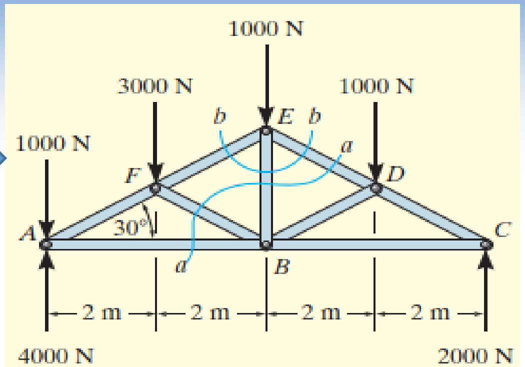
Question

- Determine the force in member EB of the roof truss shown in Fig. Indicate whether the member is in tension or compression.



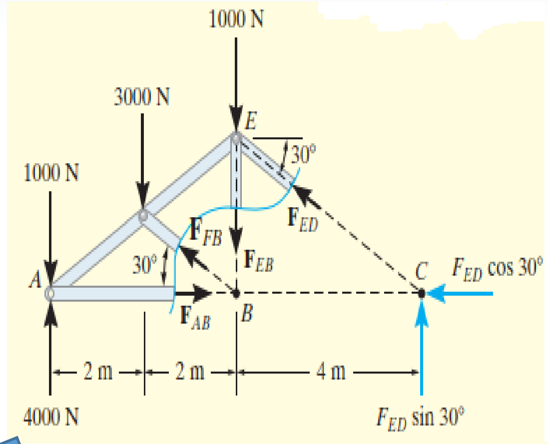
SIMPLE TRUSSES

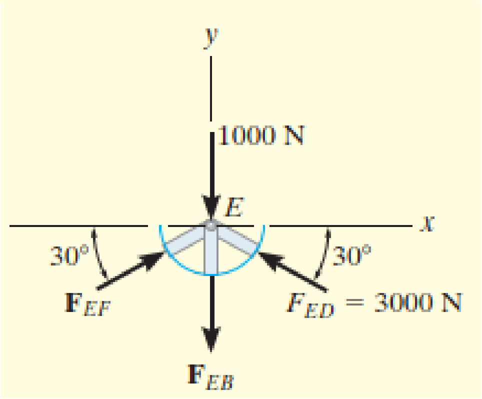
Example 6.6



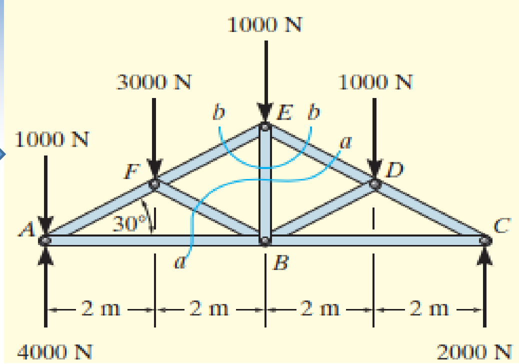
Solution

- FBD
- By the method of sections, any imaginary section that cuts through EB, Fig., will also have to cut through three other members for which the forces are unknown.
- For example, section aa cuts through ED, EB, FB, and AB.
- If a FBD of the left side of this section is considered, Fig., it is possible to obtain FED by summing moments about B to eliminate the other three unknowns;
- However, FEB cannot be determined from the remaining two equilibrium equations.



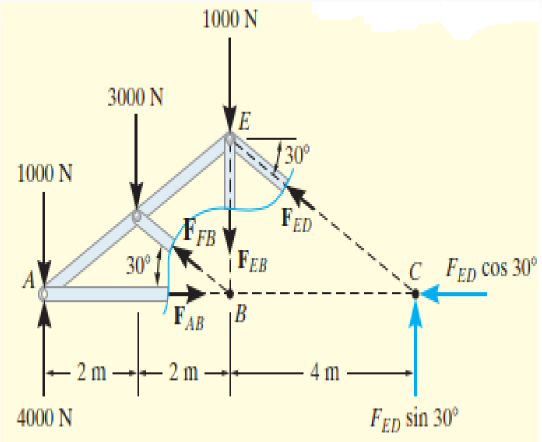


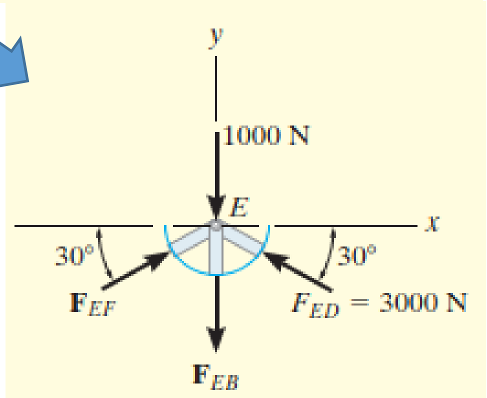
SIMPLE TRUSSES
Example 6.6



Solution

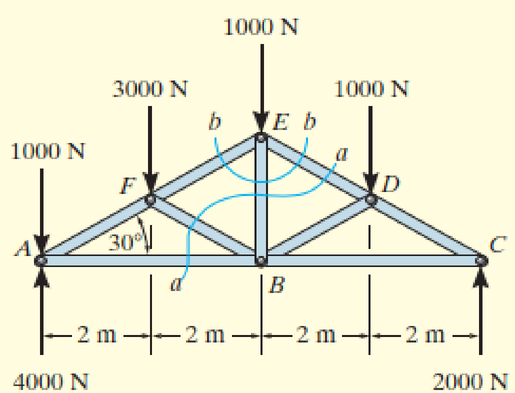
- One possible way of obtaining FEB is first to determine FED from section aa, then use this result on section bb
- Here the force system is concurrent and our sectioned free-body diagram is the same as the free-body diagram for the joint at E.
- Equations of Equilibrium.
- In order to determine the moment of FED about point B, Fig, we will use the principle of transmissibility and slide the force to point C and then resolve it into its rectangular components as shown.





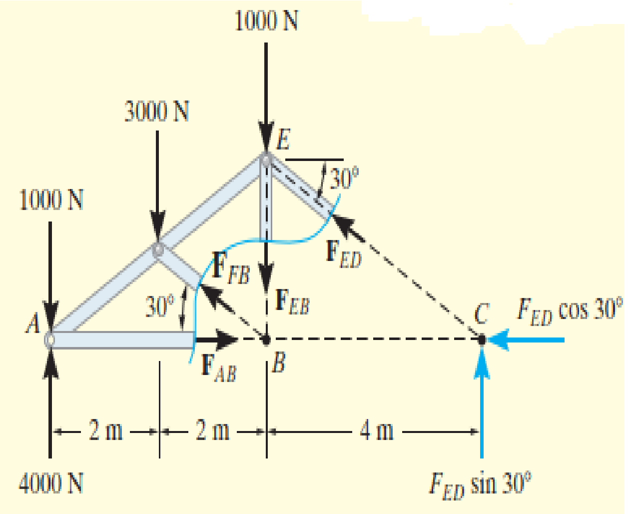
SIMPLE TRUSSES

Example 6.6



Solution

- Therefore,



$$\circlearrowleft +\Sigma M_B = 0; \qquad 1000\text{ N}(4\text{ m}) + 3000\text{ N}(2\text{ m}) - 4000\text{ N}(4\text{ m})$$

$$+ F_{ED}\sin 30°(4\text{ m}) = 0$$

$$F_{ED} = 3000\text{ N} \quad \text{(C)}$$

Considering now the free-body diagram of section *bb*, Fig. 6–18*c*, we have

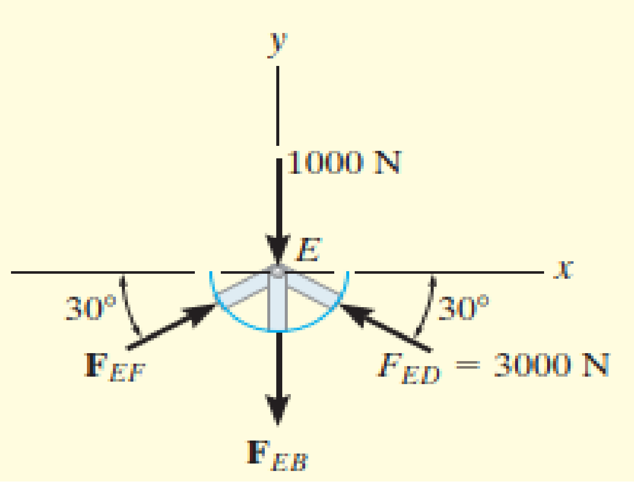
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{EF}\cos 30° - 3000\cos 30°\text{ N} = 0$$

$$F_{EF} = 3000\text{ N} \quad \text{(C)}$$

$$+\uparrow\Sigma F_y = 0; \qquad 2(3000\sin 30°\text{ N}) - 1000\text{ N} - F_{EB} = 0$$

$$F_{EB} = 2000\text{ N} \quad \text{(T)}$$

Ans.

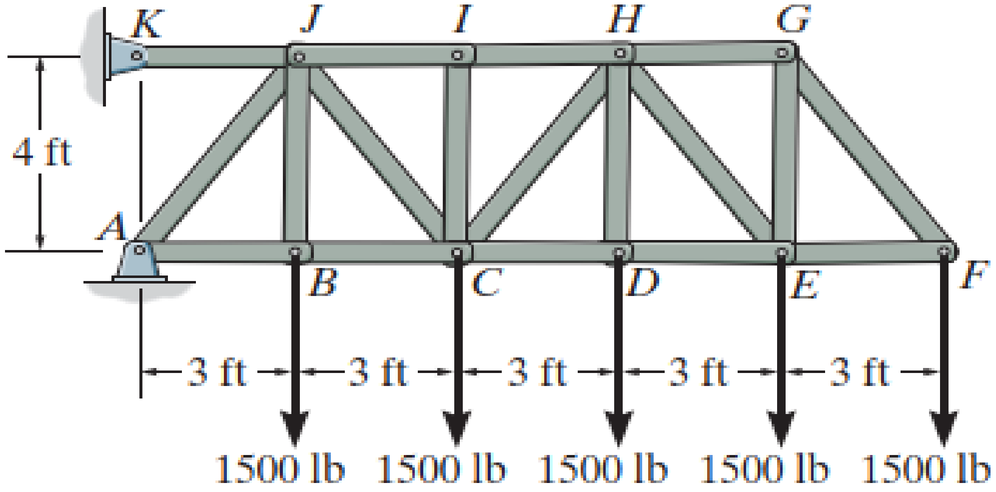


SIMPLE TRUSSES

Example 6.7

Question

• Determine the force in members HG, HE, and DE of the truss, and state if the members are in tension (T) or compression (C).



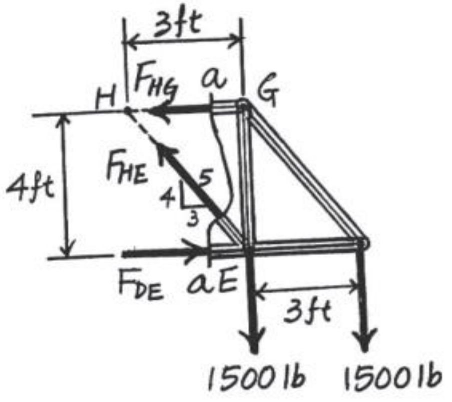
Solution

► The forces in members HG, HE, and DE are exposed by cutting the truss into two portions through section a–a and using the upper portion of the FBD, Fig. a.

► From this FBD, and can be obtained by writing the moment equations of equilibrium about points E and H, respectively.

► F_{HE} can be obtained by writing the force equation of equilibrium along the y axis.

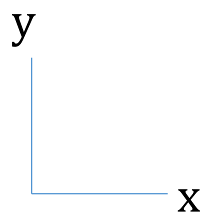
SIMPLE TRUSSES
Example 6.7



Solution

- The forces in members HG, HE, and DE are exposed by cutting the truss into two portions through section a–a and using the upper portion of the FBD, Fig. a.
- From this FBD, and can be obtained by writing the moment equations of equilibrium about points E and H, respectively.
- F_{HE} can be obtained by writing the force equation of equilibrium along the y axis.

Joint *D*: From the free-body diagram in Fig. *a*,



$\circlearrowleft + \Sigma M_E = 0; \quad F_{HG}(4) - 1500(3) = 0$

$F_{HG} = 1125 \text{ lb (T)}$

$\circlearrowleft + \Sigma M_H = 0; \quad F_{DE}(4) - 1500(6) - 1500(3) = 0$

$F_{DE} = 3375 \text{ lb (C)}$

$+\uparrow \Sigma F_y = 0; \quad F_{HE}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$

$F_{EH} = 3750 \text{ lb (T)}$

SIMPLE TRUSSES

Internal and External Redundancy

- If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute external redundancy.
- If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute internal redundancy and the truss is again statically indeterminate.
- For a truss which is statically determinate externally, there is a definite relation between the number of its members and the number of its joints necessary for internal stability without redundancy.

SIMPLE TRUSSES

Internal and External Redundancy

- For the entire truss composed of m two-force members and having the maximum of three unknown support reactions, there are in all m + 3 unknowns (m tension or compression forces and three reactions).
- Thus, for any plane truss, the equation $m + 3 = 2j$ will be satisfied if the truss is statically determinate internally.
- If $m + 3 > 2j$, there are more members than independent equations, and the truss is statically indeterminate internally with redundant members present.
- lf $m + 3 < 2j$, there is a deficiency of internal members, and the truss is unstable and will collapse under load.

SPACE (3D) TRUSSES

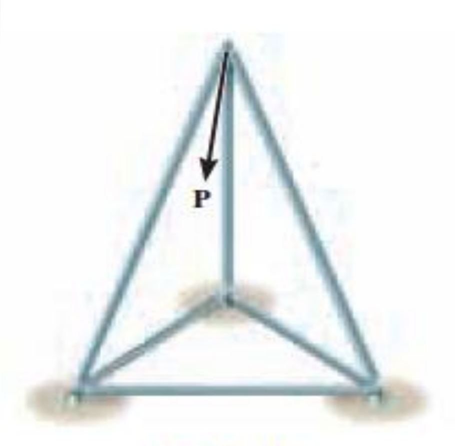
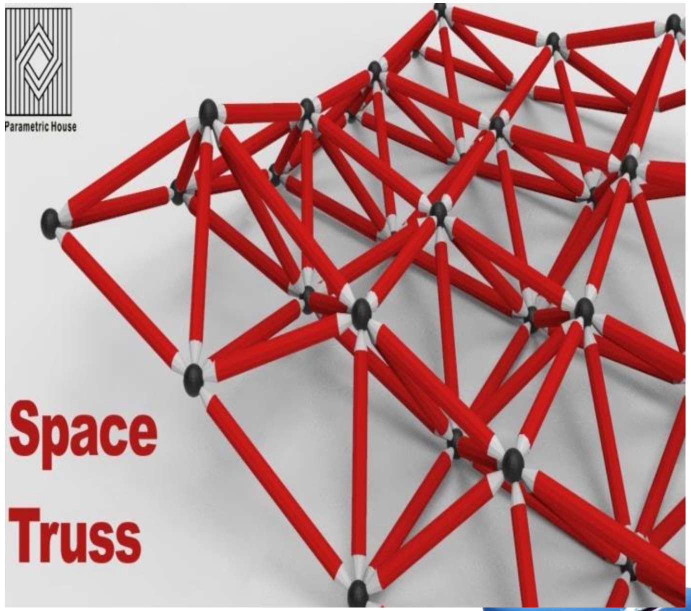


Fig. 6–19

- A space truss consists of members joined together at their ends to form a stable three-dimensional structure.
- The simplest form of a space truss is a tetrahedron, formed by connecting six members together, as shown in Fig. 6–19.
- Any additional members added to this basic element would be redundant in supporting the force P.
- A simple space truss can be built from this basic tetrahedral element by adding three additional members and a joint, and continuing in this manner to form a system of multiconnected tetrahedrons.



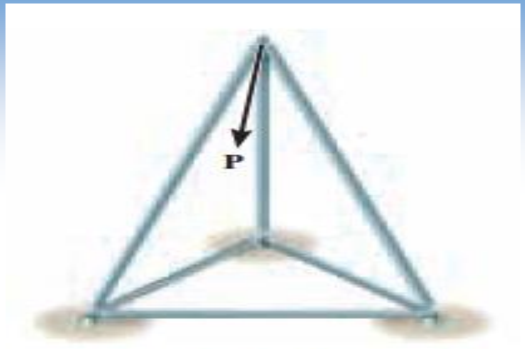


Fig. 6–19

SPACE (3D) TRUSSES

Statically Determinate Space Trusses

- When a space truss is supported externally so that it is statically determinate as an entire unit, a relationship exists between the number of its joints and the number of its member necessary for internal stability without redundancy.
- Because the equilibrium of each joint is specified by three scalar force equations, there are in all 3j such equations for a space truss with j joints.
- For the entire truss composed of m members there are m unknowns (the tensile or compressive forces in the members) plus six unknown support reactions in the general case of a statically determinate space structure.
- For any space truss, the equation $m + 6 = 3j$ will be satisfied if the truss is statically determinate internally and stable.
- A simple space truss satisfies this relation automatically

SPACE (3D) TRUSSES

Statically Determinate Space Trusses

- Starting with the initial tetrahedron, for which the equation holds, the structure is extended by adding three members and one joint at a time.
- If $m + 6 > 3j$, there are more members than there are independent equations, and the truss is statically indeterminate internally with redundant members present.
- If $m + 6 < 3j$, there is a deficiency of internal members, and the truss is unstable and subject to collapse under load.
- This relationship between the number of joints and the number of members is very helpful in the preliminary design of a stable space truss, since the configuration is not as obvious as with a plane truss, where the geometry for statically determinacy is generally quite apparent.

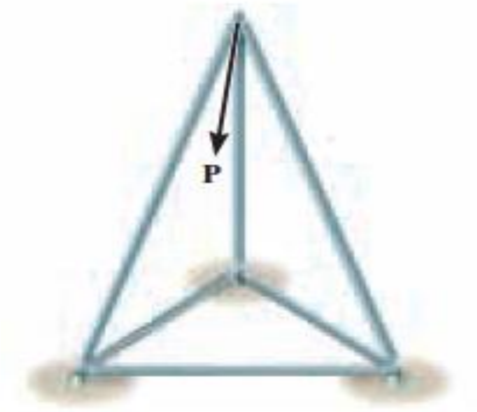


Fig. 6–19

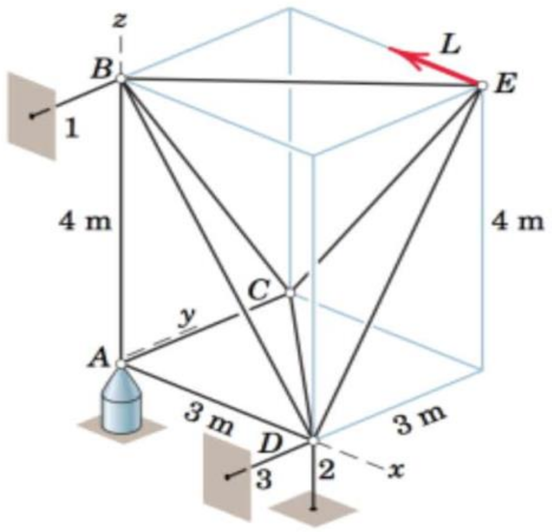
SPACE (3D) TRUSSES

- **<u>Method of Joints</u>** may also be extended directly to space trusses by satisfying the complete vector equation $\sum F = 0$ for each joint.
- We normally begin the analysis at a joint where at least one *known* force acts and not more than three unknown forces are present.
- Adjacent joints on which not more than three unknown forces act may then be analyzed in turn.
- **<u>Method of Sections for Space Trusses</u>** needs to satisfy $\sum F = 0$ & $\sum M = 0$ for any section of the truss.
- Because the two vector equations are equivalent to six scalar equations, we conclude that, a section should not be passed through more than six members whose forces are unknown.

SPACE (3D) TRUSSES
Example 6.8

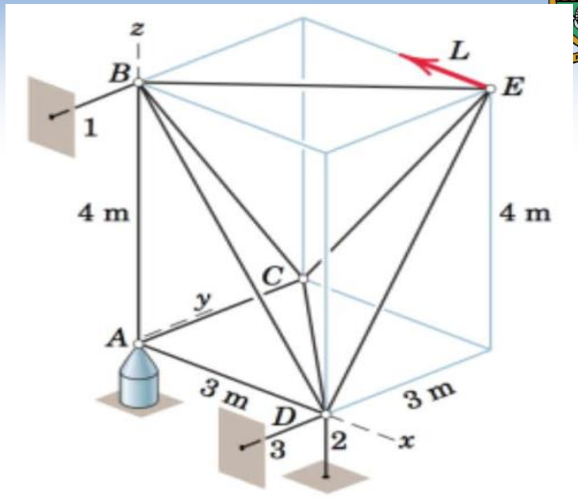
Question

- The space truss consists of the rigid tetrahedron ABCD anchored by a ball-and-socket connection at A and prevented from any rotation about the x-, y-, or z-axes by the respective links 1, 2 , and 3. The load L is applied to joint E, which is rigidly fixed to the tetrahedron by the three additional links.
- Solve for the forces in the members at joint E and indicate the procedure for the determination of the forces in the remaining members of the truss.



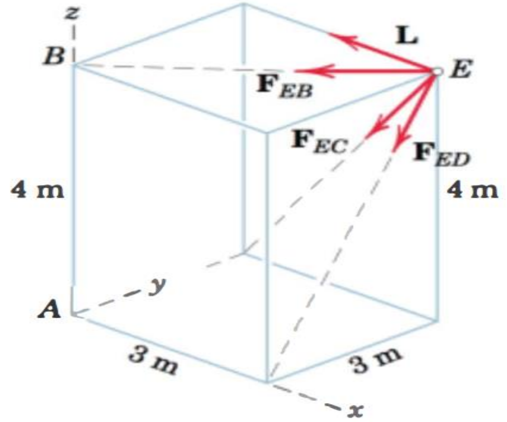
SPACE (3D) TRUSSES

Example 6.8



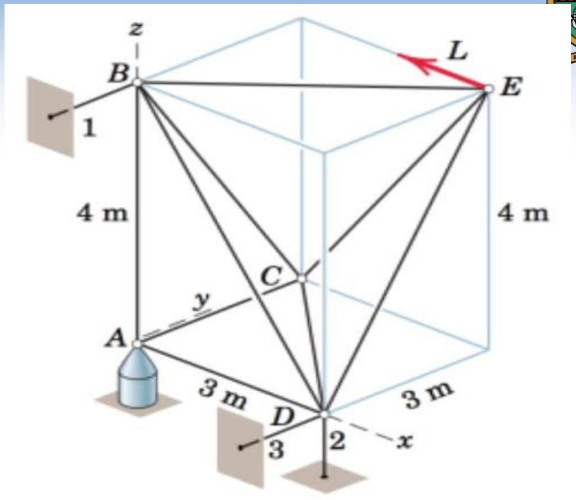
Solution

- We note first that with m = 9 members and j = 5 joints, the condition m + 6 = 3j (9+6 = 15 & 3*5 = 15) for a sufficiency of members to provide a noncollapsible structure is satisfied.
- The external reactions at A, B, and D can be calculated easily as a first step, although their values will be determined from the solution of all forces on each of the joints in succession.



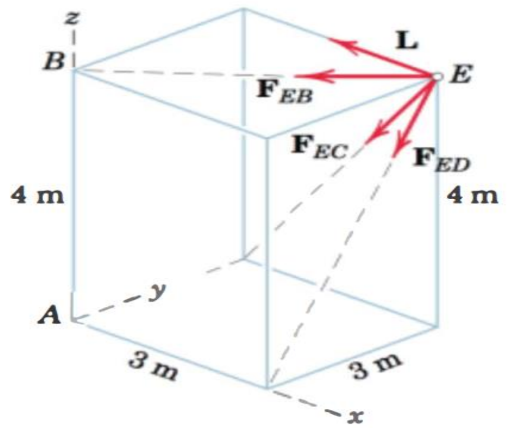
SPACE (3D) TRUSSES

Example 6.8

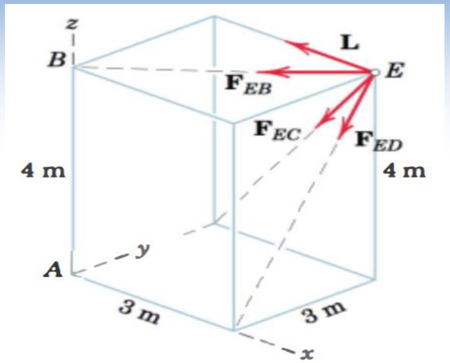


Solution

- We start with a joint on which at least one known force and not more than three unknown forces act, which in this case is joint E.
- The FBD of joint E is shown with all force vectors arbitrarily assumed in their positive tension directions (away from the joint). The vector expressions for the three unknown forces are

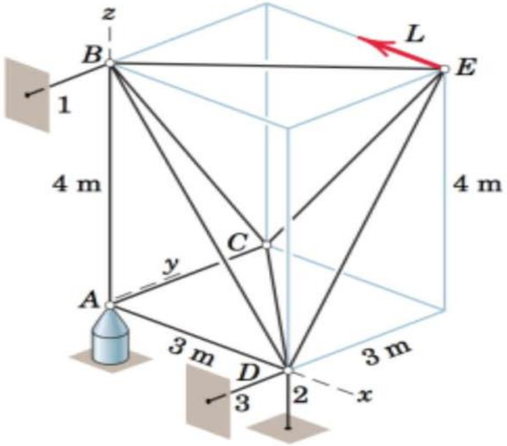


$$\mathbf{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}), \quad \mathbf{F}_{EC} = \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}), \quad \mathbf{F}_{ED} = \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k})$$



SPACE (3D) TRUSSES

Example 6.8



Solution

Equilibrium of joint E requires

$$[\Sigma \mathbf{F} = 0] \qquad \mathbf{L} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED} = \mathbf{0} \qquad \text{or}$$

$$-L\mathbf{i} + \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}) + \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}) + \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k}) = \mathbf{0}$$

Rearranging terms gives

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\mathbf{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\mathbf{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of the **i**-, **j**-, and **k**-unit vectors to zero gives the three equations

$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \qquad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \qquad F_{EC} + F_{ED} = 0$$

Solving the equations gives us

$$F_{EB} = -L/\sqrt{2} \qquad F_{EC} = -5L/6 \qquad F_{ED} = 5L/6$$

Thus, we conclude that F_{EB} and F_{EC} are compressive forces and F_{ED} is tension.

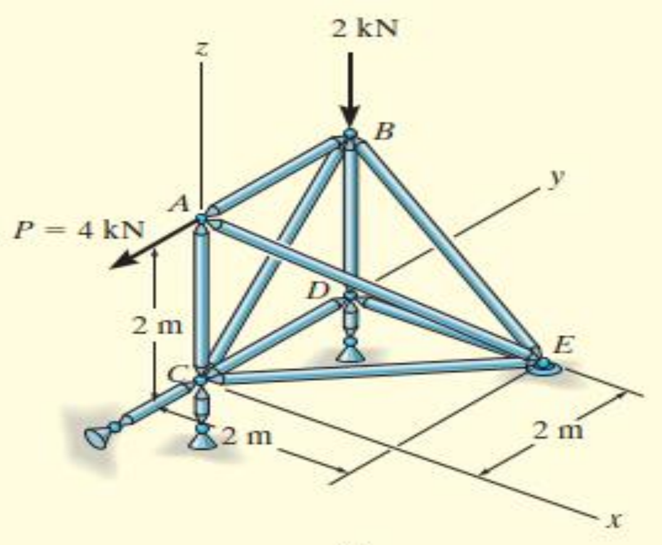
Unless we have computed the external reactions first, we must next analyze joint C with the known value of F_{EC} and the three unknowns F_{CB}, F_{CA}, and F_{CD}. The procedure is identical to that used for joint E. Joints B, D, and A are then analyzed in the same way and in that order, which limits the scalar unknowns to three for each joint. The external reactions computed from these analyses must, of course, agree with the values which can be determined initially from an analysis of the truss as a whole.

SPACE (3D) TRUSSES

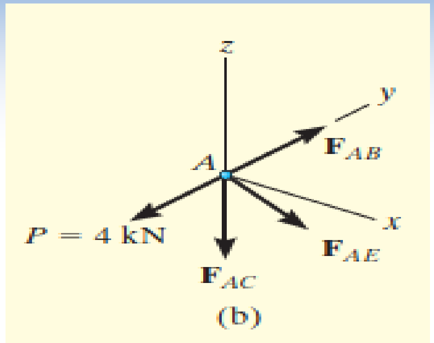
Example 6.9

Question

- Determine the forces acting in the members of the space truss shown in Fig. 6–20a. Indicate whether the members are in tension or compression.

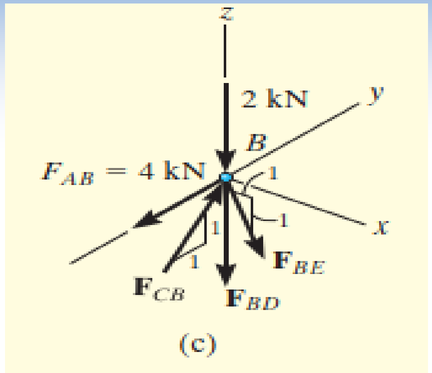


(a)

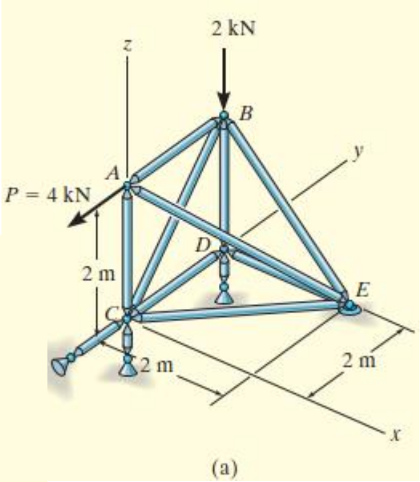


(b)

SPACE (3D) TRUSSES
Example 6.9



(c)



(a)

Solution

- Since there are one known force and three unknown forces acting at joint A, the force analysis of the truss will begin at this joint.
- Joint A. (Fig. 6–20b). Expressing each force acting on the FBD of joint A as a Cartesian vector, we have

$$\mathbf{P} = \{-4\mathbf{j}\}\ \text{kN}, \qquad \mathbf{F}_{AB} = F_{AB}\mathbf{j}, \quad \mathbf{F}_{AC} = -F_{AC}\mathbf{k},$$

$$\mathbf{F}_{AE} = F_{AE}\left(\frac{\mathbf{r}_{AE}}{r_{AE}}\right) = F_{AE}(0.577\mathbf{i} + 0.577\mathbf{j} - 0.577\mathbf{k})$$

For equilibrium,

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{P} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AE} = \mathbf{0}$$

$$-4\mathbf{j} + F_{AB}\mathbf{j} - F_{AC}\mathbf{k} + 0.577F_{AE}\mathbf{i} + 0.577F_{AE}\mathbf{j} - 0.577F_{AE}\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad 0.577F_{AE} = 0$$

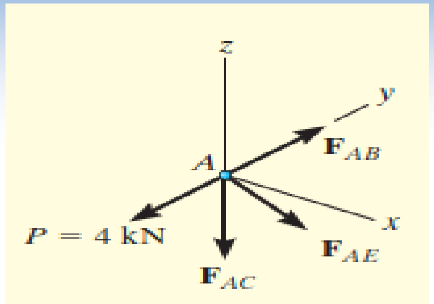
$$\Sigma F_y = 0; \qquad -4 + F_{AB} + 0.577F_{AE} = 0$$

$$\Sigma F_z = 0; \qquad -F_{AC} - 0.577F_{AE} = 0$$

$$F_{AC} = F_{AE} = 0 \qquad \textit{Ans.}$$

$$F_{AB} = 4\ \text{kN}\quad (\text{T}) \qquad \textit{Ans.}$$

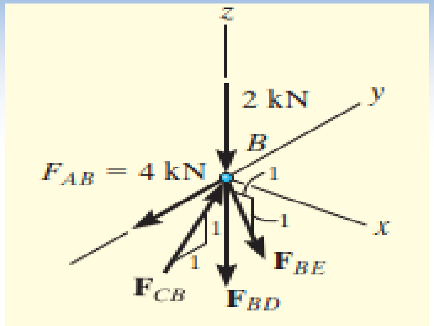
Since F_{AB} is known, joint B can be analyzed next.



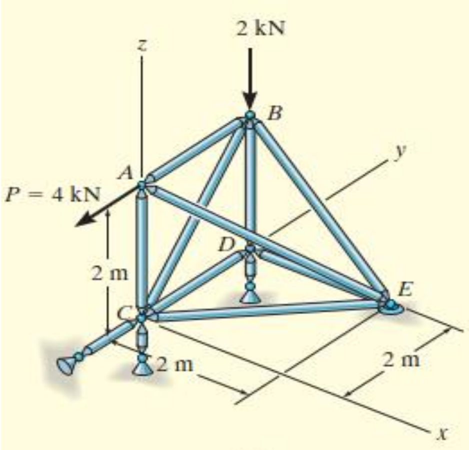
(b)

SPACE (3D) TRUSSES

Example 6.9



(c)



(a)

Solution

Joint B. (Fig. 6–20*c*).

$$\Sigma F_x = 0; \qquad F_{BE}\frac{1}{\sqrt{2}} = 0$$

$$\Sigma F_y = 0; \qquad -4 + F_{CB}\frac{1}{\sqrt{2}} = 0$$

$$\Sigma F_z = 0; \qquad -2 + F_{BD} - F_{BE}\frac{1}{\sqrt{2}} + F_{CB}\frac{1}{\sqrt{2}} = 0$$

$$F_{BE} = 0, \qquad F_{CB} = 5.65 \text{ kN (C)} \qquad F_{BD} = 2 \text{ kN (T)} \qquad \textit{Ans.}$$

The *scalar* equations of equilibrium can now be applied to the forces acting on the free-body diagrams of joints *D* and *C*. Show that

$$F_{DE} = F_{DC} = F_{CE} = 0 \qquad \textit{Ans.}$$

HOME WORK EXERCISE

6-4, 6-6, 6-10, 6-18, 6-28, 6-35, 6-44, 6-47, 6-50, 6,53, 6-55, & 6-57.

47

THE END – THANK YOU

THE END

THANK YOU