CEE 2219 – STATICS & INTRODUCTION TO MECHANICS OF MATERIALS

Lecture A3

- POSITION FORCE VECTORS (RECALL CHAPTER A2)
- DOT PRODUCT

LECTURE OBJECTIVES

- ❖To express forces and introduction of position vectors

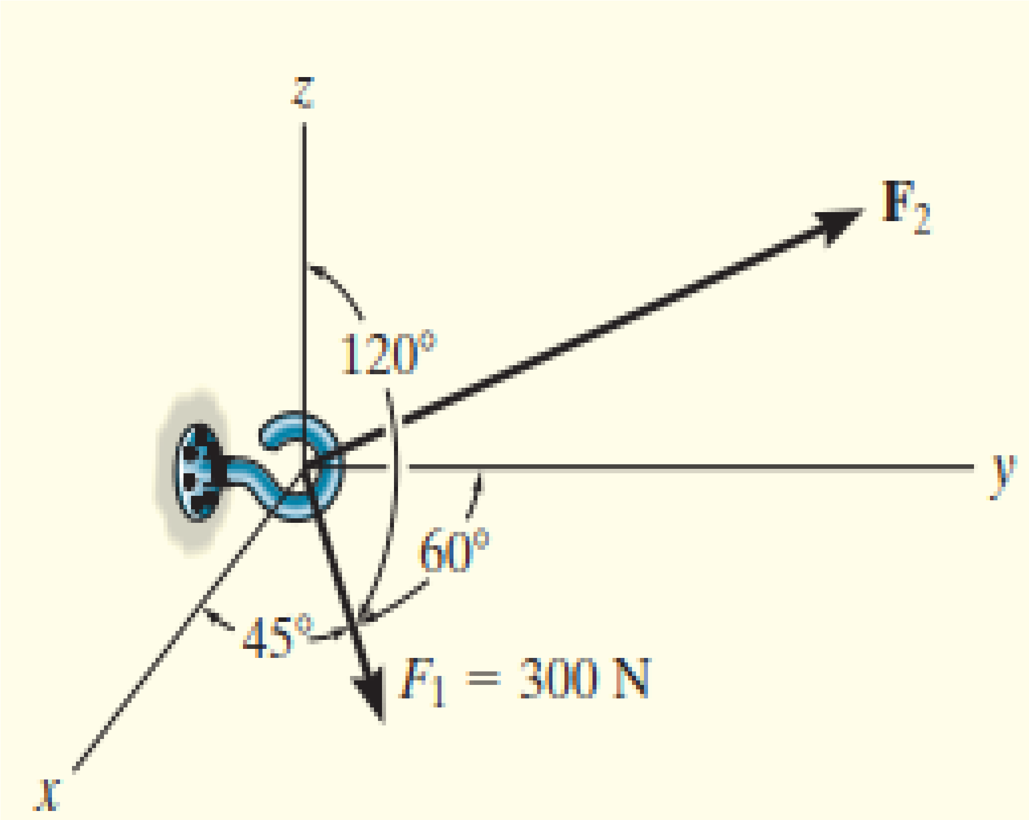
- ❖To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another.

FORCE VECTORS IN SPACE (3D)

EXAMPLES 2-6

QUESTION for HOMEWORK

- *Two forces act on the hook shown in Fig. Specify the magnitude of F_2 and its coordinate direction angles so that the resultant force F_R acts along the positive y axis and has a magnitude of 800N*



FORCE VECTORS IN SPACE (3D)

EXAMPLES 2-6

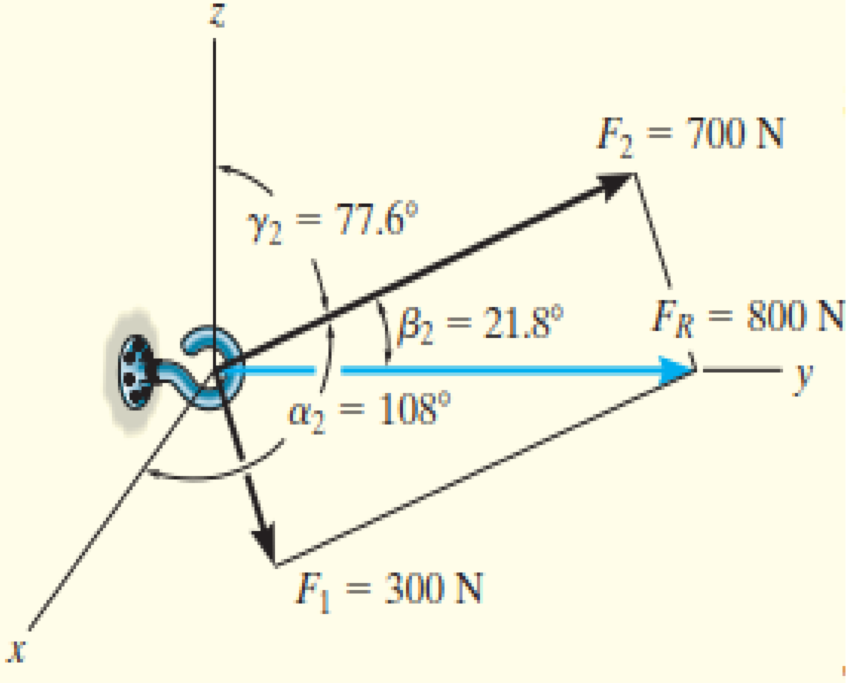
$$\mathbf{A} = A\mathbf{u}_A = A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

SOLUTION

- *To solve this problem, the resultant force F_R and its two components, F_1 and F_2, will each be expressed in Cartesian vector form.*
- *As shown in Fig, it is required that $F_R = F_1 + F_2$.*
- *Applying Eq.*

$$\mathbf{F}_1 = F_1\cos\alpha_1\mathbf{i} + F_1\cos\beta_1\mathbf{j} + F_1\cos\gamma_1\mathbf{k} = 300\cos 45^\circ\,\mathbf{i} + 300\cos 60^\circ\,\mathbf{j} + 300\cos 120^\circ\,\mathbf{k} = \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$



- *Since F_R has a magnitude of 800 N and acts in the +j direction, $F_R = (0N)(i)+(800N)(+j)+(0N)(k) = [800j]$ N*

FORCE VECTORS IN SPACE (3D)

EXAMPLES 2-6

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

SOLUTION

- *We require*

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$800\mathbf{j} = 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

$$800\mathbf{j} = (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}$$

- *To satisfy this equation the i, j, k components of F_R must be equal to the corresponding i, j, k components of (F_1+F_2) Hence,*

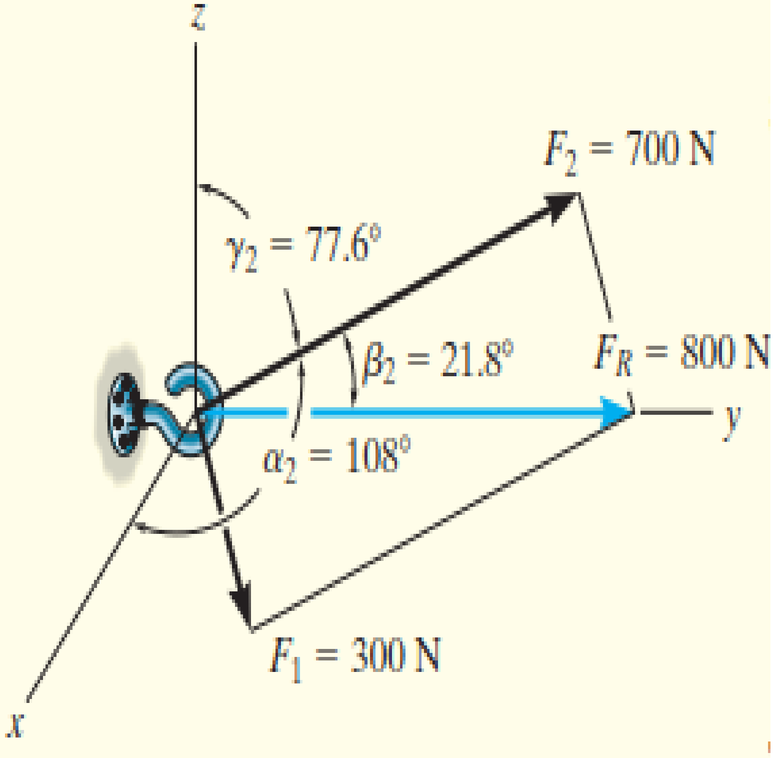
$$0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1\text{ N}$$

$$800 = 150 + F_{2y} \qquad F_{2y} = 650\text{ N}$$

$$0 = -150 + F_{2z} \qquad F_{2z} = 150\text{ N}$$

- *The magnitude of F_2 is thus*

$$F_2 = \sqrt{(-212.1\text{ N})^2 + (650\text{ N})^2 + (150\text{ N})^2} = 700\text{ N}$$



- *We can use Eq. to determine α_2, β_2, γ_2.*

$$\cos\alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ$$

$$\cos\beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ$$

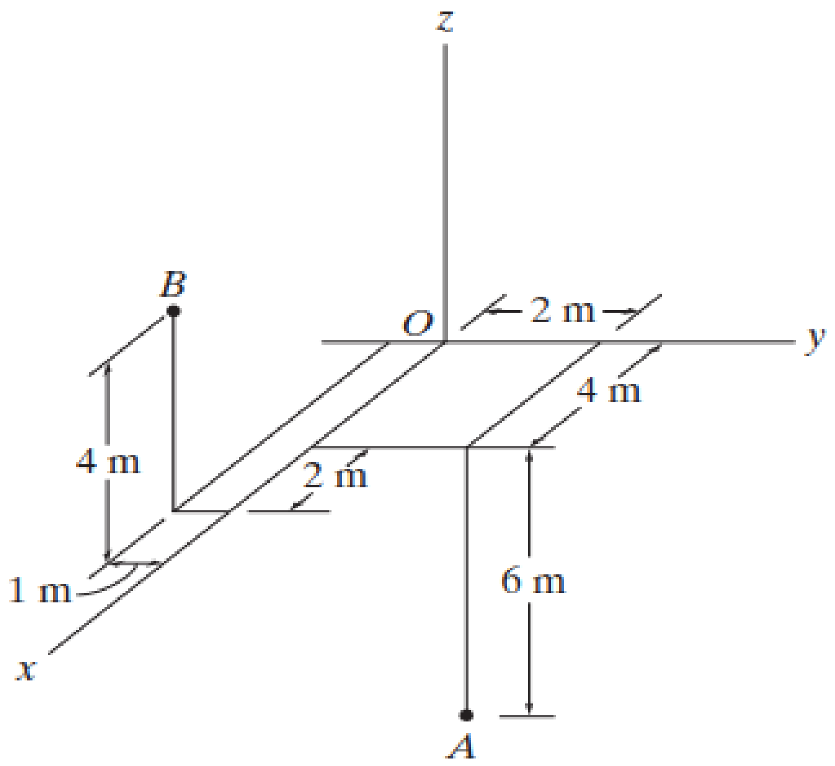
$$\cos\gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ$$

FORCE VECTORS IN SPACE (3D)

POSITION VECTORS

- *You will appreciate that the position vector is of importance in formulating a Cartesian force vector directed between two points in space*

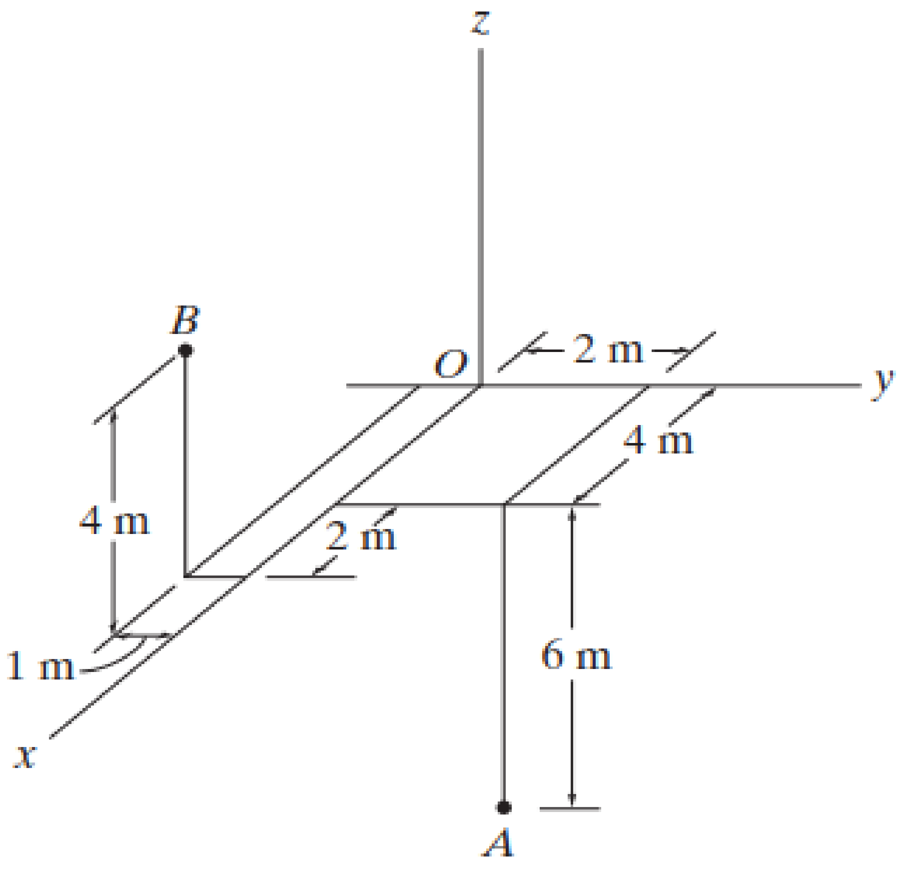
- *Later in the course you will use the position vector to calculate moment of force*



FORCE VECTORS IN SPACE (3D)
POSITION VECTORS

x, y, z Coordinates

- *Throughout the course we will use a right-handed coordinate system to reference the location of points in space.*
- *The positive z axis to be directed upward so that it measures the height of an object or the altitude of a point.*
- *The x, y axes then lie in the horizontal plane,*
- *Points in space are located relative to the origin of coordinates, O,*



FORCE VECTORS IN SPACE (3D)

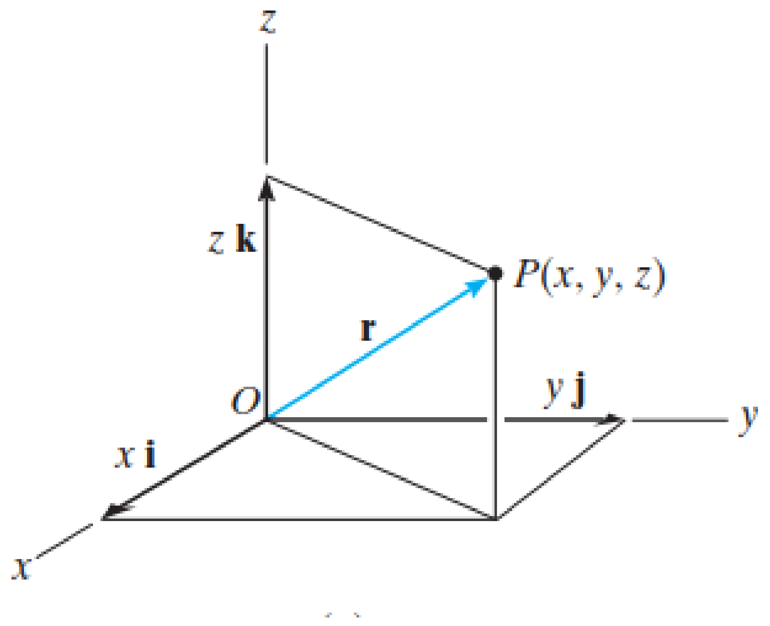
POSITION VECTORS

- *A position vector* ***r*** *is defined as a fixed vector which locates a point in space relative to another point*

- *For example, if* ***r*** *extends from the origin of coordinates, O, to point P(x, y, z)*

- *then* ***r*** *can be expressed in Cartesian vector form as*

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

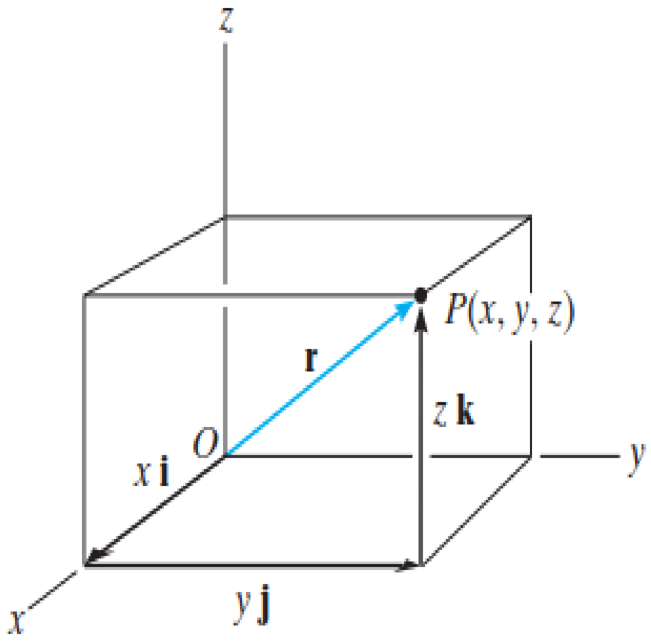


FORCE VECTORS IN SPACE (3D)

POSITION VECTORS

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

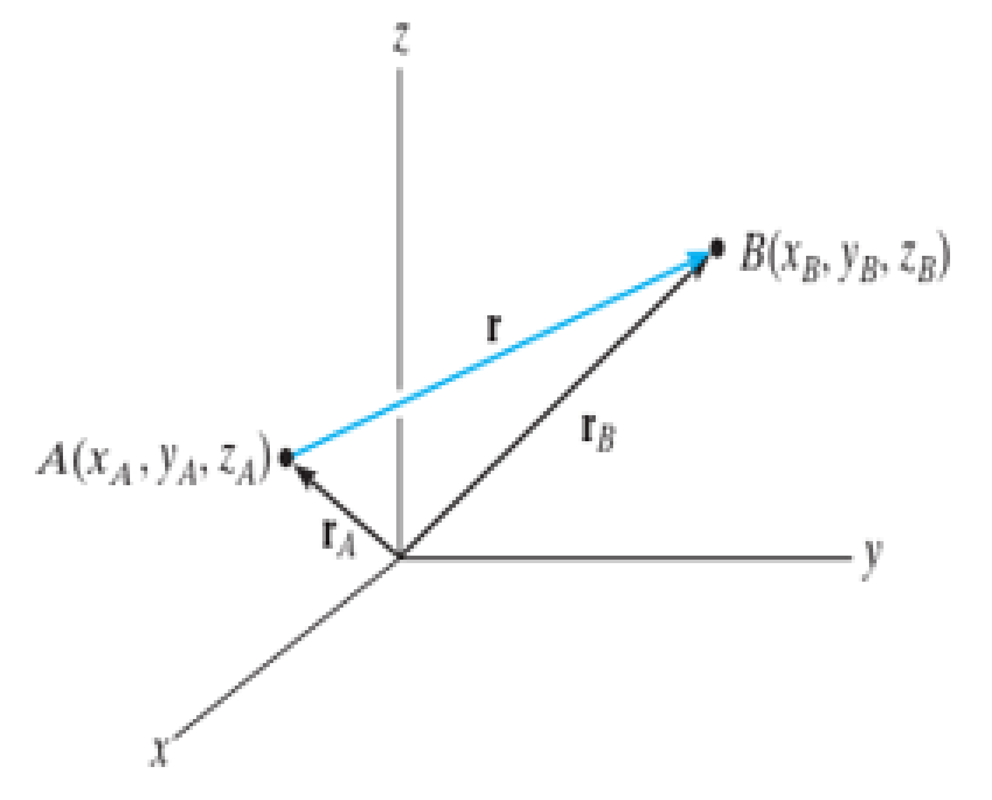
- *Note how the head-to-tail vector addition of the three components yields vector* ***r***
- *Starting at the origin O, one "travels" x in the +**i** direction, then y in the +**j** direction, and finally z in the +**k** direction to arrive at point P(x, y, z).*



FORCE VECTORS IN SPACE (3D)

POSITION VECTORS

- *In other cases, the position vector may be directed from point A to point B in space*
- *This vector is also designated by the symbol* ***r***
- *As a matter of convention, we will sometimes refer to this vector with two subscripts to indicate from and to the point where it is directed. Thus, r can also be designated as* $\boldsymbol{r}_{AB}$



FORCE VECTORS IN SPACE (3D)

POSITION VECTORS

- *From Fig. shown, by the head-to-tail vector addition, using the triangle rule, we require*

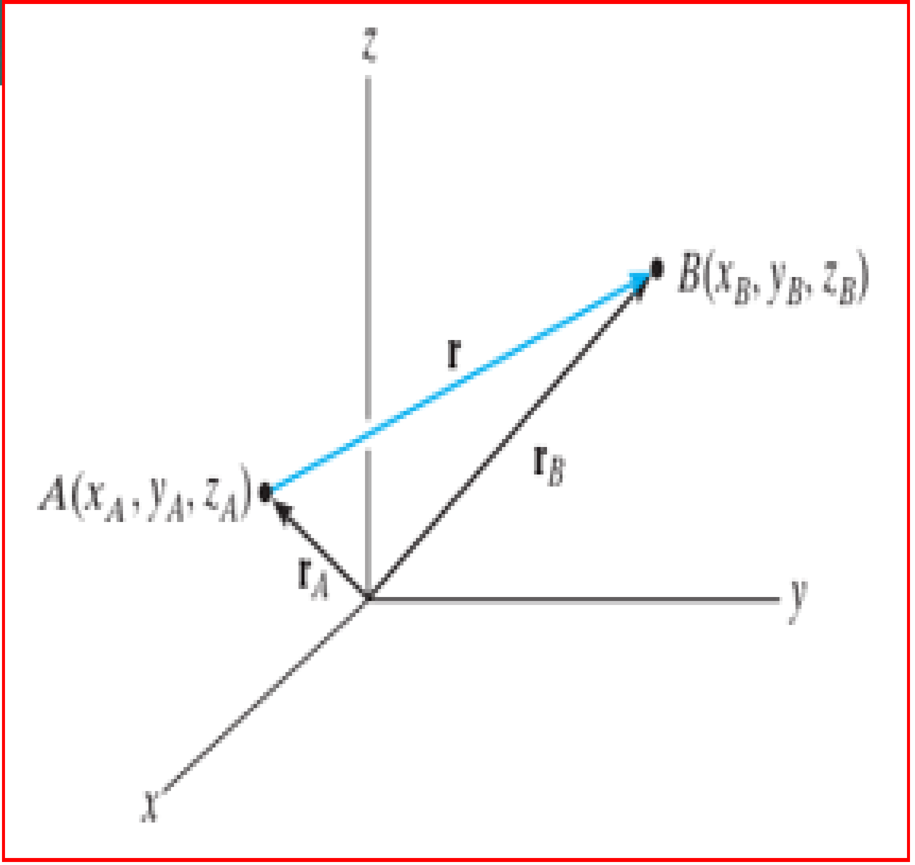
$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

- *Solving for* $\mathbf{r}$ *and expressing* $\mathbf{r}_A$ *and* $\mathbf{r}_B$ *in Cartesian vector form yields*

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

- *Or*

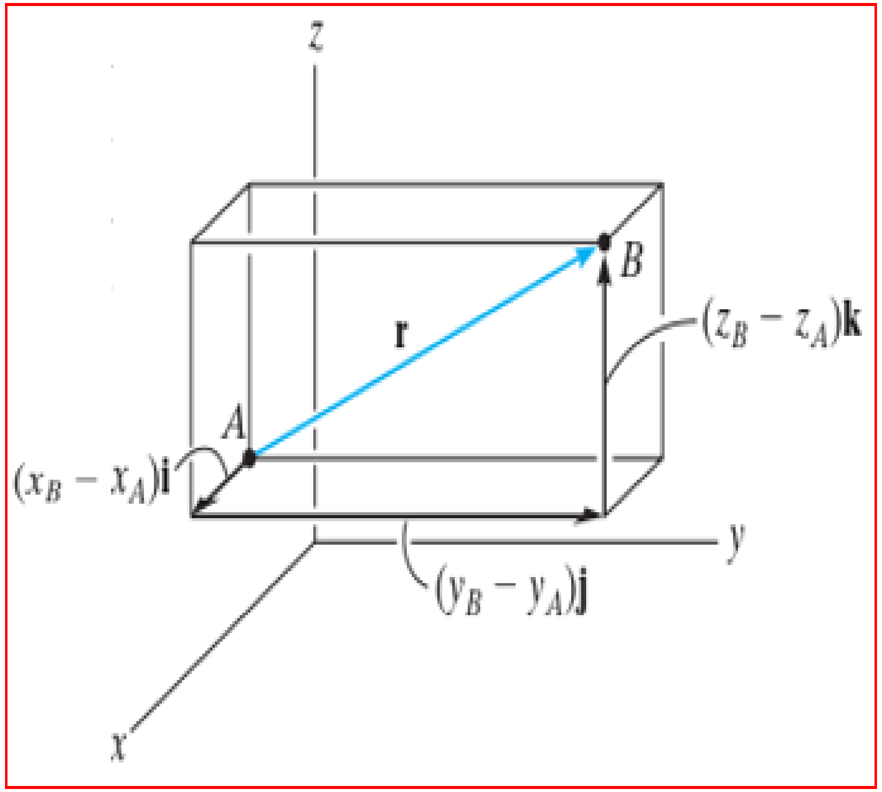
$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$



FORCE VECTORS IN SPACE (3D)

POSITION VECTORS

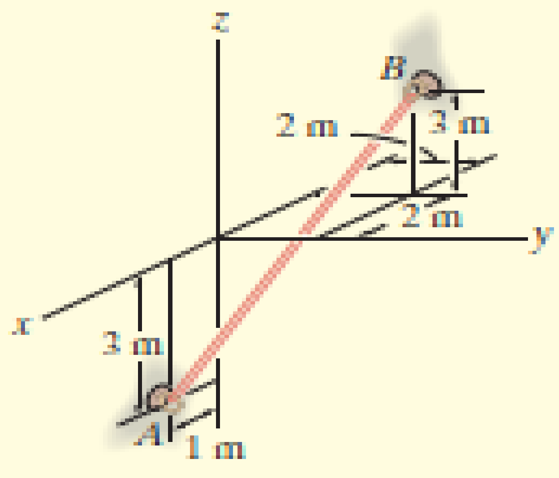
- *We can also form these components directly, by starting at A and moving through a distance of (x_B - x_A) along the positive x axis (+**i**)*
- *then (y_B - y_A) along the positive y axis (+**j**), and*
- *finally (z_B - z_A) along the positive z axis (+**k**) to get to B.*



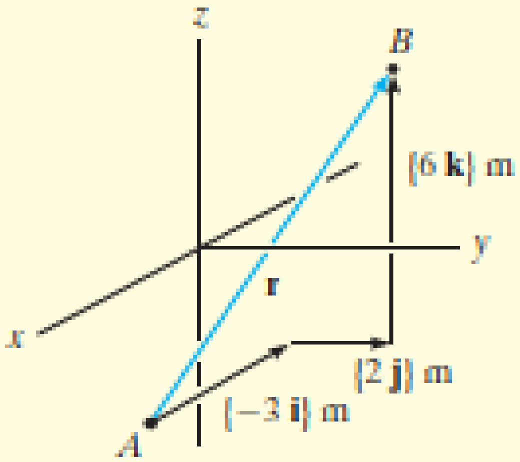
FORCE VECTORS IN SPACE (3D)
POSITION VECTORS – EXAMPLE 3.1

QUESTION

- *An elastic rubber band is attached to points A and B as shown in Fig. 2–35a. Determine its length and its direction measured from A toward B.*



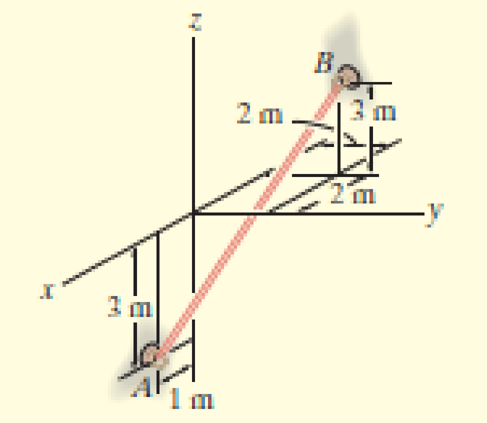
(a)



(b)

FORCE VECTORS IN SPACE (3D)

POSITION VECTORS – EXAMPLE 3.1

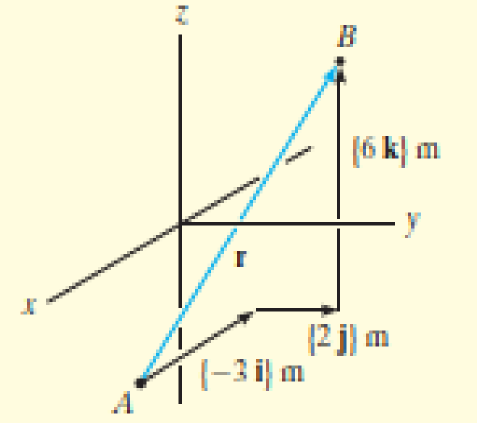


(a)

SOLUTION

- *1ST establish a position vector from A to B, Fig. 2–35b. In accordance with Eqn,* ➡ $\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$
- *the coordinates of the tail A(1 m, 0, -3 m) are subtracted from the coordinates of the head B(-2 m, 2 m, 3 m),*
- *which yields* ➡

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$



(b)

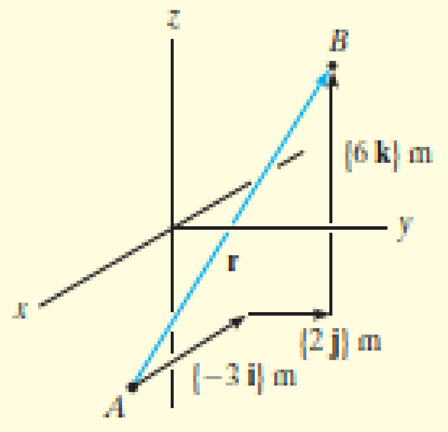
FORCE VECTORS IN SPACE (3D)

POSITION VECTORS – EXAMPLE 3.1

SOLUTION

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$

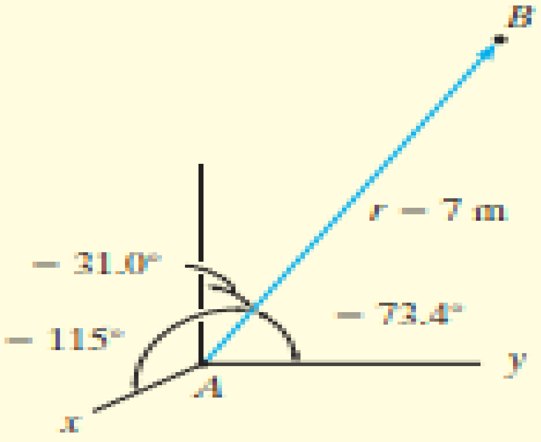
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$



(b)

- *These components of* **r** *can also be determined directly by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, m.*

- *The length of the rubber band is therefore 7m and the cosα, cosβ & cosγ = 3/7 (α=115°), 2/7 (β=73.4°) & 6/7 (γ=31.0°) resp..*



(c)

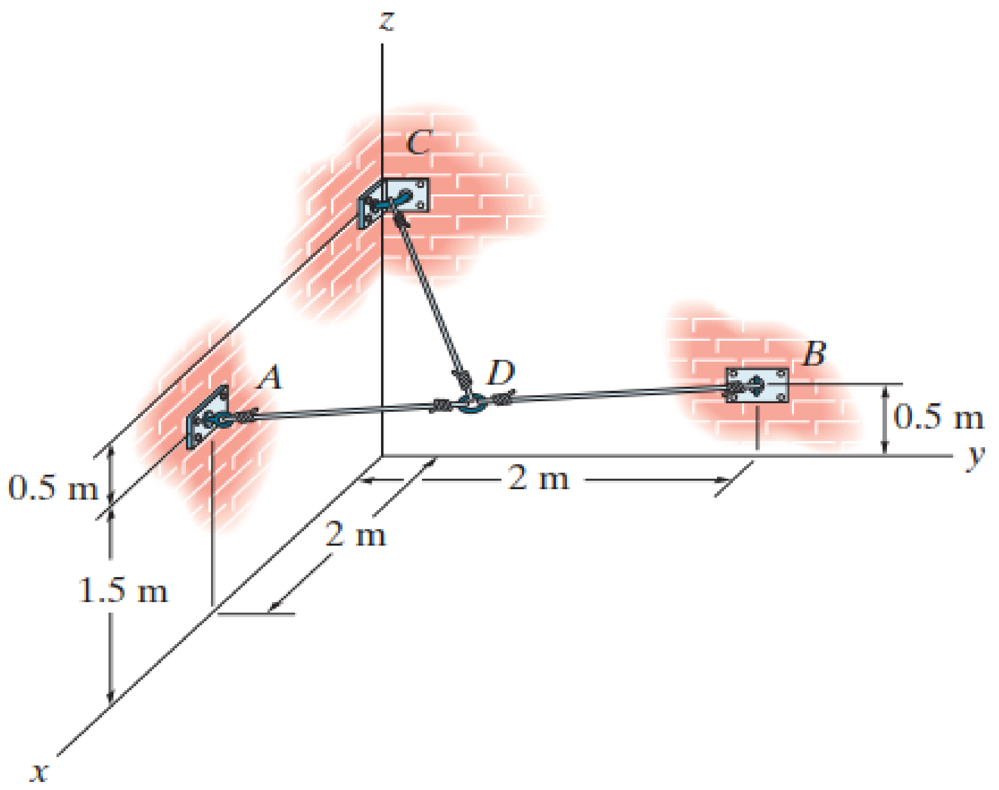
Fig. 2–35

FORCE VECTORS IN SPACE (3D)

POSITION VECTORS – EXAMPLE 3.2

QUESTION

- *Determine the lengths of wires AD, BD, and CD. The ring at D is midway between A and B*



FORCE VECTORS IN SPACE (3D)

POSITION VECTORS – EXAMPLE 3.2

SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right)\text{ m} = D(1, 1, 1)\text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$
$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

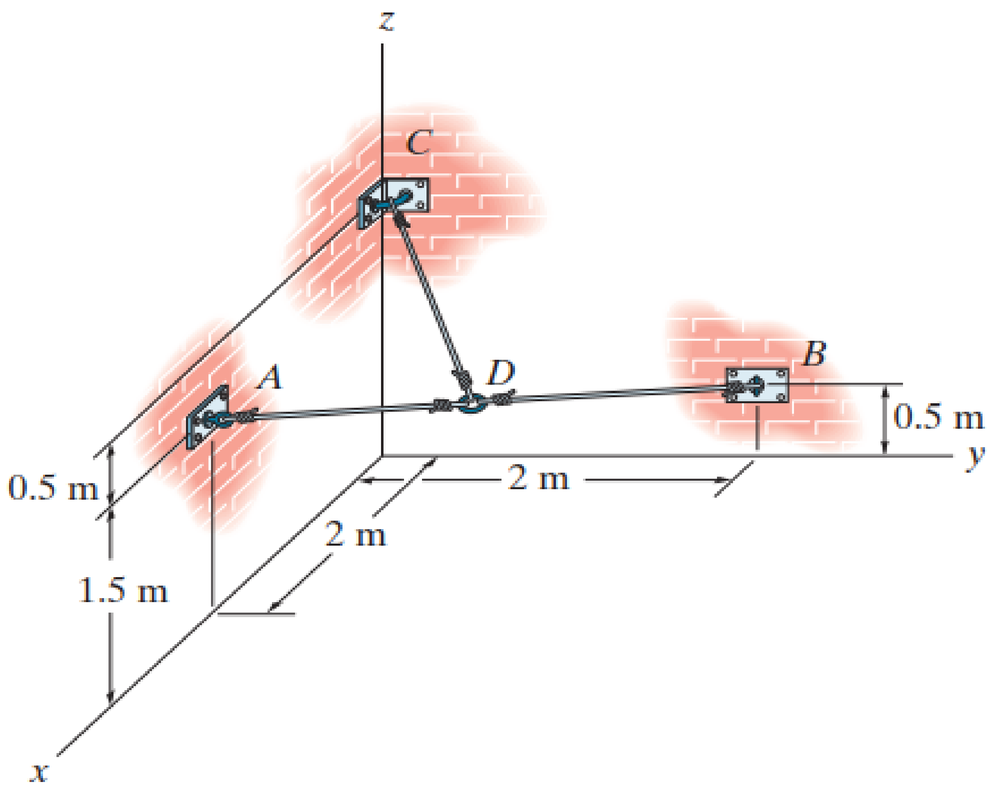
$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$
$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$
$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\text{ m}$$

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\text{ m}$$

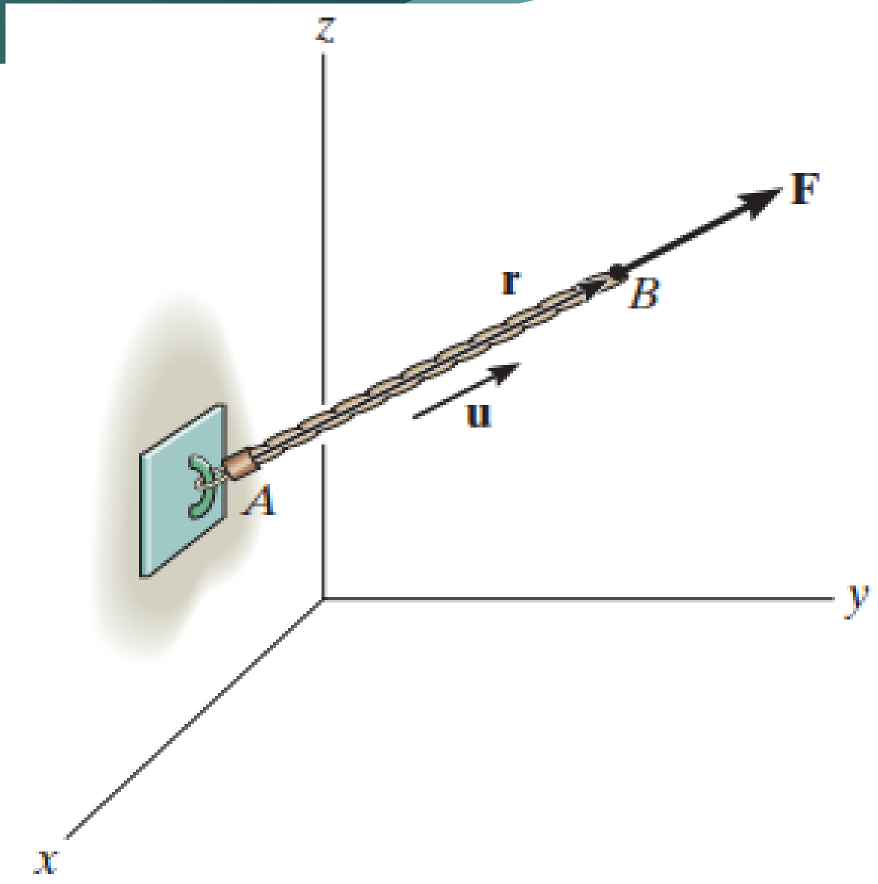
$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\text{ m}$$



FORCE VECTORS IN SPACE (3D)

Force Vector Directed Along a Line

- *Often in 3D statics problems, the direction of a force is specified by two points through which its line of action passes.*
- *Such a situation is shown, where the force F is directed along the cord AB.*
- *We can formulate F as a Cartesian vector by realizing that it has the same direction and sense as the position vector* **r** *directed from point A to point B on the cord.*



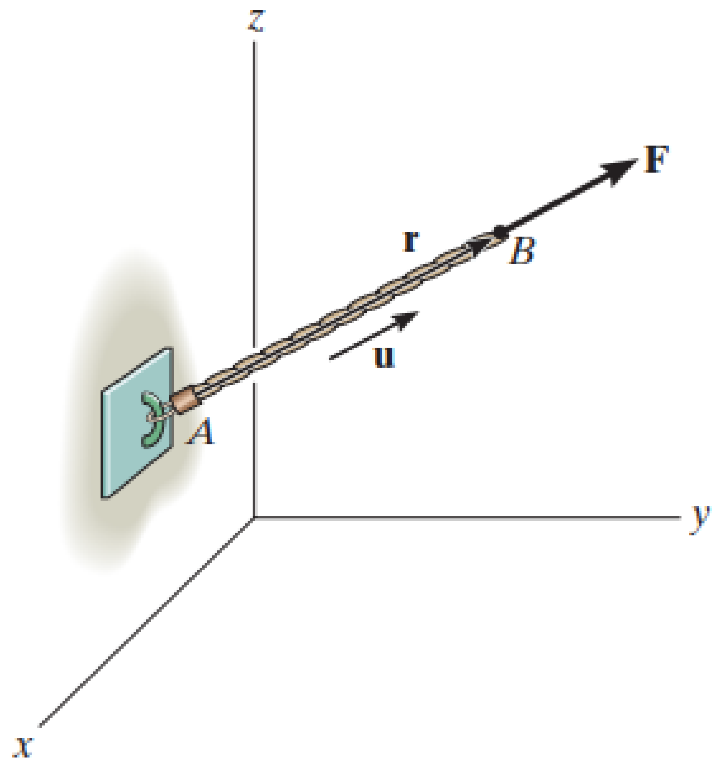
FORCE VECTORS IN SPACE (3D)

Force Vector Directed Along a Line

- *This common direction is specified by the unit vector* $\mathbf{u} = \mathbf{r}/r$.
- *Hence,*

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

- *Note that F has units of force and r has units of length.*



FORCE VECTORS IN SPACE (3D)

Force Vector Directed Along a Line

Important Points

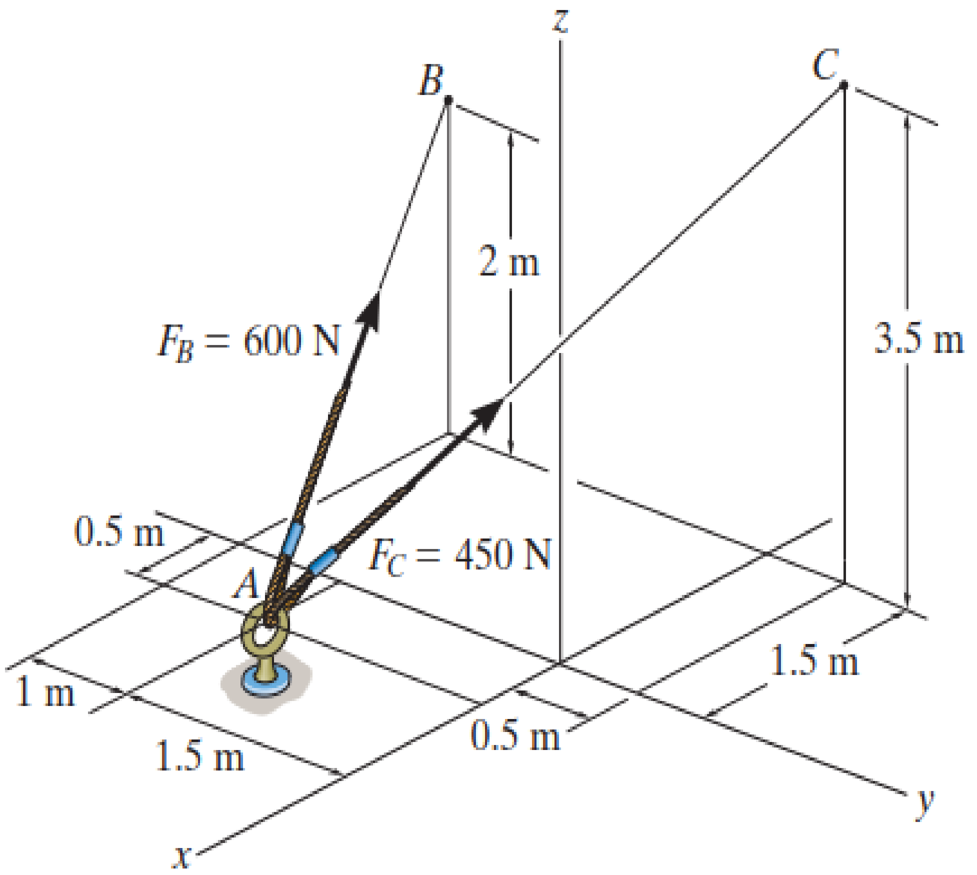
- A position vector locates one point in space relative to another point.
- The easiest way to formulate the components of a position vector is to determine the distance and direction that must be traveled along the x, y, z directions—going from the tail to the head of the vector.
- A force **F** acting in the direction of a position vector **r** can be represented in Cartesian form if the unit vector **u** of the position vector is determined and it is multiplied by the magnitude of the force, i.e., $\mathbf{F} = F\mathbf{u} = F(\mathbf{r}/r)$.

FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.3

QUESTION

- Express F_B and F_C in Cartesian vector form



FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.3

SOLUTION

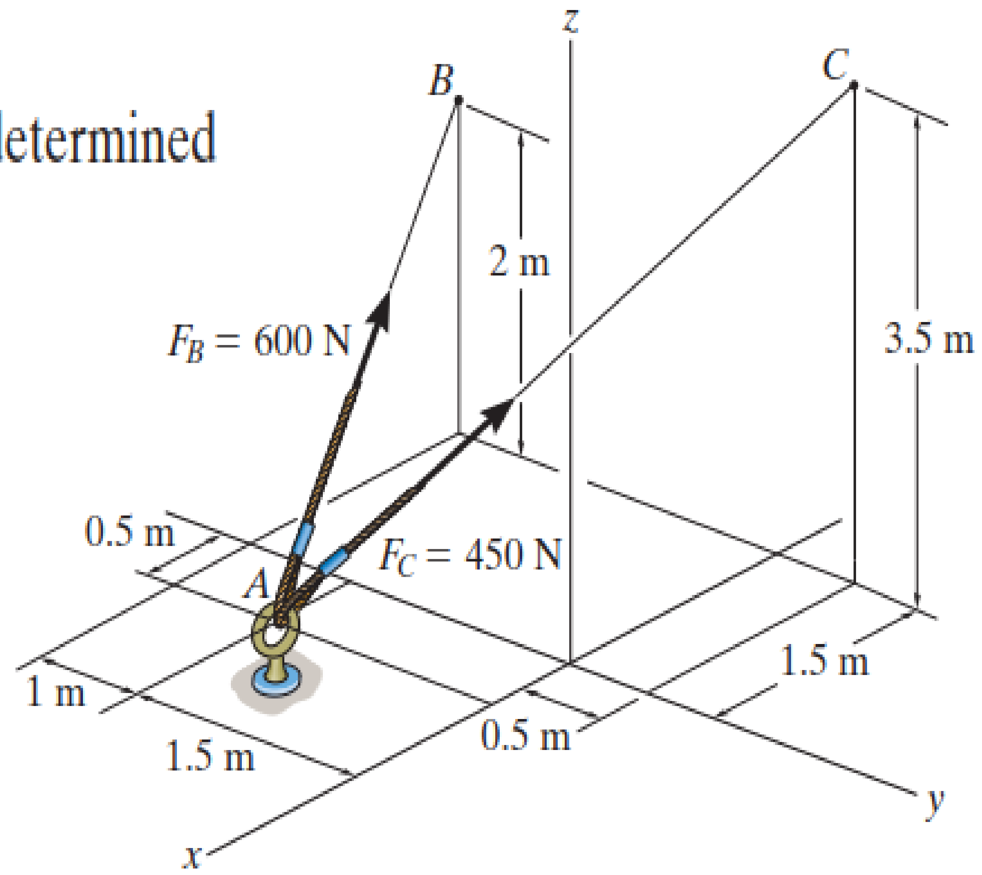
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



FORCE VECTORS IN SPACE (3D)

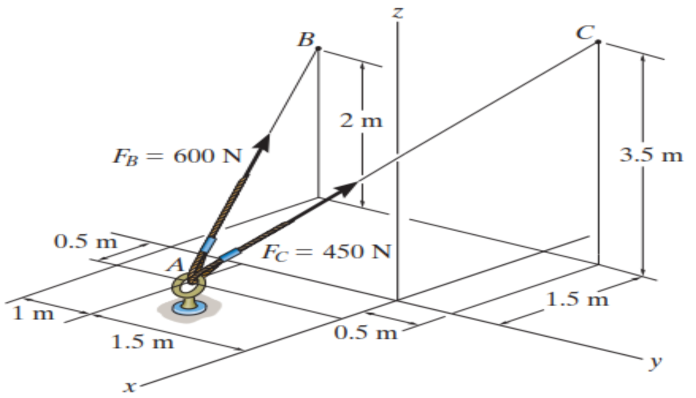
FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.3

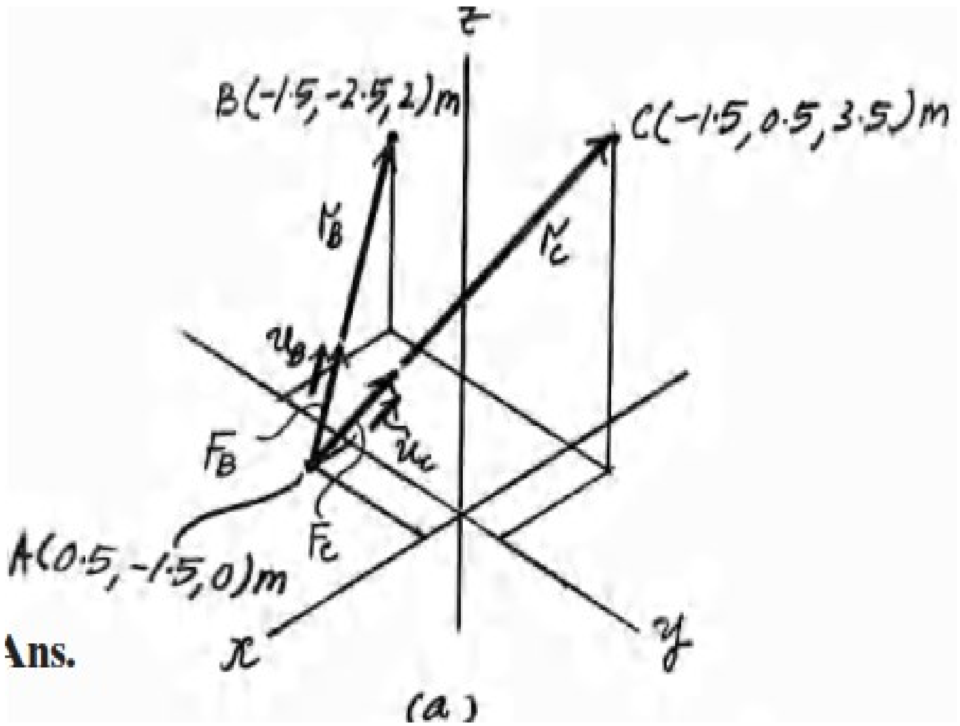
SOLUTION

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$



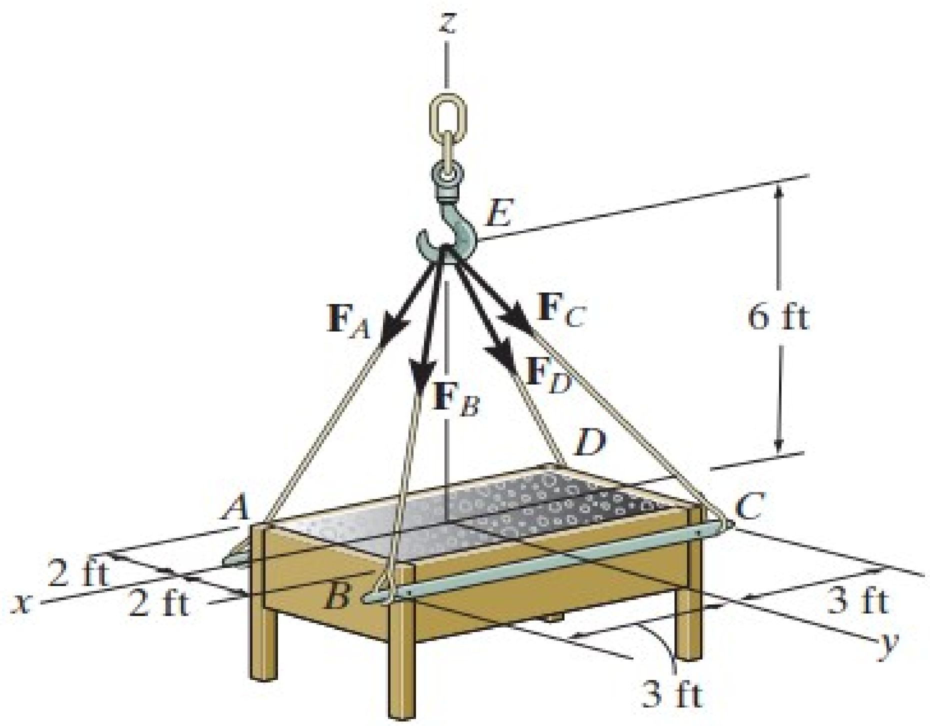


FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.4

QUESTION

- *If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.*



FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.4

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

SOLUTION

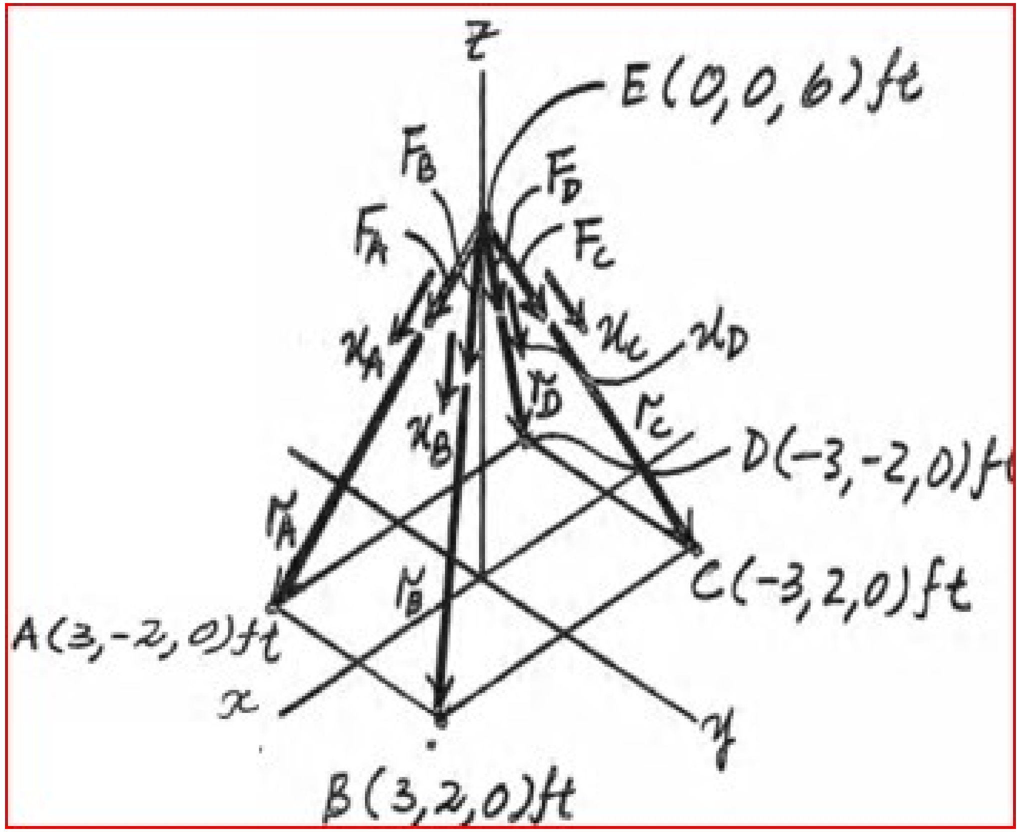
Force Vectors: The unit vectors **u**A, **u**B, **u**C and **u**D of **F**A, **F**B, **F**C and **F**D must be determined

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$



FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.4

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

SOLUTION

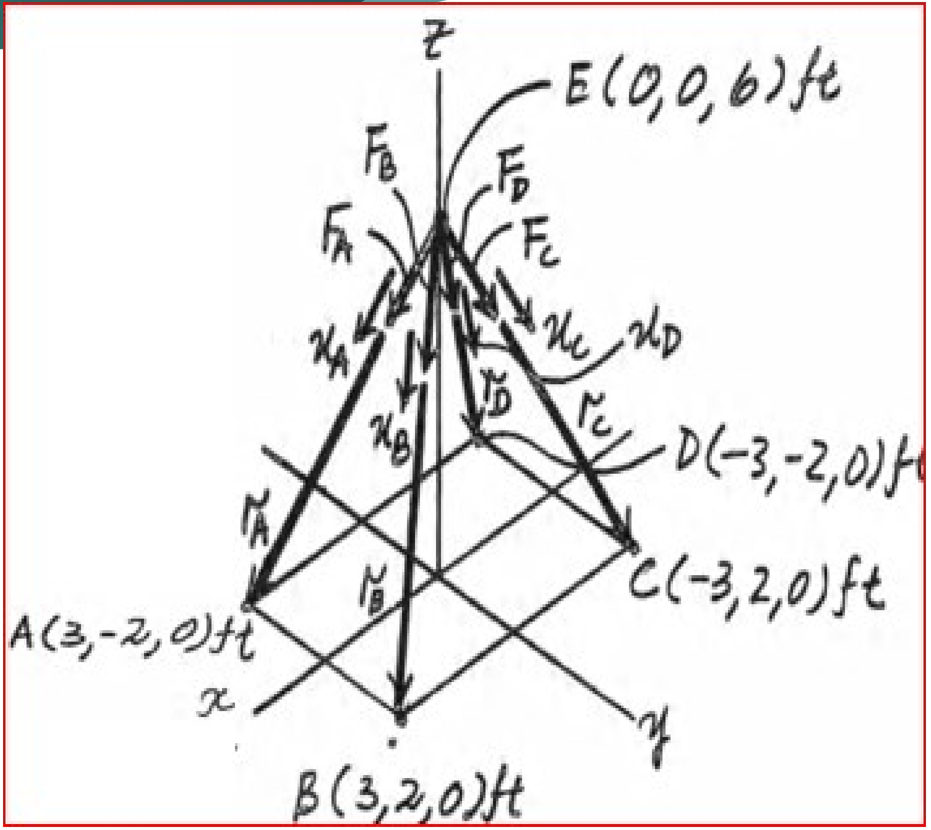
Thus, the force vectors **F**A, **F**B, **F**C and **F**D are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

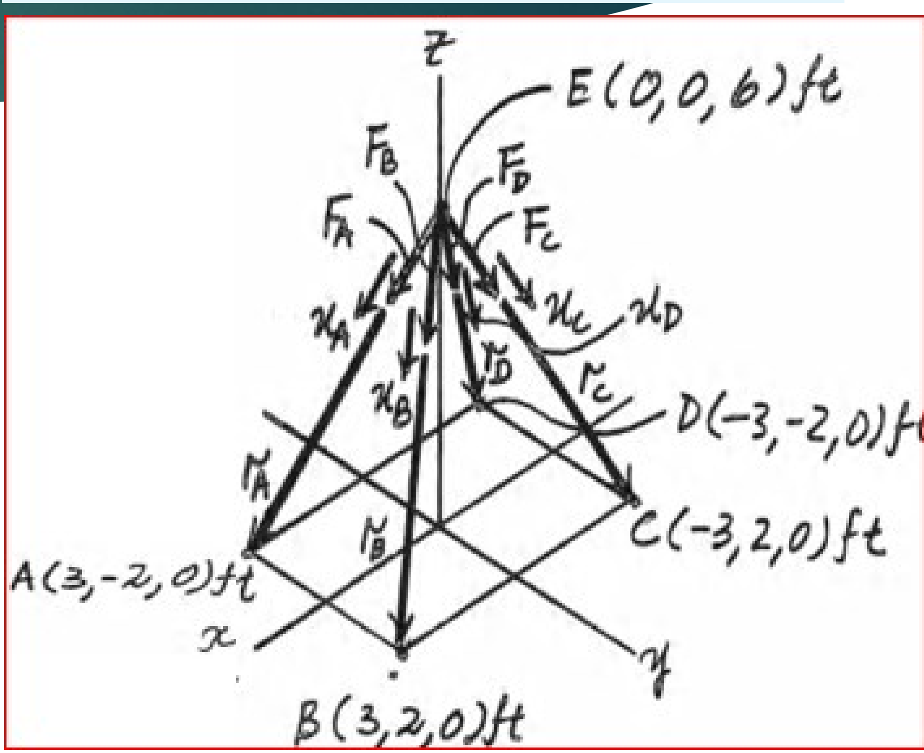
$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$



FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.4

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$



SOLUTION

Resultant force **F**R is calculated as shown

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) = \{-240\mathbf{k}\}\text{ N}$$

The magnitude of FR is $F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{0 + 0 + (-240)^2} = 240\text{ lb}$

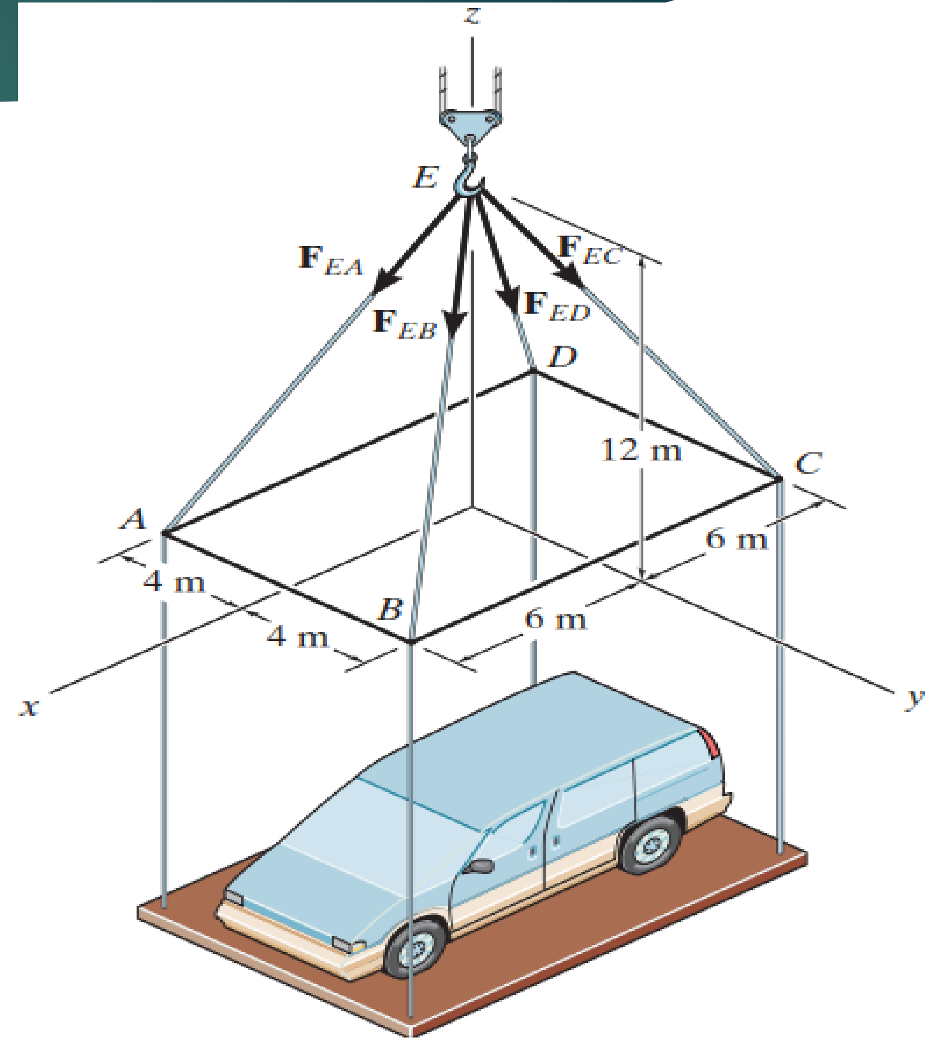
& the coordinate direction angles of FR are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ \quad \beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ \quad \gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180^\circ$$

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.5

QUESTION FOR HOME WORK

- *Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.*



FORCE VECTORS IN SPACE (3D)

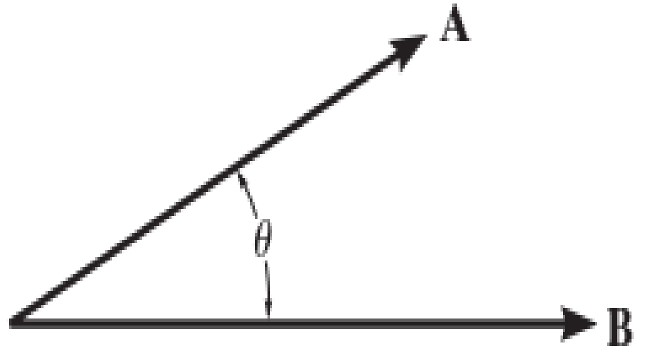
DOT PRODUCT

- *Occasionally in statics one has to find the angle between two lines or the components of a force parallel and perpendicular to a line.*
- *In two dimensions, these problems can readily be solved by trigonometry since the geometry is easy to visualize*
- *In 3D, however, this is often difficult, and consequently vector methods should be employed for the solution*
- *The **dot product**, which defines a particular method for "multiplying" two vectors, can be used to solve these problems*

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

- *The dot product of vectors A and B, written A•B is defined as the product of the* ***magnitudes*** *of A and B and the cosine of the angle θ between their tails*
- *In equation form,* $A \bullet B = AB\cos\theta$ *where* $0° \leq \theta \leq 180°$
- *The dot product is often referred to as the scalar product of vectors since the result is a* ***scalar and not a vector***



FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Laws of Operation

- *Commutative law:* $A.B = B.A$
- *Multiplication by a scalar:* $a(A.B) = (aA).B = A.(aB)$
- *Distributive law:* $A.(B+D) = (A.B) + (A.D)$

.

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Cartesian Vector Formulation

- *The Dot Product eqn must be used to find the dot product for any two Cartesian unit vector*
- *For example, i.i=(1)(1) cos0°=1 and i.j=(1)(1) cos90°=0. WHY???*

$$\vec{i}\bullet\vec{i}=1 \quad \vec{j}\bullet\vec{j}=1 \quad \vec{k}\bullet\vec{k}=1 \quad \vec{i}\bullet\vec{j}=0 \quad \vec{j}\bullet\vec{k}=0 \quad \vec{k}\bullet\vec{i}=0$$

$$\vec{P}\bullet\vec{Q}=P_xQ_x+P_yQ_y+P_zQ_z$$

$$\vec{P}\bullet\vec{P}=P_x^2+P_y^2+P_z^2=P^2$$

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Cartesian Vector Formulation

- *If we want to find the dot product of two general vectors A and B that are expressed in Cartesian vector form, then we have*

$$
\begin{aligned}
\mathbf{A} \cdot \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\
&= A_xB_x(\mathbf{i} \cdot \mathbf{i}) + A_xB_y(\mathbf{i} \cdot \mathbf{j}) + A_xB_z(\mathbf{i} \cdot \mathbf{k}) \\
&\quad + A_yB_x(\mathbf{j} \cdot \mathbf{i}) + A_yB_y(\mathbf{j} \cdot \mathbf{j}) + A_yB_z(\mathbf{j} \cdot \mathbf{k}) \\
&\quad + A_zB_x(\mathbf{k} \cdot \mathbf{i}) + A_zB_y(\mathbf{k} \cdot \mathbf{j}) + A_zB_z(\mathbf{k} \cdot \mathbf{k})
\end{aligned}
$$

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Cartesian Vector Formulation

- *Carrying out the dot-product operations, the final result becomes*

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

→ *Confirm!*

- ***Thus, to determine the dot product of two Cartesian vectors, MULTIPLY their corresponding x, y, z components and SUM these products algebraically***

- *Note that the result will be either a POSITIVE or NEGATIVE scalar, or it could be ZERO* → *SCALAR*

FORCE VECTORS IN SPACE (3D)

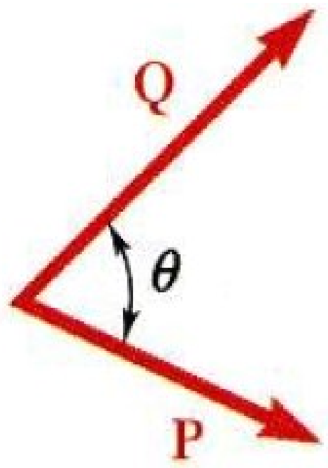
DOT PRODUCT

Applications

- *The dot product has two important applications in mechanics.*

1) *The angle formed between two vectors or intersecting lines*

- *The angle θ between the tails of vectors P and Q can be calculated using*



$$\vec{P} \bullet \vec{Q} = PQ\cos\theta = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

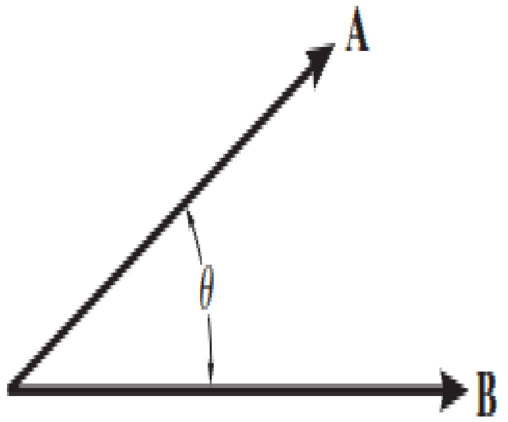
FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Applications

- *Or the angle θ between the tails of vectors A and B can be calculated using*

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) \quad 0^\circ \le \theta \le 180^\circ$$



- *Notice that if A.B = 0, θ=cos^-1(0) = 90° so that A will be perpendicular to B.*

FORCE VECTORS IN SPACE (3D)

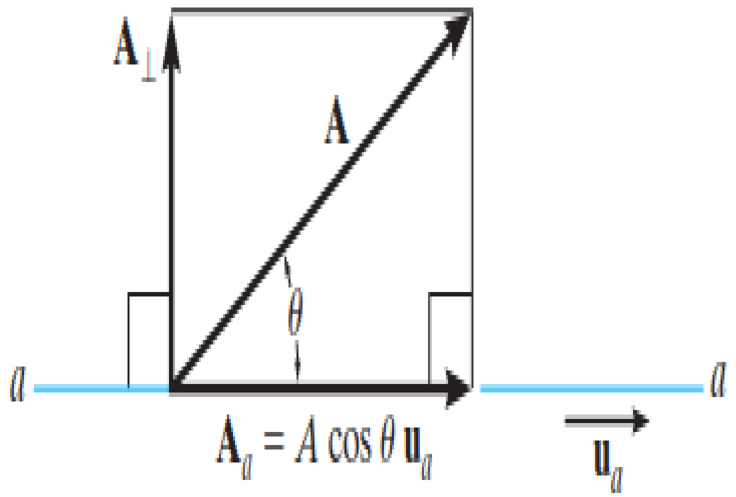
DOT PRODUCT

Applications

2) The components of a vector parallel and perpendicular to a line. Or projection of one vector on another

- *The component of vector A parallel to or collinear with the line aa in Fig shown is defined by A_a where $A_a = A\cos\theta$*
- *If the direction of the line is specified by the unit vector $\mathbf{u}_a$, then since $\mathbf{u}_a = 1$, we can determine the MAGNITUDE of A_a directly from the dot product i.e* →

$$A_a = A\cos\theta = \mathbf{A}\cdot\mathbf{u}_a$$



FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Applications

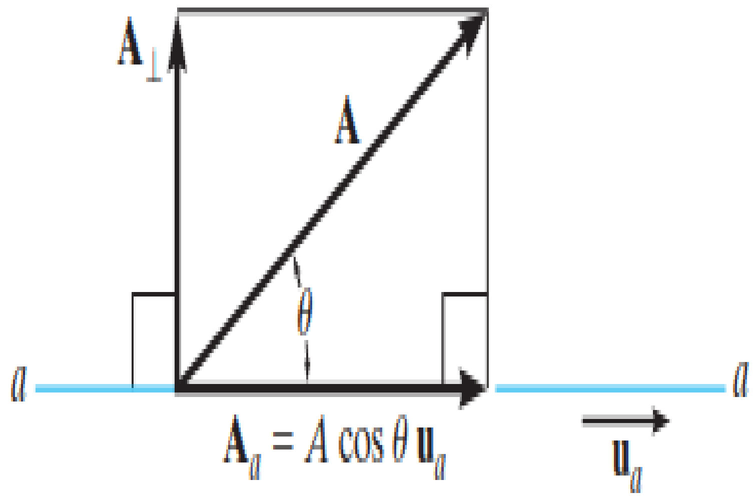
- *The perpendicular component of A can also be obtained since*

$$A = A_a + A_\perp, \text{ then } A_\perp = A - A_a$$

- *There are two possible ways of obtaining* $A_\perp$*. One way would be to determine* θ *from the dot product,*

$$\theta = \cos^{-1}(\mathbf{A}.\mathbf{u}_A/A); \text{ then } A_\perp = A \sin\theta.$$

- *Alternatively, if* A_a *is known, then by the Pythagorean theorem we can also write* →



$$A_\perp = \sqrt{A^2 - A_a^2}.$$

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT

Important Points

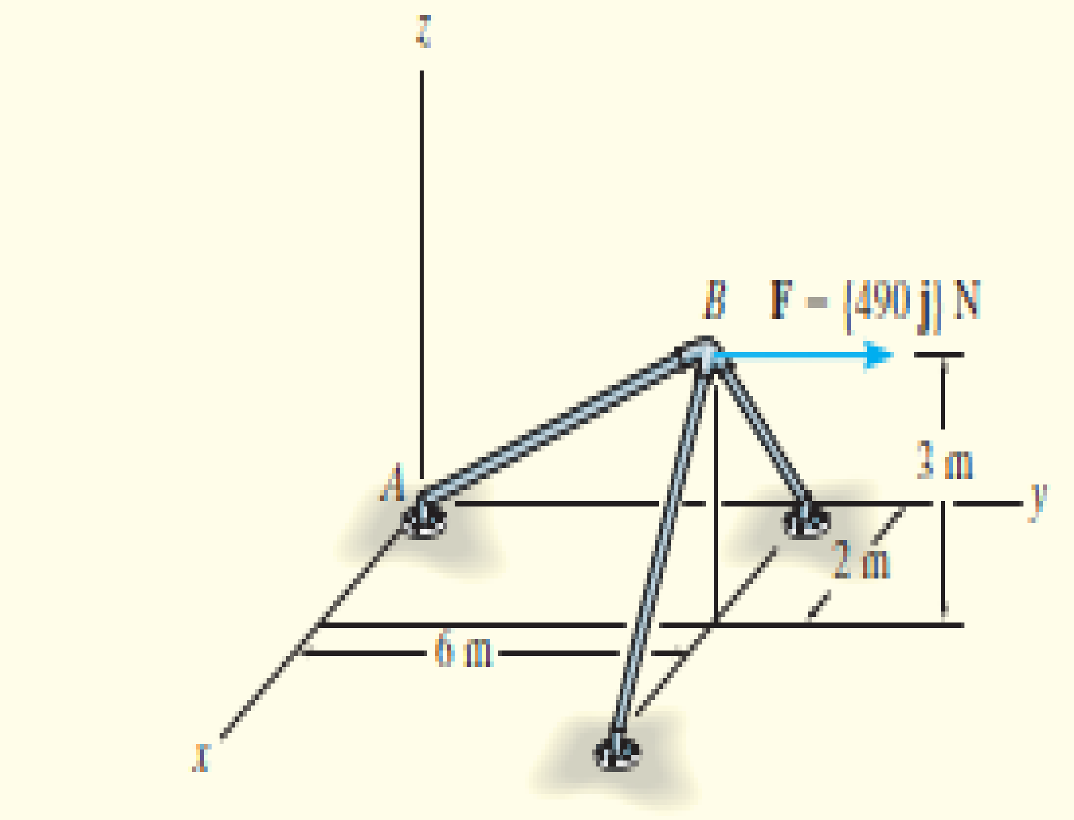
- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.
- If vectors **A** and **B** are expressed in Cartesian vector form, the dot product is determined by multiplying the respective *x*, *y*, *z* scalar components and algebraically adding the results, i.e., $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
- From the definition of the dot product, the angle formed between the tails of vectors **A** and **B** is $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{B}/AB)$.
- The magnitude of the projection of vector **A** along a line *aa* whose direction is specified by $\mathbf{u}_a$ is determined from the dot product $A_a = \mathbf{A} \cdot \mathbf{u}_a$.

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.6

QUESTION

- *The frame shown in Fig. is subjected to a horizontal force F = {490j} N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB.*



FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.6

SOLUTION

- *The magnitude of the projected component of F along AB is equal to the dot product of F and the unit vector u_B, which defines the direction of AB. Since*

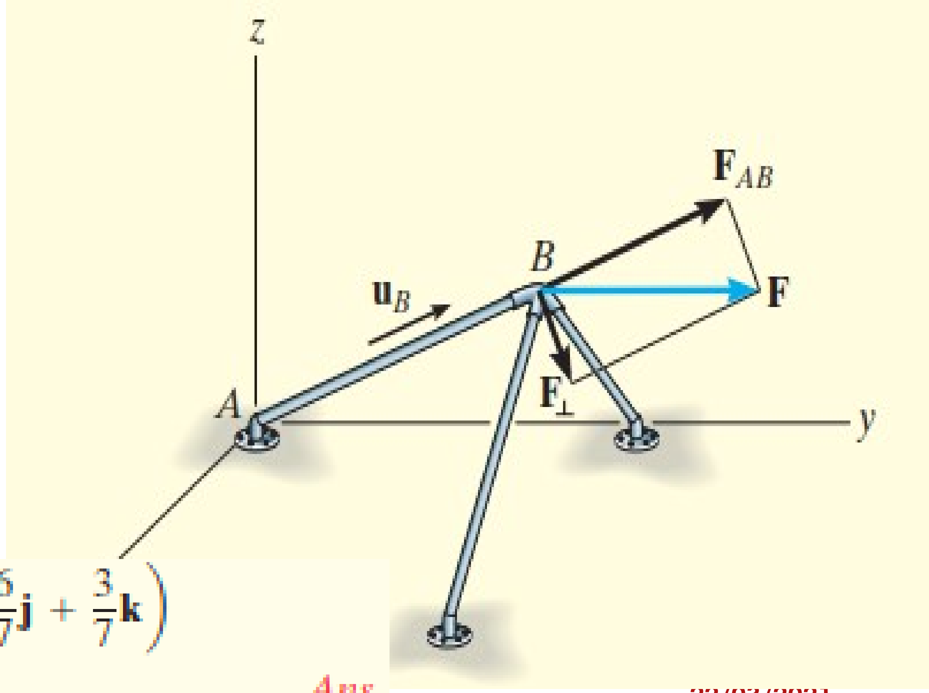
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{2^2 + 6^2 + 3^2}} = \frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

then

$$F_{AB} = F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B = (490\mathbf{j})\cdot\left(\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

$$= (0)\left(\frac{2}{7}\right) + (490)\left(\frac{6}{7}\right) + (0)\left(\frac{3}{7}\right)$$

$$= 420\text{ N}$$ *Ans.*



- *Since the result is a positive scalar, F_{AB} has the same sense of direction as $\mathbf{u}_B$. Expressing F_{AB} in Cartesian vector form, we have*

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (420\text{ N})\left(\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

$$= \{120\mathbf{i} + 360\mathbf{j} + 180\mathbf{k}\}\text{ N}$$ *Ans.*

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.6

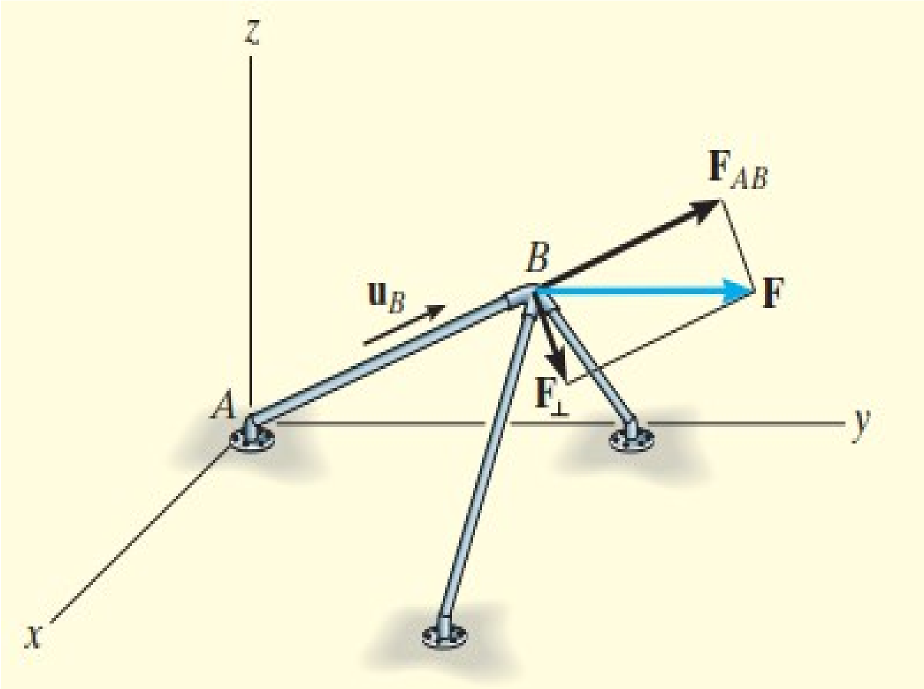
SOLUTION

- *The perpendicular component, is therefore*

$$\mathbf{F}_\perp = \mathbf{F} - \mathbf{F}_{AB} = 490\mathbf{j} - (120\mathbf{i} + 360\mathbf{j} + 180\mathbf{k}) \\ = \{-120\mathbf{i} + 130\mathbf{j} - 180\mathbf{k}\}\ \text{N}$$

- *Its magnitude can be determined either from this vector or by using the Pythagorean theorem*

$$F_\perp = \sqrt{F^2 - F_{AB}^2} = \sqrt{(490\ \text{N})^2 - (420\ \text{N})^2} \\ = 252\ \text{N}$$

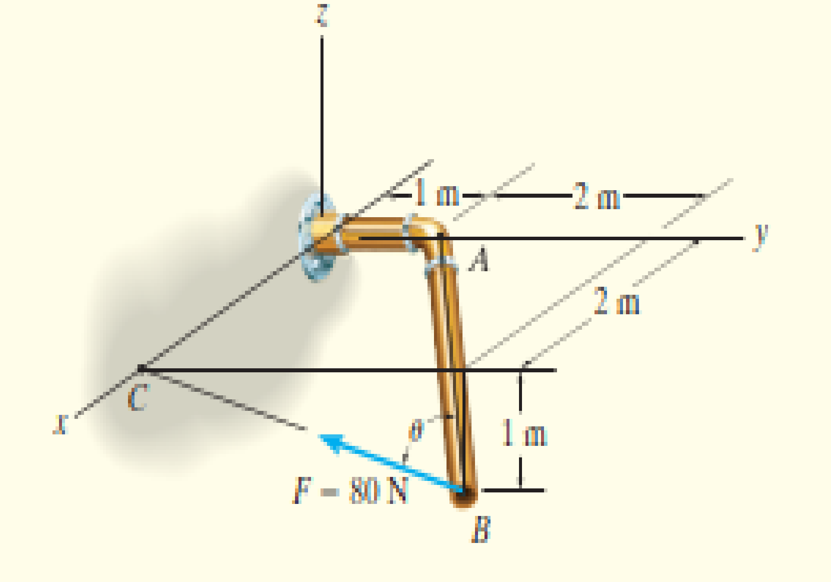


FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.7

QUESTION

- *The pipe in Fig. shown is subjected to the force of F = 80 N. Determine the angle θ between F and the pipe segment BA, and the projection of F along this segment*



(a)

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.7

SOLUTION

Angle θ. First we will establish position vectors from B to A and B to C; Fig. 2–43b. Then we will determine the angle θ between the tails of these two vectors.

$$\mathbf{r}_{BA} = \{-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}\ \text{m},\quad r_{BA} = 3\ \text{m}$$
$$\mathbf{r}_{BC} = \{-3\mathbf{j} + 1\mathbf{k}\}\ \text{m},\quad r_{BC} = \sqrt{10}\ \text{m}$$

Thus,

$$\cos\theta = \frac{\mathbf{r}_{BA}\cdot\mathbf{r}_{BC}}{r_{BA}r_{BC}} = \frac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = 0.7379$$

$$\theta = 42.5^\circ \qquad \textit{Ans.}$$

Components of F. The component of $\mathbf{F}$ along BA is shown in Fig. 2–43c. We must first formulate the unit vector along BA and force $\mathbf{F}$ as Cartesian vectors.

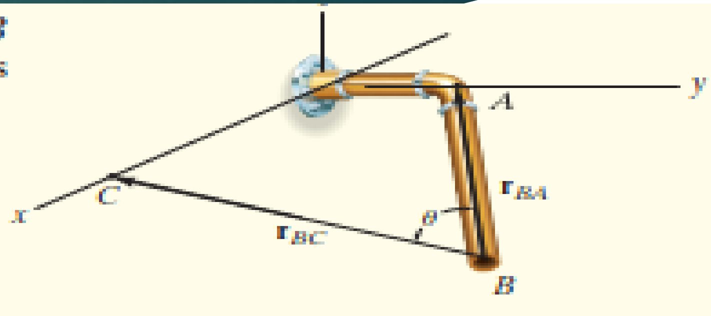
$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k})}{3} = -\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

$$\mathbf{F} = (80\ \text{N})\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = (80\ \text{N})\left(\frac{-3\mathbf{j} + 1\mathbf{k}}{\sqrt{10}}\right) = \{-75.89\mathbf{j} + 25.30\mathbf{k}\}\text{N}$$

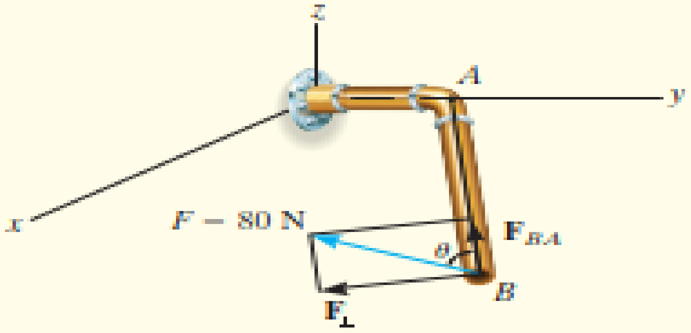
Thus,

$$F_{BA} = \mathbf{F}\cdot\mathbf{u}_{BA} = (-75.89\mathbf{j} + 25.30\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right)$$
$$= 0\left(-\frac{2}{3}\right) + (-75.89)\left(-\frac{2}{3}\right) + (25.30)\left(\frac{1}{3}\right)$$
$$= 59.0\ \text{N} \qquad \textit{Ans.}$$

NOTE: Since θ has been calculated, then also, $F_{BA} = F\cos\theta = (80\ \text{N})\cos 42.5^\circ = 59.0\ \text{N}$.



(b)



(c)

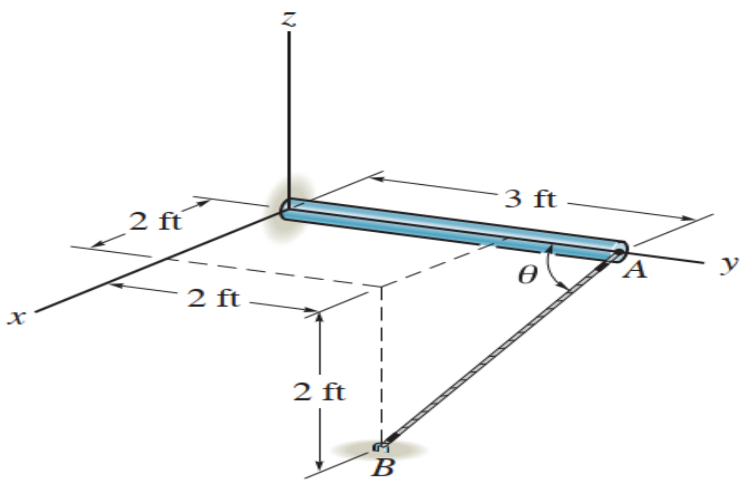
Fig. 2–43

FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.7

SOLUTION _2-116

- *Determine the angle between the y axis of the pole and the wire AB.*



FORCE VECTORS IN SPACE (3D)

DOT PRODUCT – EXAMPLE 3.6

SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\}\ \text{ft}$$

$$\mathbf{r}_{AB} = \{(2-0)\mathbf{i} + (2-3)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00\ \text{ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00\ \text{ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

$$= 0(2) + (-3)(-1) + 0(-2)$$

$$= 3$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5^\circ$$

FORCE VECTORS IN SPACE (3D)

FORCE VECTOR DIRECTED ALONG A LINE – EXAMPLE 3.5

SOLUTION FOR 3.5

$$\mathbf{F}_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

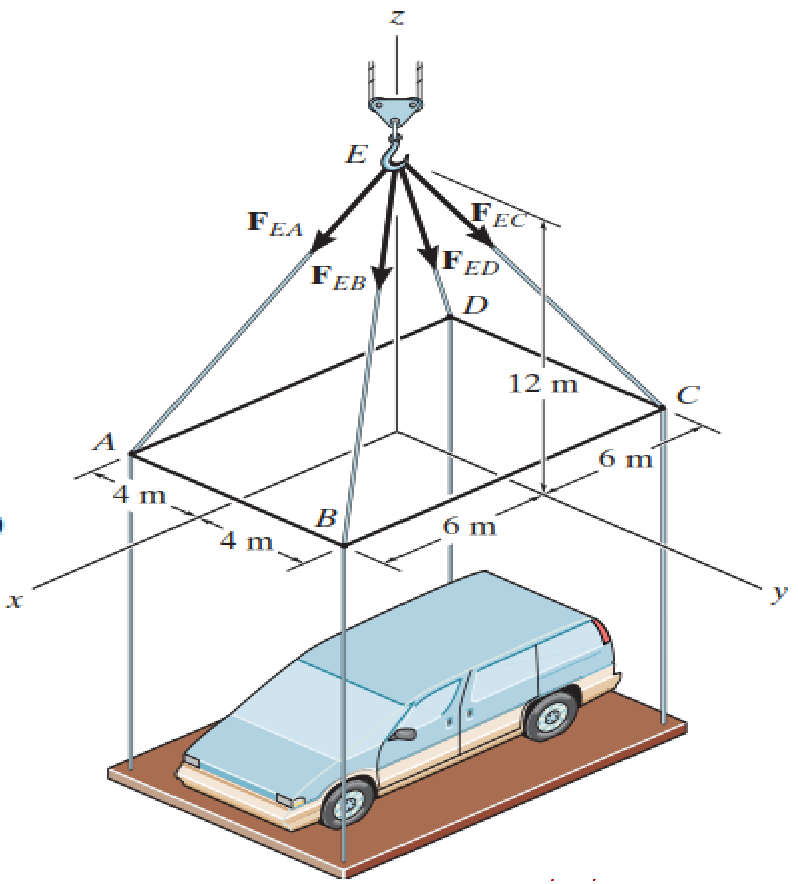
$$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

$$= \{-96\mathbf{k}\}\ \text{kN}$$



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