The University of Zambia

Department of Mathematics & Statistics

2020/2021 **Academic Year, Final Examinations (Deferred)**

MAT1110: Foundation Mathematics & Statistics For Social Sciences

Monday 14^{th} **February** $2022 - 14:00$**hrs**

Time Allowed: **3 hours**

Instructions:

1. There are **Seven** (7) questions in this examination paper. Attempt **any five (5)** questions.
2. Indicate your **computer number** on all your answer booklets.
3. **Full credit** will only be given when **all necessary working** is shown.
4. **Calculators** are **not** allowed.

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1. (a) Consider the subsets $A = [-10, 2]$, $B = (-3, 3)$ and $C = (0, 10]$ of the universal set $U = [-11, 12)$.

   Find each of the following sets and display your answer on the number line.

   i. $(A \cup C)^c$.

   ii. $A \cup (B \cap C)^c$.

   (b) i. Simplify

   $$\left(\frac{\sqrt{10}}{\sqrt{45}}\right)^{-1} + \sqrt{128},$$

   leaving your answer in the form $a\sqrt{b}$, where a is rational number and b is a natural number.

   ii. Simplify

   $$-1 + \sqrt{125} + \frac{1}{1 - \sqrt{5}},$$

   leaving your answer in the form $a + b\sqrt{c}$, where a and b are rational numbers and c is a natural number.

   (c) i. Express the rational number $0.06\overline{36}$ in the form $\frac{a}{b}$ where a and b are nonzero integers with no common factors.

   ii. Let A and B be any non-empty sets. Express the following in its simplest form:

   $$[(A \cap B)^c \cup (A - B)]^c.$$

   $[7, 8, 10]$

2. (a) Let $f(x) = x - 4$ and $g(x) = \dfrac{3}{x+1}$.

i. Find $(fog)(x)$, leaving your answer in simplest form.

ii. Solve the equation

$$(f \circ g)^{-1}(x) = 2.$$

(b) Let

$$f(x) = 2x^2 - 6x + 4.$$

i. Sketch the graph of $f(x)$.

ii. Find the values of x for which $f(x) > 0$.

iii. Is the function $f(x)$, even or odd or neither? Justify your answer.

(c) i. Solve the equation

$$4^x - 2^{x+1} - 48 = 0.$$

ii. Let $f(x) = x^3 + ax^2 + bx + 6$. Given that the remainders when $f(x)$ is divided by $x + 1$ and $x - 2$ are 20 and 8 respectively, find the value of a and of b.

[7, 10, 8]

3. (a) Find the exact value, leaving your answer in simplest form, for each of the following.

i.

$$\tan\left(\frac{-5\pi}{3}\right).$$

ii.

$$\sec\left(\frac{\pi}{12}\right).$$

iii.

$$\frac{\sin 45^\circ + \cos 45^\circ}{\cos(-45)^\circ}$$

(b) Prove each of the following identities:

i.

$$\frac{1 + \tan x}{1 - \tan x} \equiv \frac{\sin x + \cos x}{\cos x - \sin x}.$$

ii.

$$\frac{\cos^2 x}{\sin x + \sin^2 x} \equiv \frac{1 - \sin x}{\sin x}.$$

(c) Let

$$f(x) = \cos(2x - 90^\circ) \quad \text{for } 0^\circ \leq x \leq 360^\circ,$$

i. Find the amplitude, phase shift and period of the function $f(x)$.

ii. Hence, or otherwise, sketch the graph of $f(x$.

[10, 7, 8]

4. (a) i. Solve the equation

$$2\sin^3 x - \sin x = 0 \text{ for } 0 \le x \le 2\pi.$$

ii. Express

$$\log_p 12 - \left(\frac{1}{2}\log_p 9 + \frac{1}{3}\log_p 8\right)$$

as a single logarithm.

(b) i. Show that

$$\frac{1+2i}{3-i} + \frac{1-2i}{3+i}$$

is purely real.

ii. Find the partial fraction decomposition of

$$\frac{x^2}{x^3 - 6x^2 + 11x - 6}.$$

(c) The amount of money in a certain bank account is increasing exponentially. If K100,000 is present initially and K400,000 after 1 hour, how much money will be present after 210 minutes?

[9, 10, 6]

5. (a) i. Find the values of k given that $x - k$ is a factor of $p(x) = kx^3 - 3x^2 - 5kx - 9$.

ii. Use the **first principle** to differentiate

$$f(x) = \frac{1}{1 - \sqrt{x}}.$$

(b) Evaluate

i.
$$\int_3^6 \left(x - \frac{3}{x}\right)^2 \; dx.$$

ii.
$$\int_0^{\frac{\pi}{2}} \sin^3 x \; \cos x \; dx.$$

(c) Let $f(x) = \ln x$.

i. Sketch the graph of $f(x)$.

ii. Find the area of the region bounded by the graph of $f(x)$, x-axis, and the lines $x = 1$ and $x = e$.

[8, 8, 9]

6. (a) A jar contains 4 red marbles, 6 green marbles and 10 white marbles. If a marble is drawn from the jar at random, find the probability that

   i. it is green?

   ii. it is not white?

   (b) i. A card is drawn at random from an ordinary pack of playing cards. Find the probability that it is either red or diamond.

   ii. Two dice are thrown. Find the probability of scoring either the same number on both dice or scoring a sum less than 8.

   (c) Events A and B are such that $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{10}$. Find

   i. $P(A \cap B^c)$.

   ii. $P(A^c \cap B^c)$.

$[6, 11, 8]$

7. (a) i. Find the mean of the scores 88,72,65,55,43,37,21,19,12,8 obtained by students in a quiz.

   ii. Find the mean of the scores observed for 50 tosses of a coin as shown in the table below

score(x)	1	2	3	4	5	6
frequency (f)	7	15	10	3	9	6

   iii. Find the mean of the grouped frequency table

Mass(g)	$1-20$	$21-40$	$41-60$	$61-80$	$81-100$
Number of letters	10	18	24	14	18

   (b) Let

$$P(x) = x(100 - 2x),$$

   where x is the number of items sold, be the profit function in dollars, for a small scale company.

   i. Find the value of x that maximizes the profit and determine the corresponding profit.

   ii. Hence, or otherwise, sketch the graph of $P(x)$.

   (c) For the purposes of allocating first year students to different MAT 1110 lecture groups, a survey was randomly conducted on 600 students in the school of Humanities and Social Sciences of the University of Zambia. The results showed that:

   60% took ECN 1115, 50% took BBA 1110, 45% took DEM 1110. In addition, 30% to ECN 1115 and BBA 1110, 28% took ECN 1115 and DEM 1110 and 25% took BBA 1110 and DEM 1110.

   6% of the students took non of the these three courses.

   i. Illustrate this information on a Venn diagram.

   ii. Find the probability that a student chosen at random took exactly one course.

$[9, 6, 10]$