The University of Zambia

Department of Mathematics & Statistics

2018/2019 **Academic Year, Final Examinations**

MAT1110: Foundation Mathematics & Statistics for Social Sciences (Deferred)

Friday, 13 December 2019

Time Allowed: **3 hours**

Instructions:

1. There are **Seven** (7) questions in this examination paper. Attempt **any five (5)**.
2. Indicate your **computer number** on all your answer booklets.
3. **Full credit** will only be given when **all necessary working** is shown.
4. **Calculators** are **Not** allowed.

This examination has (6) **pages** of questions.

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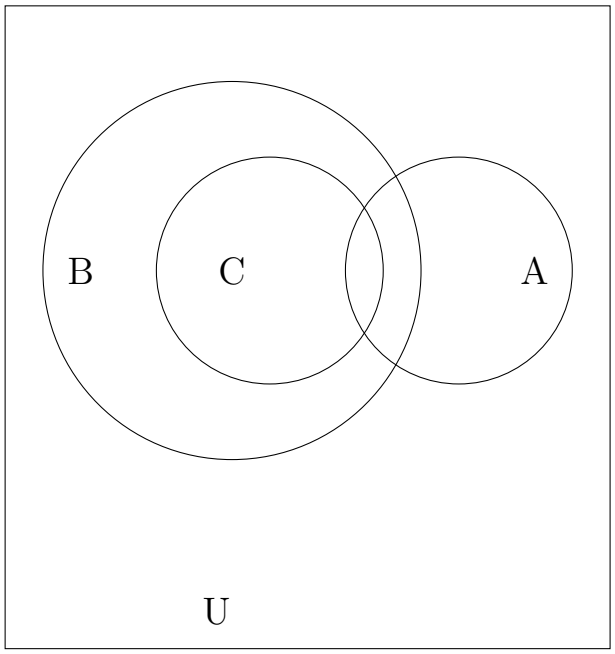
1. (a) i. Let $\mathbb{R}$ be the universal set, $A = (-\infty, 3)$ and $B = [4, \infty)$. Find the set

$$(A \cup B)^c,$$

and display your answer on the number line.

ii. Copy the Venn diagram below and shade the region represented by

$$A \cap (B \cap C^c)^c$$



(b) i. Express

$$-0 \cdot 266666 \cdots$$

in the form $\frac{a}{b}$, where a and b are integers with no common factor.

ii. Express

$$\frac{1 - \sqrt{8}}{2 + \sqrt{18}},$$

in the form $c + d\sqrt{2}$ where c and d are real numbers.

(c) Given that

$$f(x) = \frac{x + 1}{x - 1},$$

i. State the domain and range of $f(x)$.

ii. Sketch the graph of $f(x)$, labelling it clearly.

iii. Hence, or otherwise, solve the inequality

$$f(x) \geq 0.$$

[8, 7, 10]

2. (a) Let $f(x) = x^3 + ax^2 + bx + 6$. Given that $f(x)$ gives a remainder of 20 when divided by $x + 1$, and also gives a remainder of 8 when divided by $x - 2$, find the value of a and of b.

   (b) Solve the following equation;

$$9^{x+1} - 28(3^x) + 3 = 0.$$

   (c) Given that $f(x) = \frac{3}{2}$ and $g(x) = 1 + \log_{\frac{1}{4}}\left(\frac{1}{x}\right)$,

      i. Find the point of intersection of the two functions.

      ii. On the same set of axes, sketch the graph of $f(x)$ and of $g(x)$.

   (d) Evaluate

$$\lim_{x \to 3} \frac{x^3 - 27}{3 - x}.$$

[6, 6, 8, 5]

3. (a) Solve the following equation for $0 \leq x \leq 2\pi$.

$$\cos^2 x + \left(\frac{\sqrt{3} - \sqrt{2}}{2}\right)\cos x - \frac{\sqrt{6}}{4} = 0.$$

   (b) Prove each of the following identities:

      i.

$$1 - 2\cos^2 x \equiv \frac{\tan^2 x - 1}{\tan^2 x + 1}.$$

      ii.

$$\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} \equiv \frac{3 + \sin x}{1 - \sin x}.$$

   (c) Sketch the graph of

$$f(x) = 1 - \cos x$$

   for $0° \leq x \leq 270°$.

   (d) Use the **first principle** to differentiate

$$f(x) = x^3.$$

[9, 8, 4, 4]

4. (a) Given that
$$3ye^{-2x} = 4x^2 + y^2 + 2,$$
find the value of $\frac{dy}{dx}$ at $(0, 2)$, leaving your answer in simplest form.

(b) Given that $f(x) = 4x^3 + 2x^2 + 17x + 8$,

i. Express $f(x)$ in the form
$$f(x) = Q(x)(x^2 + 4) + R(x),$$
where $Q(x)$ is the quotient and $R(x)$ is the remainder when $f(x)$ is divided by $x^2 + 4$.

ii. Hence, evaluate
$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4}\ dx$$
leaving your answer in the form $a + \ln b$, where a and b are integers.

(c) Find the area of the region bounded by graph of $f(x) = \ln x$ and the lines $y = 0$, $y = 1$ and $x = 0$.

[6, 10, 9]

5. (a) i. When is a function $f(x)$ said to be continuous at a point $x = a$.

ii. Determine whether
$$f(x) := \begin{cases} 7x - 2 & \text{if } x \geq 2, \\ 3x + 5 & \text{if } x < 2, \end{cases},$$
is continuous at $x = 2$.

(b) Given that
$$f(x) = \frac{x}{9 + x^2},$$
find the coordinates of the stationary point(s), and determine whether maximum, minimum or point of inflection.

(c) In a certain University,

65% of the students are full-time students ,
55% of the students are female ,
35% of the students are male full-time students ,

Find the probability that

i. a student chosen at random from all the students in the University is a part-time student.

ii. a student chosen at random from all the students in the University is a female and a part-time student.

iii. a student chosen at random from all the female students in the University is a part-time student.

[7, 9, 9]

6. (a) It is given that 3 computers in a batch of 9 computers are faulty, but it is not known which 3. If two computers are selected at random, one at a time without replacement, find the probability that they are both faulty.

   (b) In one month, a student recorded the lengths, to the nearest minute, of each of the lectures she attended. The table below shows her data and the calculations she made before drawing a histogram to illustrate these data. Find the value of a and of b, and the interval c.

Length of lecture (minutes)	$50-53$	$54-55$	c	$60-67$
Number of lectures	20	a	30	12
Frequency density	5	13	7.5	b

   (c) i. Integrate

$$\int \frac{2x+1}{x^3-7x+6}\ dx.$$

   ii. Evaluate

$$\int_0^{\frac{\pi}{4}} \sin^2 x\ \ dx.$$

[5, 5, 15]

7. (a) i. When is a function $f(x)$ said to be even.

   ii. Determine whether

$$f(x) = \frac{x^2+1}{x^3-x},$$

   is odd, even, or neither.

   (b) Given that $f(x) = -2x^2 - 3x + 2$,

   i. Express $f(x)$ in the form

$$f(x) = a(x-h)^2 + k.$$

   ii. Determine the interval(s) where $f(x)$ is increasing and where it is decreasing.

   iii. Hence, or otherwise, sketch the graph of $f(x)$, labelling it clearly.

   (c) The following are test marks for a group of 120 MAT1110 students.

Mass(g)	$0-9$	$10-19$	$20-29$	$30-49$	$50-79$
Number of letters	8	21	53	28	10

i. Complete the frequency distribution table below.

Mass(g)	Interval width	Frequency	Frequency density
$-0.5 - 9.5$	10	8	0.8
$9.5 - 19.5$	10	21	2.1

ii. Draw a histogram to show the distribution of the test marks.

$[5, 12, 8]$