The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

September 30, 2022

MAT 1110:Foundation Mathematics and Statistics for Social Sciences

Tutorial Sheet 10: Introduction to Probability

1. (a) A label identifier for a company consist of three of the five vowels followed by four digits. How many distinct label identifiers are possible

   i. if repetitions are not allowed.

   ii. if repetitions are allowed.

   (b) A bank PIN is a string of digits e.g 19301 or 017038. A new bank wishes to generate PINs of the same length for its clients. Given that the bank has 9 million clients and that no two clients must have the same PIN, how many didgits must each PIN have to allow sufficient PINs for every client.

   (c) Our country's car license plates consist of three letter followd by four digits.

   i. How many license plates can be printed if repetition of letters and digits are allowed?

   ii. How many license plates can be printed if repetion of letters and digits are not allowed?

   iii. How many license plates begin with c and end with the digit zero?

   (d) In how many different ways can a President, Vice President, Treasurer Secretary, and Auditor be selected from a group of 39 people.

   (e) i. How many three digit numbers can be formed from the six digits 1,3,5,6,7 and 9.

   ii. How many of these numbers are less than 400?

   iii. How many are odd?

   iv. How many are even?

   v. How many are multiples of 5?

   (f) Solve for n if (i) ${}_nP_2 = 110$ (ii) ${}_nP_4 = 272{}_nP_2$.

(g) Evaluate each of the following (i) $(9-3)!$ (ii) ${}_{30}P_4$ (iii) ${}_9P_6$ (iv) ${}_7C_4$ (v) ${}_4C_0$ (vi) $\binom{10}{7}$ (vii) $\binom{100}{100}$.

2. (a) i. A die is rolled, find the probability that an prime number is obtained.

   ii. A die is rolled and a coin is tossed, find the probability that the die shows an even number and the coin shows a tail.

   (b) An ordinary die is thrown. Find the probability that the number obtained is (i) a multiple of 2. (ii) a factor of 6. (iii) less than 7.

   (c) Two dice are rolled, find the probability that the sum of the two scores is (i) equal to 13 (ii) equal to 4 (ii) less than 10 (iv) greater than 10

   (d) A jar contains 4 red marbles, 6 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that (i) this marble is green? (ii) this marble is not white?

3. (a) A die is rolled. What is the probability of getting

   i. an odd number or even number?

   ii. an even number or a number greater that 4.

   (b) In a class of 20 students, 4 of the 9 boys and 3 of the 11 girls are in the athletics team.

   A person from the class is chosen to participate in the " egg and spoon race" on Sports Day. Find the probability that the person chosen is

   i. in the athletics team.

   ii. female.

   iii. a female member of the athletics team.

   iv. a female or in the athletics team.

   (c) Events A and B are such that $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{10}$. Find (i) $P(A \cap B^c)$. (ii) $P(A^c \cap B^c)$.

4. (a) Events A and B are independent such that $P(A) = 0.3$, $P(B) = 0.5$. Find (i) $P(A \cap B)$ (ii) $P(A \cup B)$

(b) Events A and B are such that $P(A|B) = 0.4$, $P(B|A) = 0.25$ and $P(A \cap B) = 0.12$.

i. Calculate the value of $P(B)$.

ii. Explain why A and B are not independent.

iii. Calculate the value of $P(A \cap B^c)$.

5. (a) A die was thrown and the score was an odd number. What is the probability that it was a prime number?

(b) In a certain University, 65% of the students are full-time students, 55% of the students are female, 35 % of the students are male full-time students, Find the probability that

i. a student chosen at random from all the students in the University is a part-time student.

ii. a student chosen at random from all the students in the University is a female and a part-time student.

iii. a student chosen at random from all the female students in the University is a part-time student.

(c) Events X and Y are such that $P(X|Y) = \frac{2}{5}$, $P(Y) = \frac{1}{4}$ and $P(X) = \frac{1}{5}$. Find (i) $P(Y|X)$. (ii) $P(Y \cap X)$, (iii) $P(Y \cup X)$

6. A diagnostic test has a probability of 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a (false) positive when applied a non-sufferer. It is estimated that 0.5% of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart froom the fact that he/she comes from this population). Calculate the following probability:

(a) that the test result will be positive;

(b) that, given a positive result, the person is a suffurer;

(c) that, given a negative result, the person is a non-sufferer;

(d) that the person will be misclassified.