- $\int a \, dx = ax + C$
- $\int \frac{1}{x} \, dx = \ln |x| + C$
- $\int e^x \, dx = e^x + C$
- $\int a^x \, dx = \frac{e^x}{\ln a} + C$
- $\int \ln x \, dx = x \ln x - x + C$
- $\int \sin\, x \, dx = -\cos\, x + C$
- $\int \cos\, x \, dx = \sin\, x + C$
- $\int \tan \, dx = \ln |\sec\, x| + C \; or \; -\ln |\cos\, x| + C$
- $\int \cot\, x \, dx = \ln |\sin\, x| + C$
- $\int \sec\, x \, dx = \ln |\sec\, x + \tan\, x| + C$
- $\int \csc\, x \, dx = \ln |\csc\, x - \cot\, x| + C$
- $\int \sec^2\, x \, dx = \tan\, x + C$
- $\int \sec\, x\, \tan\, x \, dx = \sec\, x + C$
- $\int \csc^2\, x \, dx = -\cot\, x + C$
- $\int \tan^2\, x \, dx = \tan\, x - x + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$
- $\int \frac{dx}{\sqrt{a^2 + x^2}} = \frac{1}{a} \arcsin\left(\frac{x}{a}\right) + C$

MATH Tutorial Sheet 10 Answers | Integration & 2022.

Question 1. Substitution method.

1) a. $\int \frac{x+1}{\sqrt{2x^2+4x}}\, dx$

let $u = 2x^2+4x$

$du = 4x+4\, dx$

$dx = \frac{du}{4x+4} \rightarrow \frac{du}{4(x+1)}$

$\therefore \int \frac{\cancel{x+1}}{\sqrt{u}} \cdot \frac{du}{4\cancel{(x+1)}}$

$= \int \frac{1}{u^{1/2}} \cdot \frac{1}{4} du$

$= \frac{1}{4} \int u^{-1/2} du$

$= \frac{1}{4} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{\cancel{2} u^{\frac{1}{2}}}{\cancel{4}2} + C$

$= \frac{u^{\frac{1}{2}}}{2} + C$

$= \frac{(2x^2+4x)^{\frac{1}{2}}}{2} + C$

$\therefore \int \frac{x+1}{\sqrt{2x^2+4x}}\, dx = \frac{\sqrt{2x^2+4x}}{2} + C$

(b) $\int 2x \sin(x^2+1)\, dx$

$u = x^2+1$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$\therefore \int \cancel{2x} \sin u \cdot \frac{du}{\cancel{2x}}$

$= \int \sin u \cdot \frac{1}{\cancel{x}} du$

$= \cancel{\frac{1}{x}} \int \sin u\, du$

$= \cancel{\frac{1}{x}} \cdot -\cos u + C$

Ans $= -\cos(x^2+1) + C$

c) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\, dx$

$u = \sqrt{x} \rightarrow x^{\frac{1}{2}}$

$du = \frac{\cancel{\frac{1}{2}} x^{-\frac{1}{2}}}{\cancel{2}\cancel{\frac{1}{2}}} \rightarrow \frac{1}{2x^{\frac{1}{2}}} \rightarrow \frac{1}{2u} dx$

$dx = 2u\, du$

$\therefore \int \frac{\cos u}{\cancel{u}} \cdot 2\cancel{u} du$

$= \int \cos u \cdot 2\, du$

$= 2 \int \cos u\, dx$

$= 2 \cdot \sin u + C$

$= 2 \sin \sqrt{x} + C$

$\therefore \int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\sin\sqrt{x} + C$

1 (d) $\int 2x e^{x^2-3}\, dx$

$u = x^2 - 3$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$\therefore \int 2x e^{u} \cdot \frac{du}{\cancel{2x}}$

$= \int e^u\, du$

$= e^u + C$

$= e^{x^2-3} + C$

$\therefore \int 2x e^{x^2-3} dx = e^{x^2-3} + C$

(e) $\int \frac{\cos x}{x + \sin x} dx$.

Sorry !!! this one am dataless!

(f) $\int e^{\sin x} \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx \rightarrow dx = \frac{du}{\cos x}$

$\therefore \int e^u \cos x \cdot \frac{du}{\cos x}$

$= \int e^u\, du$

$= e^u + C$

$= e^{\sin x} + C$

$\therefore \int e^{\sin x} \cos x\, dx = e^{\sin x} + C$

9) $\int_0^5 x^3 \sqrt{x^4+1}\, dx$

$u = x^4 + 1$

$du = 4x^3 dx \rightarrow dx = \frac{du}{4x^3}$

$\int_0^5 \cancel{x^3} \sqrt{u} \cdot \frac{du}{4\cancel{x^3}}$

$\int_0^5 u^{\frac{1}{2}} \cdot \frac{1}{4} du$

$= \frac{1}{4} \int_0^5 u^{\frac{1}{2}} du$

$= \frac{1}{\cancel{4}_2} \cdot \frac{\cancel{2} u^{\frac{3}{2}}}{3} + c \Big]_0^5$

$= \frac{u^{\frac{3}{2}}}{6} + c \Big]_0^5$

$= \frac{(x^4+1)^{\frac{3}{2}}}{6} + c \Big]_0^5$

$= \left[\frac{(5^4+1)^{\frac{3}{2}}}{6}\right]^5 - \left[\frac{(0^4+1)^{\frac{3}{2}}}{6}\right]_0$

$= \frac{(626)^{\frac{3}{2}}}{6} - \frac{1}{6}$

$= \frac{(\sqrt{626})^3 - 1}{6}$ units.

$\therefore \int_0^5 x^3 \sqrt{x^4+1} dx = \frac{(\sqrt{626})^3 - 1}{6}$ unit

(e) $\int \frac{\cos x}{1 + \sin x} dx$ $u = 1 + \sin x$

$du = \cos x\, dx$

$dx = \frac{du}{\cos x}$

$\int \frac{\cos x}{u} \cdot \frac{du}{\cos x}$

$= \int \frac{1}{u} du$

$= \int u^{-1}$

$= \ln |u| + C$

$= \ln |1 + \sin x| + C$

$\therefore \int \frac{\cos x}{1 + \sin x} dx = \ln |1 + \sin x| + C$

Question two

Show that

(a) $\int_1^2 \frac{x^7-2x+1}{4x^3}\,dx = \frac{223}{160}$

$= \frac{x^7}{4x^3} - \frac{2x}{4x^3} + \frac{1}{4x^3}$

$= \frac{x^4}{4} - \frac{1}{2x^2} + \frac{1}{4x^3}$

$= \frac{x^4}{4} - \frac{x^{-2}}{2} + \frac{x^{-3}}{4}$

$\int_0^2 \frac{x^4}{4} - \frac{x^{-2}}{2} + \frac{x^{-3}}{4}\,dx$

$= \frac{x^5}{20} + \frac{x^{-1}}{2} - \frac{x^{-2}}{8} + c\Big]_1^2$

$= \frac{x^5}{20} + \frac{1}{2x} - \frac{1}{8x^2} + c\Big]_1^2$

$= \left[\frac{2^5}{20} + \frac{1}{4} - \frac{1}{32}\right]^2 - \left[\frac{1}{20} + \frac{1}{2} - \frac{1}{8}\right]_{+1}$

$= \frac{32}{20} + \frac{1}{4} - \frac{1}{32} - \left(\frac{1}{20} + \frac{1}{2} - \frac{1}{8}\right)$

$= \frac{256+40-5}{160} - \left(\frac{8+80-20}{160}\right)$

$= \frac{291}{160} - \frac{68}{160}$

$= \frac{291-68}{160}$

$= \frac{223}{160}$ units hence shown

(b) $\int_0^1 \frac{x}{(2x^2+1)^3}\,dx = \frac{1}{9}$

$u = 2x^2+1$

$du = 4x\,dx$

$dx = \frac{du}{4x}$

$\int_0^1 \frac{\cancel{x}}{(u)^3} \cdot \frac{du}{4\cancel{x}}$

$= \int_0^1 \frac{1}{u^3} \cdot \frac{1}{4}du$

$= \frac{1}{4}\int_0^1 u^{-3}\,du$

$= \frac{1}{4} \cdot \frac{-u^{-2}}{2} + c\Big]_0^1$

$= \frac{-u^{-2}}{8} + c\Big]_0^1$

$= \frac{-1}{8u^2} + c\Big]_0^1$

$= \frac{-1}{8(2x^2+1)^2} + c\Big]_0^1$

$= \left.\frac{-1}{8(2+1)^2}\right]^1 - \left[\frac{-1}{8(0+1)^2}\right]_0$

$= \frac{-1}{8(9)} + \frac{1}{8}$

$= \frac{-1}{72} + \frac{1}{8}$

$= \frac{-1+9}{72}$

$= \frac{8}{72}$

$= \frac{1}{9}$ units, Hence Shown

b) $\int_0^{\frac{\pi}{2}} \cos x \sin x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int_0^{\frac{\pi}{2}} \cos x \, u \frac{du}{\cos x}$

$= \int_0^{\frac{\pi}{2}} u \, du$

$= \left. \frac{u^2}{2} + c \right]_0^{\frac{\pi}{2}}$

$= \left. \frac{(\sin x)^2}{2} + c \right]_0^{\frac{\pi}{2}}$

$= \frac{(\sin 90)^2}{2} - \frac{(\sin 0)^2}{2}$

$= \frac{1}{2}$ units

$\therefore \int_0^{\frac{\pi}{2}} \cos x \sin x \, dx = \frac{1}{2}$ units

1) (i) $\int_{-1}^{1} (3x-1)\sqrt{3x^2-2x+3} \, dx$

$u = 3x^2 - 2x + 3$

$du = 6x - 2 \, dx$

$dx = \frac{du}{6x-2}$

$dx = \frac{du}{2(3x-1)}$

$\therefore \int_{-1}^{1} (3x-1)\sqrt{u} \cdot \frac{du}{2(3x-1)}$

$\int_{-1}^{1} \sqrt{u} \cdot \frac{1}{2} du$

$= \frac{1}{2} \int_{-1}^{1} u^{\frac{1}{2}} du$

$= \left. \frac{1}{2} \cdot \frac{2u^{\frac{3}{2}}}{3} + c \right]_{-1}^{1}$

$= \left. \frac{u^{\frac{3}{2}}}{3} + c \right]_{-1}^{1}$

$= \left. \frac{(3x^2-2x+3)^{\frac{3}{2}}}{3} + c \right]$

$= \left. \frac{(3-2+3)^{\frac{3}{2}}}{3} \right]^{1} - \left[\frac{(3+2+3)^{\frac{3}{2}}}{3} \right]_{-1}$

$= \frac{(\sqrt{4})^3}{3} - \frac{(\sqrt{8})^3}{3} \rightarrow (\sqrt{4} \cdot \sqrt{2})^3 = (2\sqrt{2})^3$

$= \frac{8 - (2\sqrt{2})^3}{3}$

$= \frac{8 - 8\sqrt{2}^3}{3} \rightarrow \sqrt{2} \times \sqrt{2} \times \sqrt{2}$

$= \frac{8 - 16\sqrt{2}}{3}$ units

$\therefore \int_{-1}^{1} (3x-1)\sqrt{3x^2-2x+3} \, dx = \frac{8-16\sqrt{2}}{3}$ units

Question two

Show that

(a) $\int_1^2 \frac{x^7-2x+1}{4x^3}\,dx = \frac{223}{160}$

$= \frac{x^7}{4x^3} - \frac{2x}{4x^3} + \frac{1}{4x^3}$

$= \frac{x^4}{4} - \frac{1}{2x^2} + \frac{1}{4x^3}$

$= \frac{x^4}{4} - \frac{x^{-2}}{2} + \frac{x^{-3}}{4}$

$\therefore \int_0^2 \frac{x^4}{4} - \frac{x^{-2}}{2} + \frac{x^{-3}}{4}\,dx$

$= \frac{x^5}{20} + \frac{x^{-1}}{2} - \frac{x^{-2}}{8} + c\Big]_1^2$

$= \frac{x^5}{20} + \frac{1}{2x} - \frac{1}{8x^2} + c\Big]_1^2$

$= \frac{2^5}{20} + \frac{1}{4} - \frac{1}{32}\Big]^2 - \Big[\frac{1}{20} + \frac{1}{2} - \frac{1}{8}\Big]_{+1}$

$= \frac{32}{20} + \frac{1}{4} - \frac{1}{32} - \left(\frac{1}{20} + \frac{1}{2} - \frac{1}{8}\right)$

$= \frac{256+40-5}{160} - \left(\frac{8+80-20}{160}\right)$

$= \frac{291}{160} - \frac{68}{160}$

$= \frac{291-68}{160}$

$= \frac{223}{160}$ units hence Shown

(b) $\int_0^1 \frac{x}{(2x^2+1)^3}\,dx = \frac{1}{9}$

$u = 2x^2+1$

$du = 4x\,dx$

$dx = \frac{du}{4x}$

$\int_0^1 \frac{\cancel{x}}{(u)^3} \cdot \frac{du}{4\cancel{x}}$

$= \int_0^1 \frac{1}{u^3} \cdot \frac{1}{4} du$

$= \frac{1}{4} \int_0^1 u^{-3}.\,du$

$= \frac{1}{4} \cdot \frac{-u^{-2}}{2} + c\Big]_0^1$

$= \frac{-u^{-2}}{8} + c\Big]_0^1$

$= \frac{-1}{8u^2} + c\Big]_0^1$

$= \frac{-1}{8(2x^2+1)^2} + c\Big]_0^1$

$= \frac{-1}{8(2+1)^2}\Big]^1 - \Big[\frac{-1}{8(0+1)^2}\Big]_0$

$= \frac{-1}{8(9)} + \frac{1}{8}$

$= \frac{-1}{72} + \frac{1}{8}$

$= \frac{-1+9}{72}$

$= \frac{8}{72}$

$= \frac{1}{9}$ units, Hence Shown

Question two

c) $\int_0^{\frac{\pi}{2}} \cos x \sin^5 x \, dx = \frac{1}{6}$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int_0^{\frac{\pi}{2}} \cos x \, u^5 . \frac{du}{\cos x}$

$= \int_0^{\frac{\pi}{2}} u^5 \, du$

$= \left. \frac{u^6}{6} + c \right]_0^{\frac{\pi}{2}}$

$= \left. \frac{(\sin x)^6}{6} + c \right]_0^{\frac{\pi}{2}}$

$= \frac{(\sin 90)^6}{6} - \frac{(\sin 0)^6}{6}$

$= \frac{1}{6} - 0$

$= \frac{1}{6}$ units, hence Shown

d) $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx = \frac{1}{2}$

$u = \sec x$

$du = \sec x \tan x \, dx$

$dx = \frac{du}{\sec x \tan x}$

$\therefore \int_0^{\frac{\pi}{4}} \tan x \, u^2 . \frac{du}{\tan x . u}$

$\int_0^{\frac{\pi}{4}} u^2 . \frac{du}{u}$

$\int_0^{\frac{\pi}{4}} u \, du$

$\left. \frac{u^2}{2} + c \right]_0^{\frac{\pi}{4}}$

$\left. \frac{(\sec x)^2}{2} \right]_0^{\frac{\pi}{4}}$

$\frac{(\sec 45)^2}{2} - \frac{(\sec 0)^2}{2}$

$= \frac{\left(\frac{1}{\cos 45°}\right)^2}{2} - \frac{\left(\frac{1}{\cos 0°}\right)}{2}$

$= \frac{\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}{2} - \frac{1}{2}$

$= \frac{\frac{1}{\frac{1}{2}}}{2} - \frac{1}{2}$

$= \frac{1}{1} \div \frac{2}{2} - \frac{1}{2}$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ units, hence Shown

Question 3 integration by part

a) (i) $\int x^2 \cos x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = x^2$, $dv = \cos x$

$du = 2x$ $\quad v = \sin x$

$= x^2 \sin x - \int 2x \sin x$

$= x^2 \sin x - [-2x\cos x - \int 2 \cdot -\cos x]$ $\quad u = 2x$

$= x^2 \sin x + 2x \cos x - \int 2\cos x$ $\quad du = 2.$

$dv = \sin x$

$v = -\cos x$

$= x^2 \sin x + 2x \cos x - 2\sin x + C$

(ii) $\int x^3 \sin x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = x^3$ $\quad dv = \sin x$

$du = 3x^2$ $\quad v = -\cos x$

$= -x^3 \cos x - \int -3x^2 \cos x$

$= -x^3 \cos x + \int 3x^2 \cos x$

$u = 3x^2$ $\quad dv = \cos x$

$du = 6x$ $\quad v = \sin x$

$= -x^3 \cos x + [3x^2 \sin x - \int 6x \sin x]$

$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x$

$u = 6x$ $\quad dv = \sin x$

$du = 6.$ $\quad v = -\cos x$

$= -x^3 \cos x + 3x^2 \sin x - [-6x\cos x - \int -6\cos x]$

$= -x^3 \cos x + 3x^2 \sin x + 6x\cos x - \int 6\cos x$

$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x + C$

iii) $\int \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln x$ $\quad dv = 1dx$

$du = \frac{1}{x}$ $\quad v = x$

$\int \ln x \, dx = x\ln x - \int x \cdot \frac{1}{x}$

$= x \ln x - \int 1$

$\int \ln x \, dx = x\ln x - x + C$

iv) $\int \frac{\ln x}{x^2} dx$

$= \int \frac{1}{x^2} \cdot \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln x$ $\quad dv = x^{-2} \rightarrow \frac{1}{x^2}$

$du = \frac{1}{x}$ $\quad v = -x^{-1} \rightarrow \frac{-1}{x}.$

$\therefore \int \frac{\ln x}{x^2} dx = \frac{-\ln x}{x} - \int \frac{1}{x} \cdot \frac{-1}{x}$

$= \frac{-\ln x}{x} - \int -\frac{1}{x^2}$

$= \frac{-\ln x}{x} + \int \frac{1}{x^2} \rightarrow \int x^{-2}$

$= \frac{-\ln x}{x} - \frac{1}{x} + C$

$\therefore \int \frac{\ln x}{x^2} dx = \frac{-\ln x}{x} - \frac{1}{x} + C$

3 (a) (v) $\int x^2 \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \cancel{x^2} \ln x \quad dv = \cancel{\ln} x^2$

$du = \cancel{2x}\frac{1}{x} \quad v = \frac{x^3}{3}$

$\int x^2 \ln x \, dx = \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}$

$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3}$

$\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

vi) $\int (x+1)^2 \ln 3x \, dx$

$u = \ln 3x \quad dv = (x+1)^2$

$du = \frac{1}{x} \quad v = \frac{x^3}{3} + x^2 + x + C$

$= \ln 3x\left(\frac{x^3}{3} + x^2 + x\right) - \int \frac{1}{x} \cdot \frac{x^3}{3} + x^2 + x$

$= \ln 3x\left(\frac{x^3}{3} + x^2 + x\right) - \int \frac{1}{\cancel{x}} \cdot \cancel{x}\left(\frac{x^2}{3} + x + 1\right)$

$= \ln 3x\left(\frac{x^3}{3} + x^2 + x\right) - \int \frac{x^2}{3} + x + 1$

$= \ln 3x\left(\frac{x^3}{3} + x^2 + x\right) - \frac{x^3}{9} - \frac{x^2}{2} - x + C$

vii) $\int x e^{2x} \, dx$

$\int u\,dv = uv - \int v\,du$

$u = x \quad dv = e^{2x}$

$du = 1 \quad v = \frac{e^{2x}}{2}$

$= \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2}$

$= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$

viii) $\int x^2 e^{4x} \, dx$

$u = x^2 \quad dv = e^{4x}$

$du = 2x \quad v = \frac{e^{4x}}{4}$

$= \frac{x^2 e^{4x}}{4} - \int \frac{2x e^{4x}}{4}$

$= \frac{x^2 e^{4x}}{4} - \int \frac{x e^{4x}}{2}$

$u = x \quad dv = \frac{e^{4x}}{2}$

$du = 1 \quad v = \frac{e^{4x}}{8}$

$= \frac{x^2 e^{4x}}{4} - \left[\frac{x e^{4x}}{4} - \int \frac{e^{4x}}{8}\right]$

$= \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{4} + \int \frac{e^{4x}}{8}$

$= \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{4} + \frac{e^{4x}}{32} + C$

$\therefore \int x^2 e^{4x} \, dx = \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{4} + \frac{e^{4x}}{32} + C$

3 ⓐ (ix) $\int e^{2x} \cos dx$

$u = e^{2x}$ $\quad dv = \cos x$

$du = 2e^{2x}$ $\quad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= e^{2x}\sin x - \int 2e^{2x}\sin x$

$u = 2e^{2x}$ $\quad dv = \sin x$

$du = 4e^{2x}$ $\quad v = -\cos x$

$= e^{2x}\sin x - \left[-2e^{2x}\cos x - \int 4e^{2x}\cos x\right]$

$= e^{2x}\sin x + 2e^{2x}\cos x - \int 4e^{2x}\cos x$

$\therefore \int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x$

$1\int e^{2x}\cos x + 4\int e^{2x}\cos x = e^{2x}\sin x + 2e^{2x}\cos x$

$\frac{5\int e^{2x}\cos x}{5} = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5}$

$\int e^{2x}\cos x = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5} + c$

x) $\int e^{-x}\sin 4x\,dx$

$u = \sin 4x$ $\quad dv = e^{-x}$

$du = 4\cos 4x$ $\quad v = -e^{-x}$

$\int u\,dv = uv - \int v\,du$

$= -e^{-x}\sin 4x - \int -e^{-x}\cdot 4\cos 4x$

$= -e^{-x}\sin 4x + 4\int e^{-x}\cos 4x$

$u = \cos 4x$ $\quad dv = e^{-x}$

$du = -4\sin 4x$ $\quad v = -e^{-x}$

$= -e^{-x}\sin 4x + 4\left[-e^{-x}\cos 4x - \int -e^{-x}\cdot -4\sin 4x\right]$

$= e^{-x}\sin 4x - 4e^{-x}\cos 4x - 4\int 4e^{-x}\sin 4x$

$= -e^{-x}\sin 4x - 4e^{-x}\cos 4x - 4\times 4\int e^{-x}\sin 4x$

$= -e^{-x}\sin 4x - 4e^{-x}\cos 4x - 16\int e^{-x}\sin 4x$

$\therefore \int e^{-x}\sin 4x\,dx = -e^{-x}\sin 4x - 4e^{-x}\cos 4x - 16\int e^{-x}$

$1\int e^{-x}\sin 4x + 16\int e^{-x}\sin 4x = -e^{-x}\sin 4x - 4e^{-x}\cos 4x$

$\frac{17\int e^{-x}\sin 4x}{17} = \frac{-e^{-x}\sin 4x - 4e^{-x}\cos 4x}{17}$

$\therefore \int e^{-x}\sin 4x = \frac{-e^{-x}\sin 4x - 4e^{-x}\cos 4x}{17} + c$

3 (b) (i) $\int_0^{\frac{1}{2}} x e^{2x} dx$

$\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^{2x}$

$du = 1 \qquad v = \frac{e^{2x}}{2}$

$= \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}$

$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} \Big]_0^{\frac{1}{2}}$

$= \left(\frac{1}{2}\frac{e^{2(\frac{1}{2})}}{2} - \frac{e^{2(\frac{1}{2})}}{4}\right)^{\frac{1}{2}} - \left(0 - \frac{e^0}{4}\right)_0$

$= \frac{e}{4} - \frac{e}{4} - \left(-\frac{1}{4}\right)$

$= \frac{1}{4}$ units

ii) $\int_0^{\frac{\pi}{4}} x \sin 2x \, dx$

$u = x \qquad dv = \sin 2x$

$du = 1 \qquad v = \frac{-\cos 2x}{2}$

$\int u\,dv = uv - \int v\,du$

$= -\frac{x\cos 2x}{2} - \int -\frac{\cos 2x}{2}$

$= \frac{-x\cos 2x}{2} + \int \frac{\cos 2x}{2}$

$= \frac{-x\cos 2x}{2} + \frac{\sin 2x}{4} \Big]_0^{\frac{\pi}{4}}$

$= \left(-\frac{\pi}{4} \cdot \frac{\cos \cancel{2}(\frac{\pi}{4})}{2} + \frac{\sin \cancel{2}(\frac{\pi}{4})}{4}\right)^{\frac{\pi}{4}} - (0)$

$= -\frac{\pi}{4} \cdot 0 + \frac{1}{4} \rightarrow \frac{1}{4}$ units.

$\therefore \int_0^{\frac{\pi}{4}} x \sin 2x \, dx = \frac{1}{4}$ units

iii) $\int_{\frac{1}{2}}^{1} x^4 \ln 2x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln 2x \qquad dv = x^4$

$du = \frac{1}{x} \qquad v = \frac{x^5}{5}$

$= \frac{x^5}{5} \ln 2x - \int \frac{x^5}{5} \cdot \frac{1}{x}$

$= \frac{x^5}{5} \ln 2x - \int \frac{x^4}{5}$

$= \frac{x^5}{5} \ln 2x - \frac{x^5}{25} + C \Big]_{\frac{1}{2}}^{1}$

$= \left[\frac{1}{5} \ln 2(1) - \frac{1}{25}\right]^1 - \left[\frac{1}{10} \ln 1 - \frac{1}{50}\right]_{\frac{1}{2}}$

$\ln 1 = 0$

$= \frac{1}{5} \ln 2 - \frac{1}{25} + \frac{1}{50}$

$= \frac{1}{5} \ln 2 - \frac{1}{50}$

iv) $\int_0^{\pi} 3x^2 \cos\left(\frac{x}{2}\right) dx$

$u = 3x^2 \qquad dv = \cos\left(\frac{x}{2}\right)$

$du = 6x \qquad v = 2\sin\left(\frac{x}{2}\right)$

$= 3x^2 \cdot 2\sin\left(\frac{x}{2}\right) - \int 6x \cdot 2\sin\left(\frac{x}{2}\right)$

$= 6x^2 \sin\left(\frac{x}{2}\right) - \int 12x \sin\left(\frac{x}{2}\right)$

$u = 12x \qquad dv = \sin\left(\frac{x}{2}\right)$

$du = 12 \qquad v = -2\cos\left(\frac{x}{2}\right)$

$= 6x^2 \sin\left(\frac{x}{2}\right) - \left[-24x\cos\left(\frac{x}{2}\right) - \int -24\cos\left(\frac{x}{2}\right)\right]$

$= 6x^2 \sin\left(\frac{x}{2}\right) + 24x\cos\left(\frac{x}{2}\right) - 48\sin\left(\frac{x}{2}\right)$

$= \left[6\pi^2 \sin 90 + 24\pi \cos 90 - 48 \sin 90\right]^{\pi} - [0]_0$

$= 6\pi^2 + 0 - 48(1)$

$= 6\pi^2 - 48$

$\therefore \int_0^{\pi} 3x^2 \cos\left(\frac{x}{2}\right) dx = 6\pi^2 - 48$ units

Question 4 intergration by partial

ⓐ (i) $\int \frac{x}{(2-x)(3+x)} dx$

$\frac{x}{(2-x)(3+x)} = \frac{A}{2-x} + \frac{B}{3+x}$

$x = A(3+x) + B(2-x)$

when $x = 2$ | when $x = -3$

$2 = 5A$ | $-3 = 5B$

$A = \frac{2}{5}$ | $B = \frac{-3}{5}$

$\therefore \int \frac{x}{(2-x)(3+x)} dx = \int \frac{\frac{2}{5}}{2-x} + \int \frac{-\frac{3}{5}}{3+x} dx$

$= \frac{2}{5}\int \frac{1}{2-x} - \frac{3}{5}\int \frac{1}{3+x} dx$

$\int \frac{x}{(2-x)(3+x)} dx = \frac{2}{5} \ln|2-x| - \frac{3}{5} \ln|3+x| + C$

(ii) $\int \frac{6x+13}{x^2+5x+6} dx$

$\frac{6x+13}{(x+3)(x+2)} dx = \frac{A}{x+3} + \frac{B}{x+2}$

$6x+13 = A(x+2) + B(x+3)$

When $x = -2$ | when $x = -3$

$1 = B$ | $-5 = -A$, $A = 5$

$\therefore \int \frac{x}{x^2+5x+6} dx = \int \frac{5}{x+3} + \int \frac{1}{x+2} dx$

$= 5\int \frac{1}{x+3} + 1\int \frac{1}{x+2}$

$\int \frac{x}{x^2+5x+6} = 5\ln|x+3| + \ln|x+2| + C$

iii) $\int \frac{3x+4}{(x+2)^2} dx$

$\frac{3x+4}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$

$3x+4 = A(x+2) + B$

when $x = -2$ | when $x = 1$

$-2 = B$ | $7 = 3A + B$

$7 = 3A - 2$

$3A = 7+2$

$3A = 9$

$A = 3$

$\therefore \int \frac{3x+4}{(x+2)^2} dx = \int \frac{3}{x+2} + \int \frac{-2}{(x+2)^2}$

$= 3\int \frac{1}{x+2} - 2\int \frac{1}{(x+2)^2}$

$= 3\ln|x+2| - 2\left(\frac{1}{-(x+1)}\right) + C$

$\int \frac{3x+4}{(x+2)^2} = 3\ln|x+2| + \frac{2}{x+1} + C$

v) $\int \frac{x+5}{(x+3)^3} dx$

$\frac{x+5}{(x+3)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$

$x+5 = A(x+3)^2 + B(x+3) + C$

when $x = -3$ | when $x = 0$

$2 = C$ | $5 = 9A + 3b + C$

$5 = 9A + 3b + 2$

when $x = 1$ | $5-2 = 9A + 3b$

$6 = 16A + 4B + C$ | $3 = 9A + 3b$ (i)

$6 = 16A + 4B + 2$ | $1 = 3A + B$ (i)

$4 = 16A + 4B$ (ii) $\rightarrow 1 = 4A + B$ (ii)

~~$3 = 9A + 3B =$~~ ii - i

$1 = 4A + B$ | $1 = B$

$-\ 1 = 3A + B$

$A = 0$

$\therefore \int \frac{x+5}{(x+3)^3} dx = \int \frac{1}{(x+3)^2} + \int \frac{2}{(x+3)^3}$

a) iv) Continiuation

$$\int \frac{x+5}{(x+3)^3}\,dx = \int \frac{1}{(x+3)^2} + \int \frac{2}{(x+3)^3}$$

$$= \int (x+3)^{-2} + 2\int (x+3)^{-3}$$

$$= \frac{(x+3)^{-1}}{-1} + \frac{\cancel{2}\cdot(x+3)^{-2}}{-\cancel{2}}$$

$$\therefore \int \frac{x+5}{(x+3)^3}\,dx = \frac{-1}{x+3} - \frac{1}{(x+3)^2} + C$$

v) $\cancel{\int \frac{x+5}{(x+3)^3}\,dx}$ $\int \frac{x^2+x+5}{x^3+2x^2+x}\,dx$

$$\frac{x^2+x+5}{x(x^2+2x+1)}$$

$$\frac{x^2+x+5}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2+x+5 = A(x+1)^2 + Bx(x+1) + Cx$$

when $x=0$ when $x=-1$

$5 = A$ $5 = -C \rightarrow C = -5$

when $x = 1$

$7 = 4A + 2B + C \rightarrow 7 = 20 + 2B - 5$

$7 - 15 = 2B$

$-8 = 2B$

$B = -4$

$$\therefore \int \frac{x^2+x+5}{x^3+2x^2+x}\,dx = \int \frac{5}{x} + \int \frac{-4}{x+1} + \int \frac{-5}{(x+1)^2}$$

$$= 5\int \frac{1}{x} - 4\int \frac{1}{x+1} - 5\int \frac{1}{(x+1)^2}$$

$$\int \frac{x^2+x+5}{x^3+2x^2+x}\,dx = 5\ln|x| - 4\ln|x+1| + \frac{5}{x+1} + C$$

4 (b) (i) $\int_1^2 \frac{3}{x(x+1)}\,dx$

$$\frac{3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$3 = A(x+1) + Bx$$

when $x = -1$ when $x = 0$

$3 = -B$ $3 = A$

$B = -3$

$$\therefore \int_1^2 \frac{3}{x(x+1)}\,dx = \int \frac{3}{x} + \int \frac{-3}{x+1}\,dx$$

$$= 3\int \frac{1}{x}\,\cancel{d} - 3\int \frac{1}{x+1}$$

$$= 3\ln|x| - 3\ln|x+1| \Big]_1^2$$

$$= \Big[3\ln 2 - 3\ln 3\Big]^2 - \Big[3\ln 1 - 3\ln 2\Big]_1$$

$$= 3\ln 2 - 3\ln 3 + 3\ln 2$$

$$= 6\ln 2 - 3\ln 3$$

$$= \ln 2^6 - \ln 3^3$$

$$= \ln \frac{2^6}{3^3}$$

$$= \ln \left(\frac{2^2}{3}\right)^3$$

$$= \ln \left(\frac{4}{3}\right)^3$$

$$= 3\ln\left(\frac{4}{3}\right) \text{ units}$$

$$\therefore \int_1^2 \frac{3}{x(x+1)}\,dx = 3\ln\left(\frac{4}{3}\right) \text{ units}$$

b (ii) $\int_1^2 \frac{1}{x(3x-1)^2}\,dx$

$$\frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{3x-1} + \frac{C}{(3x-1)^2}$$

$$1 = A(3x-1)^2 + Bx(3x-1) + Cx$$

when $x = 0$ $\quad$ when $x = \frac{1}{3}$

$1 = A$ $\quad$ $1 = \frac{C}{3} \rightarrow C = 3$

when $x = 1$

$1 = 4A + 2B + C \rightarrow 1 - 4 - 3 = 2B$

$-6 = 2B \rightarrow B = -3$

$$\therefore \int_1^2 \frac{1}{x(3x-1)^2}\,dx = \int \frac{1}{x} + \int \frac{-3}{3x-1} + \int \frac{3}{(3x-1)^2}$$

$$= \int \frac{1}{x} - 3\int \frac{1}{3x-1} + 3\int \frac{1}{(3x-1)^2}$$

$$= \ln x - 3\ln|3x-1| + 3\left(\frac{-1}{3x-1}\right)$$

$$= \ln x - 3\ln|3x-1| - \frac{3}{3x-1} + C$$

$$= \left[\ln 2 - 3\ln 5 - \frac{3}{5}\right]^2 - \left[\ln 1 - 3\ln 2 - \frac{3}{2}\right]_1$$

$$= \ln 2 - 3\ln 5 - \frac{3}{5} + 3\ln 2 + \frac{3}{2}$$

$$= \ln 2 + 3\ln 2 - 3\ln 5 - \frac{3}{5} + \frac{3}{2}$$

$$= 4\ln 2 - 3\ln 5 - \frac{9}{10}$$

$$= \ln 2^4 - \ln 5^3 - \frac{9}{10}$$

$$= \ln \frac{2^4}{5^3} - \frac{9}{10}$$

$$\therefore \int_1^2 \frac{1}{x(3x-1)^2}\,dx = \ln\left(\frac{4}{5}\right) + \frac{3}{10}$$

iii) $\int_0^2 \frac{1}{(x+1)(x+2)}$

$$\frac{1}{(x+1)(x+2)} \equiv \frac{A}{x+1} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+1)$$

when $x = -2$ $\quad$ when $x = -1$

$1 = -B$ $\quad$ $1 = A$

$B = -1$

$$\int_0^2 \frac{1}{(x+1)(x+2)} = \int \frac{1}{x+1} + \int \frac{-1}{x+2}$$

$$= \int \frac{1}{x+1} - 1\int \frac{1}{x+2}$$

$$= \ln|x+1| - \ln|x+2|\Big]_0^2$$

$$= \left[\ln|2+1| - \ln|2+2|\right]^2 - (\ln 1 - \ln 2)_0$$

$$= \ln 3 - \ln 4 + \ln 2$$

$$= \ln \frac{3}{\cancel{4}_2} \times \cancel{2}$$

$$= \ln \frac{3}{2}$$

$$\therefore \int_0^2 \frac{1}{(x+1)(x+2)}\,dx = \ln \frac{3}{2}.$$

iv) $\int_0^2 \frac{4-2x}{(x+1)(x+3)(2x+1)}\,dx$

$$\frac{4-2x}{(x+1)(x+3)(2x+1)} = \frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{2x+1}$$

$$\int \frac{4-2x}{(x+1)(x+3)(2x+1)} = \int \frac{-3}{x+1} + \int \frac{1}{x+3} + \int \frac{6/5}{2x+1}$$

$$= -3\ln|x+1| + \ln|x+3| + \frac{6}{5}\ln|2x+1|$$

$$= \left[-3\ln 3 + \ln 5 + \frac{6}{5}\ln 5\right]^2 - \left[\ln 4 + \frac{6}{5}\ln 1\right]_0$$

$$= -3\ln 3 + \frac{11}{5}\ln 5 - \ln 4$$

$$= \ln(5)^{\frac{11}{5}} - \ln 3^3 - \ln 4$$

$$= \ln(5)^{\frac{11}{5}} - \ln \frac{27}{4}$$

$$\therefore \int_0^2 \frac{4-2x}{(x+1)(x+3)(2x+1)} = \frac{11}{5}\ln 5 - \ln \frac{27}{4}$$

a) iv) Continuation

$$\int \frac{x+5}{(x+3)^3}\,dx = \int \frac{1}{(x+3)^2} + \int \frac{2}{(x+3)^3}$$

$$= \int (x+3)^{-2} + 2\int (x+3)^{-3}$$

$$= \frac{(x+3)^{-1}}{-1} + \frac{2\cdot(x+3)^{-2}}{-2}$$

$$\therefore \int \frac{x+5}{(x+3)^3}dx = \frac{-1}{x+3} - \frac{1}{(x+3)^2} + C$$

v) $\cancel{\int \frac{x+5}{(x+3)^3}dx}$ $\int \frac{x^2+x+5}{x^3+2x^2+x}\,dx$

$$\frac{x^2+x+5}{x(x^2+2x+1)}$$

$$\frac{x^2+x+5}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2+x+5 = A(x+1)^2 + Bx(x+1) + Cx$$

when $x=0$: $5 = A$

when $x=-1$: $5 = -C \rightarrow C = -5$

when $x=1$:

$7 = 4A + 2B + C \rightarrow 7 = 20 + 2B - 5$

$7 - 15 = 2B$

$-8 = 2B$

$B = -4$

$$\therefore \int \frac{x^2+x+5}{x^3+2x^2+x}dx = \int \frac{5}{x} + \int \frac{-4}{x+1} + \int \frac{-5}{(x+1)^2}$$

$$= 5\int \frac{1}{x} - 4\int \frac{1}{x+1} - 5\int \frac{1}{(x+1)^2}$$

$$\int \frac{x^2+x+5}{x^3+2x^2+x}dx = 5\ln|x| - 4\ln|x+1| + \frac{5}{x+1} + C$$

4 (b) (i) $\int_1^2 \frac{3}{x(x+1)}\,dx$

$$\frac{3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$3 = A(x+1) + Bx$$

when $x=-1$: $3 = -B$, $B = -3$

when $x=0$: $3 = A$

$$\therefore \int_1^2 \frac{3}{x(x+1)}dx = \int \frac{3}{x} + \int \frac{-3}{x+1}\,dx$$

$$= 3\int \frac{1}{x} - 3\int \frac{1}{x+1}$$

$$= 3\ln|x| - 3\ln|x+1| \Big]_1^2$$

$$= \left[3\ln 2 - 3\ln 3\right] - \left[3\ln 1 - 3\ln 2\right]$$

$$= 3\ln 2 - 3\ln 3 + 3\ln 2$$

$$= 6\ln 2 - 3\ln 3$$

$$= \ln 2^6 - \ln 3^3$$

$$= \ln \frac{2^6}{3^3}$$

$$= \ln\left(\frac{2^2}{3}\right)^3$$

$$= \ln\left(\frac{4}{3}\right)^3$$

$$= 3\ln\left(\frac{4}{3}\right) \text{ units}$$

$$\therefore \int_1^2 \frac{3}{x(x+1)}dx = 3\ln\left(\frac{4}{3}\right) \text{ units}$$

Question 5

(a) $\int \sin^2 2x \, dx$, let $A = 2x$

from double angle formula

$\cos 2A = 1 - 2\sin^2 A$

$\frac{2\sin^2 A}{2} = \frac{1-\cos 2A}{2}$

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\therefore \int \sin^2 A = \frac{1}{2}\int (1-\cos 2A)\,dx$

$= \frac{1}{2}\left(\int 1 - \int \cos 2(2x)\right)$

$= \frac{1}{2}\left(x - \frac{\sin(4x)}{4}\right) + c$

$\therefore \int \sin^2 2x = \frac{1}{2}x - \frac{\sin 4x}{8} + C$

(b) $\int \sin 3x \cos 3x \, dx$

$u = \sin 3x$

$du = 3\cos 3x \, dx$

$dx = \frac{du}{3\cos 3x}$

$\int u \cos 3x \cdot \frac{du}{3\cos 3x}$

$= \frac{1}{3}\int u \, du$

$= \frac{1}{3} \cdot \frac{u^2}{2} + C$

$= \frac{1}{3} \frac{(\sin 3x)^2}{2} + C$

$= \frac{\sin^2 3x}{6} + C$

c) $\int \sin 3x \cos 2x \, dx$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

substitute

$\sin 3x \cos 2x = \frac{1}{2}[\sin(3x+2x) + \sin(3x-2x)]$

$= \frac{1}{2}[\sin(5x) + \sin x]$

$= \frac{1}{2}\int \sin 5x + \frac{1}{2}\int \sin x$

$= \frac{1}{2} \cdot -\frac{\cos 5x}{5} + \frac{1}{2} \cdot -\cos x$

$= \frac{-\cos 5x}{10} - \frac{\cos x}{2} + C$

$\therefore \int \sin 3x \cos 2x \, dx = \frac{-\cos 5x}{10} - \frac{\cos x}{2} + C$

(d) $\int (\sec x + \tan x)^2 dx$

$= (\sec x + \tan x)(\sec x + \tan x)$

$= (\sec^2 x + 2\tan x \sec x + \tan^2 x)$

$= (\sec^2 x + 2\tan x \sec x + (\sec^2 x - 1))$

$= (2\sec^2 x + 2\tan x \sec x - 1)$

$= 2\int \sec^2 x + 2\int \tan x \sec x - \int 1$

$= 2\tan x + 2\sec x - x + C$

$\therefore \int (\sec x + \tan x)^2 dx = 2\tan x + 2\sec x - x + C$

#; Solved By SINAMBILI

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