

The University of Zambia

Department of Mathematics & Statistics

2017/2018 Academic Year, Final Examinations

MAT1110: Foundation Mathematics & Statistics for Social Sciences

Tuesday 20<sup>th</sup> November 2018 (09 : 00 – 12 : 00)

Time Allowed: **3 hours**

**Instructions:**

1. There are **seven questions** in this examination. Attempt **any five (5)**.
2. Indicate your **computer number** on all your answer booklets.
3. **Full credit** will only be given when necessary working is shown.
4. **Calculators** are not allowed.

This examination has (4) **pages** of questions.

- 
1. (a) Let  $\mathbb{R}$  be the universal set and let  $A = [-7, 3)$ ,  $B = \{x : x \leq 10, x \in \mathbb{R}\}$ .  
Find the set

$$B - A,$$

and display your answer on the number line.

- (b) i. Simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form  $a\sqrt{2}$ , where  $a$  is an integer.

- ii. Hence, otherwise, simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your in answer in the form  $b + c\sqrt{2}$ , where  $b$  and  $c$  are integers.

- (c) Express the following in its simplest form:

$$[(A \cap B)^c \cup (A - B)]^c.$$

(d) Express

$$\frac{(-2 + 3i)^2}{1 + 2i},$$

in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ .

[5, 6, 4, 5]

2. (a) Given that

$$f(x) = x - 4 \text{ and } g(x) = \frac{3}{x + 1},$$

- i. Find the domain and range of  $g(x)$ .
- ii. Find  $(f \circ g)^{-1}(2)$  leaving your answer in simplest form.

(b) Given that  $f(x) = 2x^2 - 4x + 5$ ,

- i. Write  $f(x)$  in the form

$$f(x) = a(x - h)^2 + k,$$

where  $(h, k)$  is the turning point.

- ii. Hence, otherwise, sketch the graph of  $f(x)$ .

(c) Given that  $(x - 1)$  and  $(x + 1)$  are factors of

$$f(x) = ax^3 + bx^2 - 3x - 7,$$

- i. find the value of  $a$  and of  $b$ .
- ii. factorize  $f(x)$  completely.

[7, 6, 7]

3. (a) Solve the equation

$$\sqrt{3} \tan x + 1 = 0, \text{ for } 0^\circ \leq x \leq 360^\circ.$$

(b) Prove the following identities:

i.

$$\frac{\cos x}{1 - \sin x} - \tan x = \sec x.$$

ii.

$$\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x.$$

(c) Given that

$$f(x) = 3 \sin(x + 60^\circ) \text{ for } 0^\circ \leq x \leq 360^\circ,$$

- i. Find the period, amplitude and phase shift.

ii. Hence, otherwise, sketch the graph of  $f(x) = 3 \sin(x + 60^\circ)$ .

[4, 8, 8]

4. (a) Given that

$$f(x) = \frac{x^2 + 2}{(x + 2)^2 (x + 3)},$$

Write down the partial fraction decomposition of  $f(x)$ .

(b) Solve each of the following equations

i.

$$\left(\frac{1}{2}\right)^x = 0.125.$$

ii.

$$\log_4(-x) + \log_4(6 - x) = 2.$$

(c) Sketch the graph of

$$y = 3^x - 1.$$

(d) Solve the equation

$$4x^3 - 12x^2 + 5x + 6 = 0.$$

[6, 7, 3, 4]

5. (a) Evaluate each of the following:

i.

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{2x^2 - 5x + 2}.$$

ii.

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+3} - \sqrt{3}}.$$

(b) Differentiate

i.

$$y = \sqrt{3x^2 + 4x}.$$

ii.

$$y = \frac{3x^2 + 6x - 7}{(x + 1)^2}, \quad x \neq -1,$$

(c) Integrate

i.

$$\int x e^{x^2} dx$$

ii.

$$\int x^2 \cos x dx$$

.

[7, 6, 7]

6. (a) Given that

$$f(x) = x(x^2 - 4),$$

- i. Find the turning point(s) and determine whether maximum or minimum.
- ii. Hence, otherwise, sketch the graph of  $f(x)$ .

(b) i. Evaluate

$$\int_0^1 \frac{x}{(2x^2 + 1)^2} dx.$$

- ii. Find the area bounded by the  $x$ -axis and the curve

$$y = x^2 - 4x + 3.$$

(c) Events  $A$  and  $B$  are independent such that  $P(A) = \frac{3}{10}$  and  $P(B) = \frac{1}{2}$ . Find

$$P(A \cup B).$$

[8, 9, 3]

7. (a) A jar contains 3 red marbles, 4 green marbles and 8 white marbles. If a marble is drawn from the jar at random, what is the probability that it is not red?

(b) Two dice are thrown. What is the probability of scoring a double given that the total on the two dice is greater than 10?

(c) In one month, a student recorded the lengths, to the nearest minute, of each of the lectures she attended. The table below shows her data and the calculations she made before drawing a histogram to illustrate these data. Find the value of  $a$  and of  $b$ .

Length of lecture (minutes)	50 – 53	54 – 55	56 – 59	60 – 67
Number of lectures	20	$a$	30	12
Frequency density	5	13	7.5	$b$

(d) Find the mean of the following data:

Class interval	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	6	4	7

[3, 6, 5, 6]