1 a (i) Each label identifier is of the form $x_1 x_2 x_3 d_1 d_2 d_3 d_4$, where for $i = 1, 2, 3$, x_i is a vowel and for $i = 1, 2, 3, 4$, d_i is a digit.

There are 5 choices for x_1, i.e $\binom{5}{1}$, 4 choices for x_2 i.e $\binom{4}{1}$, and 3 choices for x_3 i.e $\binom{3}{1}$.

There are 10 i.e $\binom{10}{1}$ choices for d_1
9 i.e $\binom{9}{1}$ " " d_2
8 for d_3 and 7 for d_4

Therefore, by the multiplication principle, there are

$5 \times 4 \times 3 \times 10 \times 9 \times 8 \times 7 =$

302,400 distinct label identifiers possible.

(ii) There are $5 \times 5 \times 5 \times 10 \times 10 \times 10 \times 10$ distinct label identifiers possible.

b) Let n be the number of digit of each PIN. The number of PINS is therefore

$$\underbrace{10 \times 10 \times 10 \times \cdots \times 10}_{n \text{ times}} = 10^n$$

We want $10^n \geq 9{,}000{,}000$

Thus $n \geq \dfrac{\log 9{,}000{,}000}{\log 10}$

$= 6.95$. It follows that each PIN must have at least 7 digits

1 c (i) $26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10$

(ii) $26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$

(iii) $1 \times 26 \times 26 \times 10 \times 10 \times 10 \times 1$

Plates begin with C and end with 0

d) $39 \times 38 \times 37 \times 36 \times 35 = {}^{39}P_5$

e (i) When need the number of 3 tuples of the form (a, b, c) where $a, b, c \in \{1, 3, 5, 6, 7, 9\}$. This is

$\binom{6}{1} \times \binom{6}{1} \times \binom{6}{1} = 6 \times 6 \times 6 = 216$

(ii) A number abc is less than 400 if ~~a<4~~, a is either 1 or 3. Thus are $2 \times 6 \times 6 = \binom{2}{1} \times \binom{6}{1} \times \binom{6}{1}$

$= 72$ numbers

(ii) Here c is in $\{1, 3, 5, 7, 9\}$.

Thus there are $6 \times 6 \times 5 = 180$

odd numbers

(iii) Here c must be 6, Thus there

are $6 \times 6 \times 1 = 36$ numbers.

(iv) Here c must be 5 and

nothing else, Thus there

are $6 \times 6 \times 1 = 36$ such

numbers.

1f (i) ${}_nP_2 = 110$

$$\frac{n!}{(n-2)!} = 110 \Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 110$$

$\Rightarrow n(n-1) = 110 \Rightarrow n^2 - n = 110$

$\Rightarrow n^2 - n - 110 = 0 \Rightarrow (n+10)(n-11) = 0$

$n = -10$ Not applicable

$\therefore n = 11$

(ii) ${}^{n}P_4 = 272\ {}^{n}P_2$

$$\frac{n!}{(n-4)!} = 272\left(\frac{\cancel{n!}}{(n-2)!}\right)$$

$$\frac{(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} = 272$$

$n^2 - 3n - 2n + 6 = 272$

$n^2 - 5n - 266 = 0$

$(n+14)(n-19) = 0$

Hence $n = 19$

2 a (i) Let A be the event of obtaining a prime number, then

$A = \{2, 3, 5\}$

$n(A) = 3$

S = Sample space $= \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = 3/6 = 1/2$

(ii) Let A be the event of an even number and having a tail, then $A = \{(2, T), (4, T), (6, T)\}$, $n(A) = 3$

Sample space $S = \{(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T)$

$(5,H), (5,T), (6,H), (6,T) \}$

Thus $n(S) = 12 = \binom{6}{1} \times \binom{2}{1} = 6 \times 2$

$\therefore P(A) = \frac{3}{12} = \frac{1}{6}$

2(B)(i) ~~Let~~ See example in notes

Let A be the event of having a multiple of 2

then $A = \{2, 4, 6\}$, $S = \{1,2,3,4,5,6\}$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) Let A be event of a factor of 6, then $A = \{1,2,3,6\}$

$\therefore P(A) = \frac{4}{6}$

(iii) Let A be event of having number less than 7, then

$A = \{1,2,3,4,5,6\}$, $\therefore P(A) = 1$

2 Q(i) Let A be the event that the sum of two scores is equal to 13, then

~~A = {~~ $A = \{\ \}$

$\therefore P(A) = 0$

(ii) Let A be the event that the sum of two scores is equal to 4, then

$A = \{(1,3), (2,2), (3,1)\}$

$n(A) = 3$

$n(S) = 6 \times 6 = \binom{6}{1} \times \binom{6}{1} = 36$

$\therefore P(A) = \frac{3}{36} = \frac{1}{12}$

C (iii) Let A be the event that the sum of two scores is less than 10, then

$A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1),$
$(4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3),$
$(5,4), (6,1), (6,2), (6,3)\}$

$n(A) = 30$

$n(S) = 36$

$\therefore P(A) = \frac{30}{36} = \frac{10}{12} = \frac{5}{6}$

(iv) Let A be event of the sum being greater ~~than~~ 10, the

$A = \{(5,6), (6,5), (6,6)\}, \; n(A) = 3$

$n(s) = 36$

$\therefore P(A) = \frac{3}{36} = \frac{1}{12}$

2d (i) $P(A) = \frac{6}{4+6+10} = \frac{6}{20} = \frac{3}{10}$

Where A is the event of being green and S, sample space is all the marbles.

(ii) Let W be the event of a marble being white, then

$P(W) = \frac{10}{20} = \frac{1}{2}$

$P(W') = 1 - P(W) = 1 - \frac{1}{2} = \frac{1}{2}$

3 (a)(i) Let A be the event of having an odd number and B be the event of having an ~~odd~~ even number

then $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$

We seek $P(A \cancel{\text{or}} B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

Now $n(A) = 3$, $\therefore P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{\cancel{2}6} = \frac{1}{2}$

$n(A \cap B) = 0$, $\therefore P(A \cap B) = 0$

Hence $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{2} - 0$

$= \underline{1}$

3a ii) Let A be event of an even number, then $A = \{2, 4, 6\}$

and $n(\cancel{A}) = 3 \Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

Let B be the event of getting a number greater than 4, then

$B = \{5, 6\}$, $n(B) = 2$ and $P(B)$

$= \frac{2}{6} = \frac{1}{3}$. We seek

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

~~and $\frac{1}{2} + \frac{1}{3}$~~

$A \cap B = \{6\}$, $\Rightarrow n(A \cap B) = 1$

and $P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6}$

$= \frac{4}{6} = \frac{2}{3}$

3 b)

	Athlete (A)	Non-Athlete N	Total
Boys (B)	4	5	9
Girls (G)	3	8	11
Total	7	13	20

b(i) Number of ~~elem~~ outcomes in the event A of being an athlete is 7. Thus

$P(A) = 7/20$

(ii) Number of outcomes in the event G of being female is 11

$\therefore\ P(G) = 11/20$

(iii) We seek $P(G/A)$

$= 3/7$, since athlete has already occured or $P(G/A) = \frac{P(G \cap A)}{P(A)} = \frac{3/20}{7/20}$

$= 3/20 \times \frac{20}{7} = 3/7$.

(iv) $P(G \cup A) = P(G) + P(A) - P(G \cap A)$

$= \frac{11}{20} + \frac{7}{20} - \left(\frac{3}{20}\right)$

$= \frac{11+7-3}{20} = \frac{15}{20} = \frac{3}{4}$

3 C(i) $P(A \cap B^c) = P(B^c/A)P(A)$

but $P(B^c/A) = 1 - P(B/A)$

Now, $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/10}{3/10}$

$= \frac{1}{3}$, Thus $P(B^c/A) = 1 - 1/3 = 2/3$

Hence $P(A \cap B^c) = (2/3)(3/10) = \frac{2}{10} = \frac{1}{5}$

3a (ii) $P(A^c \cap B^c) = P((A \cup B)^c)$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [\cancel{0.3 + 0.4} \; \frac{3}{10} + \frac{2}{5}$

$- \frac{1}{10}] = 1 - \left[\frac{3+4-1}{10}\right]$

$= 1 - \left(\frac{6}{10}\right) = \frac{4}{10}$

4a(i) $P(A \cap B) = P(A/B)\,P(B)$

$= P(A)\,P(B)$

$= (0.3)(0.5)$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (0.3) + (0.5) - 0.15$

4b) (i) $\cancel{P(B)} \cancel{=}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow$$

$$P(B) = \frac{P(A \cap B)}{P(A/B)} = \frac{0.12}{0.4}$$

$$= \frac{12}{40} = \frac{6}{20} = \frac{3}{10}$$

(ii) Events A and B are not independent since

$$0.30 = P(B) \neq P(B/A) = 0.25$$

(iii) $$P(A \cap B^c) = \frac{P(B^c/A)\,P(A)}{\cancel{P(A)}}$$

Now $$P(B^c/A) = 1 - P(B/A)$$

$$= 1 - 0.25$$

$$= 0.75$$

$$P(A) = \frac{P(A \cap B)}{P(B/A)} = \frac{0.12}{0.25} = \frac{12}{25}$$

$$\therefore P(A \cap B^c) = (0.75)\left(\frac{12}{25}\right)$$

5 a and c Try on your own. Don't do 5 b because the concept there was not taught.