MAT1110 2022 TEST 1 SOLUTIONS

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QUESTION ONE

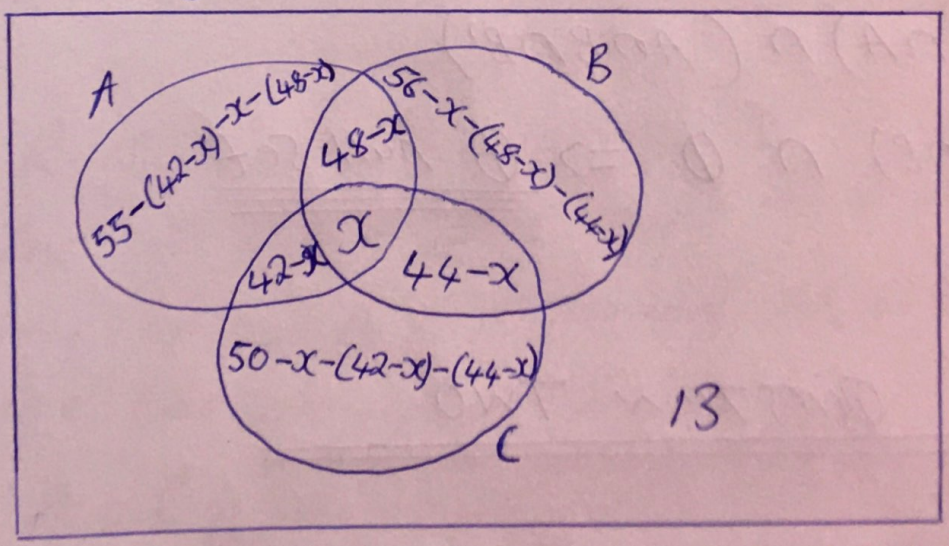
(a) let set A be for ECN students
let set B be for BBA students
let set C be for Dev students

$A = 55, B = 56, C = 50$

$A \cap B = 48, A \cap C = 42, B \cap C = 44, (A \cup B \cup C)^c = 13$

note that the number of students who take all the courses wasn't given

(i)



(ii) $55-(42-x)-x-(48-x)+56-x-(48-x)-(44-x)+$
$50-x-(42-x)-(44-x)+48-x+42-x+$
$44-x+x = 67$

$27+x = 67, \therefore x = \underline{40}$ (take all 3 courses)

(iii) only $A = 5$, only $B = 4$, only $C = 4$. $\therefore$ 13 take only one course

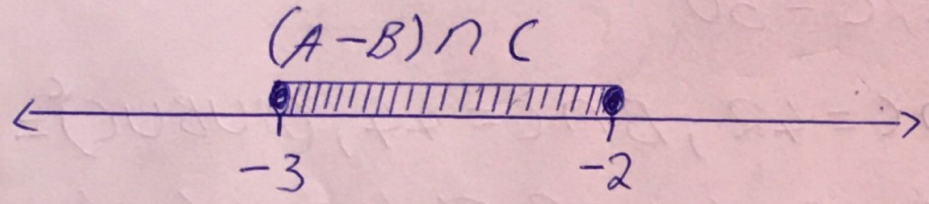
(b) $A = (-4, 2]$, $B = (-2, 4)$, $C = [-3, 3]$

$(A - B) \cap (X \cap C')'$

$\Rightarrow (A - B) \cap C$

but $A - B = (-4, -2]$

$\therefore (A - B) \cap C = [-3, -2]$



(c) $(A \cap B) \cap (A \cap B')$

$(A \cap B \cap A) \cap (A \cap B \cap B')$

$(A \cap B) \cap \emptyset \Rightarrow \underline{\emptyset \text{ null set}}$

QUESTION TWO

(a)

$$\frac{(1+2i)(3+i) + (1-2i)(3-i)}{(3+i)(3-i)} = \frac{3+i+6i-2+3-i-6i+2}{9+2}$$

$$= \frac{6}{11}$$

(b) ~~$z+2 = x+iy+2 =$~~

$z-2 = x+iy-2 = x-2+iy$

$\overline{z-2} = x-2-iy$

$2z = 2x+2iy$

$x-2-iy+2x+2iy = 1-i$

$(3x-2)+(2y-y)i = 1-i$

$\therefore \quad 3x-2=1 \qquad y = \underline{\underline{-1}}$

$3x=3$

$x = \underline{\underline{1}}$

(c) (i) R is a function because there is no single member of set X that maps onto different members of set y. A member of set X maps only one one member of set y

(ii) $R_f = \{4, 3, 3\}$

QUESTION THREE

(a) (i) $f(x) = \dfrac{-x}{x - x^3}$

$f(-x) = \dfrac{x}{-x + x^3}$, $f(x)$ is not an even function

$= -1\left(\dfrac{-x}{-x+x^3}\right)$

$= -1\left(\dfrac{x}{x-x^3}\right)$

$f(x)$ is not an odd function either

neither

(ii) $f(x)$ is said to be a one to one function if $[f(a)] = [f(b)]$

that is if $a = b$, or else it's not

$f(a) = \dfrac{-a}{a-a^3}$, $f(b) = \dfrac{-b}{b-b^3}$

$\dfrac{-a}{a-a^3} = \dfrac{-b}{b-b^3}$, $-a(b-b^3) = -b(a-a^3)$

$ab - a^3b = ab - ab^3$, $a^3b = ab^3$, $a^2 = b^2$

$\therefore f(x)$ is not a one to one function

(b) $f(x) = x^2 + (k-3)x + k - 4$

If Discriminant $(D) = 0$, then $f(x)$ has equal roots (one real root)

$\Rightarrow$ we know that $D = b^2 - 4ac$

$= (k-3)^2 - 4(k-4)$

$= k^2 - 6k + 9 - 4k + 16$

$= k^2 - 10k + 25$

$= (k-5)(k-5)$

$\therefore (k-5)(k-5) = 0$

$k = \underline{\underline{5}}$

(c) $p(x) = -2x^2 + 220x + 8000$

$(-2x^2 + 220x) + 8000$

$-2(x^2 - 110x) + 8000$

$-2(x^2 - 110x + 3025 - 3025) + 8000$

$-2(x^2 - 110x + 3025) + 8000 + 6050$

$-2[x - 55]^2 + 14050$

t.p $(55, 14050)$

(i) 55 items

(ii) $p(55) = \underline{\underline{14050}}$

QUESTION FOUR

(a) $f(x) = -2x^2+4x-5$

$\Rightarrow a<0$, $f(x)$ has a maximum turning point

$\Rightarrow$

$$-2(x^2-2x)-5$$

$$-2(x^2-2x+1-1)-5$$

$$-2(x^2-2x+1)-5+2$$

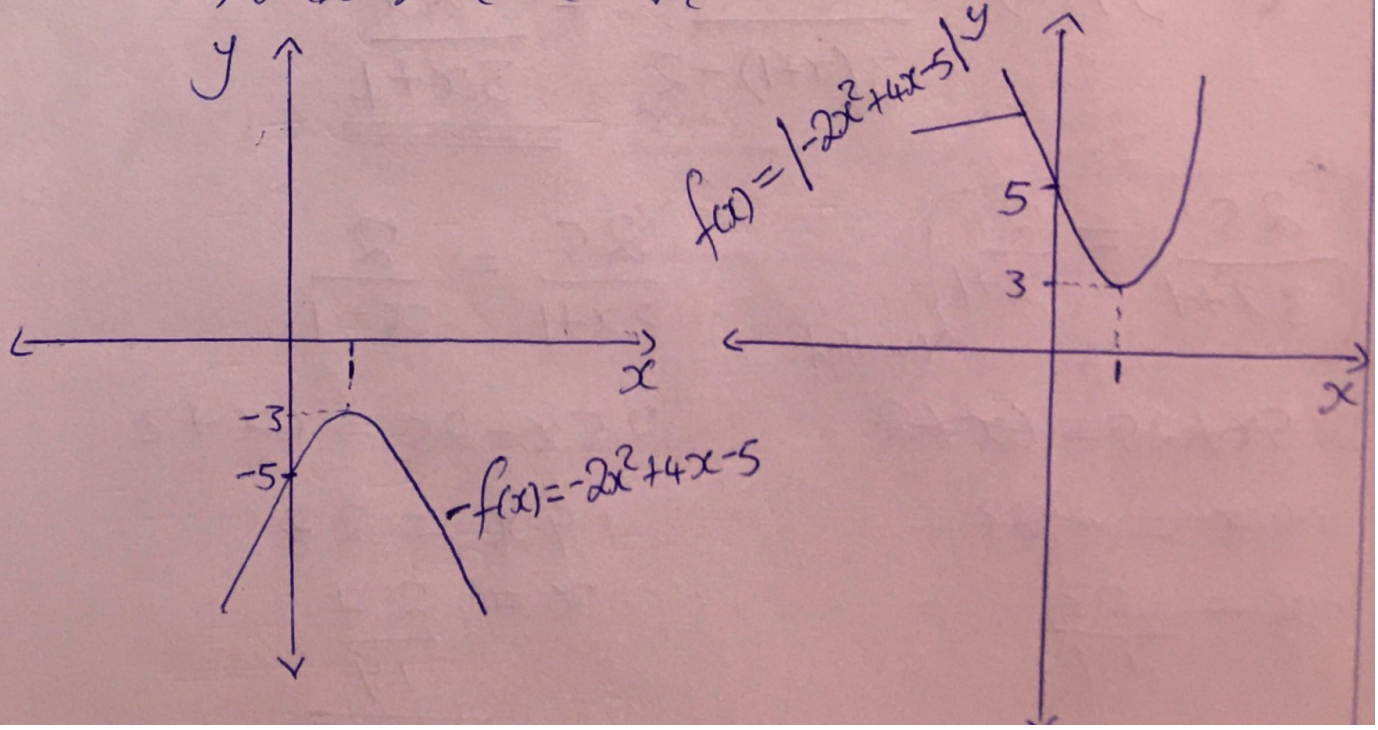
$$-2(x^2-2x+1)-3$$

$$-2(x-1)^2-3$$

$$t.p(1,-3)$$

$\Rightarrow$ Y-intercept $\Rightarrow (0,-5)$

$\Rightarrow D=(4)^2-4(-2x-5)=16-40=-24$, $\therefore$ The roots/ X-intercepts do not exist



$$x = \frac{25}{3y-2}$$

$$3xy - 2x = 25$$

$$3xy = 25 + 2x$$

$$y = \frac{25+2x}{3x}$$

$$\therefore f^{-1}(x) = \frac{25+2x}{3x}$$

(ii) $3x - 2 \neq 0$

$3x \neq 2$

$x \neq \frac{2}{3}$

$\therefore$ The Domain of $f(x)$ is all real numbers except $\frac{2}{3}$

$D_f = \{x : x \in \mathbb{R}, x \neq \frac{2}{3}\}$

$R_F \Rightarrow$ All reals except $f(\frac{2}{3})$

(iii) $(f \circ g)(x) = \frac{25}{3(x+1)-2} = \frac{25}{3x+1}$

$\frac{25}{3x+1} = \frac{2}{x+1}$ cancelled

~~$25x + 25 = 6x + 2$~~

~~$19x = -23$~~

~~$x = \frac{-23}{19}$~~

$\frac{25}{3x+1} = \frac{2}{x-1}$

$25x - 25 = 6x + 2$

$19x = 27$

$x = \frac{27}{19}$

(c)

(i) $f(-2) = 0$

$-8 + 4K + 8 + 12 = 0$

$4K = -12$

$K = \underline{-3}$

$P(x) = x^3 - 3x^2 - 4x + 12$

(ii)

$$\begin{array}{c|cccc} -2 & 1 & -3 & -4 & 12 \\ & \downarrow & -2 & 10 & -12 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$(x^2 - 5x + 6)$

$(x-3)(x-2)$

$\therefore p(x) = (x+2)(x-3)(x-2)$

If $p(x) = 0$ then $\underline{x+2 = 0, x = -2}$

$\underline{x-3 = 0, x = 3}$

$\underline{x-2 = 0, x = 2}$