

Department of Mathematics & Statistics
2020/2021 Academic Year, Final Examinations
MAT1110: Foundation Mathematics & Statistics For Social Sciences
Monday 13th December 2021 – 14 : 00hrs

Time Allowed: 3 hours

Instructions:

1. There are Seven (7) questions in this examination paper. Attempt any five (5) questions.
2. Indicate your computer number on all your answer booklets.
3. Full credit will only be given when all necessary working is shown.
4. Calculators are not allowed.

1. ✓(a) A fast food restaurant conducted a survey on its 185 customers. The results came out as follows; 70 liked pizza, 60 liked fries, and 50 liked shawarma. In addition, 30 liked pizza and fries, 20 liked pizza and shawarma, 35 liked fries and shawarma. 17 liked all the three meals.

- i. Illustrate this information on a Venn diagram..
- ii. Find the number of customers who did not like any of these available meals?
- iii. Find the number of customers who liked exactly one meal?

(b) Simplify each of the following leaving your answer in the form $a\sqrt{b}$, where a and b are natural numbers.

- i. $\sqrt{48} + \sqrt{108}$
- ii. $(\frac{\sqrt{3}}{21})^{-1} + \sqrt{27}$.

(c) i. Let $\mathbb{R}$ be the universal set, $A = [-2, 3)$ and $B = (-3, 3]$. Find the set

$$B \cap A^c,$$

and display your answer on the number line.

ii. Let X and Y be any non-empty sets. Express the following in its simplest form:

$$[(X \cap Y^c) \cup (X \cap Y)^c]^c.$$

[8, 7, 10]

2. (a) Express

$$\frac{-1+i}{3-2i}$$

in the form $a + bi$, where a and b are real numbers.

ii. By completing the square method, express $f(x) = 2x^2 - 6x + 4$ in the form $f(x) = a(x-h)^2 + k$, where (h, k) is the turning point.

(b) Let

$$f(x) = \frac{4x+1}{2x+1}$$

i. Find f^{-1} .

ii. Find the domain, and range of f .

iii. Sketch the graph of f .

✓ (c) Let P be the polynomial given by $P(x) = x^4 + 6x^3 + 2x^2 + 3x + 1$.

i. Find the remainder when P is divided by $2x + 1$.

ii. Express P in the form

$$P(x) = (x^2 + 1)Q(x) + R(x),$$

where Q is the quotient and R the remainder when P is divided by $x^2 + 1$.

[7, 10, 8]

3. (a) Solve each of the following equations:

i.

$$\left(\frac{1}{32}\right)^{-2x} = 4^{4x+2}$$

ii.

$$\log_3(3x+9) = 4$$

(b) Prove each of the following identities:

i.

$$\frac{\cos^2 x + 4 \cos x + 3}{\sin^2 x} = \frac{3 + \cos x}{1 - \cos x}$$

ii.

$$\tan^2 x + 1 + \tan x \sec x = \frac{1}{1 - \sin x}$$

$$1 + \cot^2 x = \csc^2 x$$

(c) Let

$$f(x) = \frac{1}{2} \sin(x + 30^\circ) \text{ for } 0^\circ \leq x \leq 360^\circ$$

i. Solve the equation $f(x) = 0$.

ii. Hence, or otherwise, sketch the graph of $f(x) = \frac{1}{2} \sin(x + 30^\circ)$.

[8]

Handwritten marks: a large 'X' and the number '20'.

4. (a) Evaluate each of the following:

i.

$$\lim_{x \rightarrow 3} \frac{27 - x^3}{x - 3}$$

ii.

$$\lim_{x \rightarrow 0} \frac{\sqrt{3} - \sqrt{x+3}}{x}$$

(b) Let f be a polynomial given by

$$f(x) = x^3 + 2x^2 - 3x$$

i. Sketch the graph of f .

ii. Hence, or otherwise, find the area of the region bounded by the graph of f and the x -axis.

(c) The amount of money in a certain bank account is increasing exponentially. If K100,000 is present initially and K400,000 after 1 hour, how much money will be present after 210 minutes?

{9, 10, 6

5. (a) i. Use the first principle to differentiate

$$f(x) = \frac{1}{1-x}$$

ii. Let $y = (x^2 + 1)^{\frac{1}{2}}$. Show that

$$\frac{d^2y}{dx^2} = \frac{a - x^2}{b(x^2 + 1)^c},$$

where a , b and c are real numbers.

(b) Let $f(x) = x^4 - x^2$.

i. Determine the intervals where f is increasing or decreasing.

ii. Hence, or otherwise, sketch the graph of f .

(c) Evaluate

i.

$$\int_1^4 \left(\frac{2}{x^3} - 3\sqrt{x} \right) dx$$

ii.

$$\int_0^{\frac{\pi}{2}} \sin x e^{(1+\cos x)} dx$$

6. (a) Let

$$f(x) = \frac{1}{x^3 + 5x}$$

i. Find the partial fraction decomposition of f .

ii. Hence integrate

$$\int \frac{1}{x^3 + 5x} dx$$

- (b) i. In a group of 40 college students, 16 are on self sponsorship. Find the probability that a student chosen at random is not on self sponsorship?
- ii. Two dice are thrown. Find the probability of scoring either the same number on both dice or scoring a sum greater than 8.

(c) The table below shows the scores observed for 50 tosses of a die.

score(x)	1	2	3	4	5	6
frequency (f)	7	15	10	3	9	6

- i. Find the mean, $\bar{x}$.
- ii. Find the median.
- iii. Find the mode.

[8, 10, 7]

7e[✓] (a) Events A and B are independent such that $P(A) = y$, $P(B) = y + \frac{1}{5}$, and $P(A \cap B) = \frac{3}{20}$. Find

- i. the value of y .
- ii. $P(A \cup B)$.

(b) Integrate

i.

$$\int \frac{x}{6} e^{3x^2} dx.$$

ii.

$$\int x \cos x dx.$$

(c) The grouped frequency distribution below shows the weights, to the nearest gram, of 84 letters.

Mass(g)	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100
Number of letters	10	18	24	14	18

- i. Complete the frequency distribution table below.

Mass(g)	Interval width	Frequency	Frequency density
$0.5 \leq x < 20.5$	20	10	0.5

- ii. Find the interval representing the median class.
- iii. Draw a histogram.

[7]